MINIMUM BALLISTIC FACTOR MISSILE SHAPES FOR VARIABLE SKIN-FRICTION COEFFICIENT

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Minimum ballistic factor for slender axisymmetric power law bodies have been obtained by taking a variable skin-friction coefficient for the cases when any two of the three quantities length, diameter and surface area have been pre-prescribed.

In all the analyses devoted to the problem of determination of slender axisymmetric power law bodies having minimum ballistic factor, the skin-friction coefficient was assumed to be constant[1-5]. Assuming that the distribution of skin-friction coefficient versus the abscissa is represented by power law, the author has studied here the effect of the variable skin-friction on shapes of missiles of minimum ballistic factor using both Newtonian drag law and Newton-Busemann drag law for the cases when length-thickness, surface area-thickness and surface area-length are given and has compared these results with those obtained earlier[4-5].

PROBLEM FORMULATION

Assuming the distribution of skin-friction coefficient as

$$C_f = A/x^a$$

where A and a being constant*, the expression for drag of an axisymmetric body in hypersonic flow at zero angle of attack[6] is given by

$$D/4\pi q = \int_0^l y \left[y'^3 + k \frac{yy'y''}{2} + C_{fa} \frac{(1-a)}{2} (x/l)^{-a} \right] dx \tag{1}$$

Here C_{fa} denotes the average value of the skin-friction coefficient over the entire length of the body and $k = 0$ for Newtonian law and $k = 1$ for Newton-Busemann law.

The volume and surface area of a body are given by

$$v = \pi \int_0^l y^2 \, d\, x \tag{2}$$

$$s = 2\pi \int_0^l y \, d\, x \tag{3}$$

and the power law body to be investigated is given by

$$y = (d/2)\,(x/l)^n \tag{4}$$

where n is a constant.

*Typical values of the constant a are 0 for the idealized model in which the skin-friction coefficient is constant, 1/5 for the turbulent flow model, and 1/2 for the laminar flow model.

The ballistic factor of a missile is proportional to the ratio D/qv which will be represented by C. In the three cases to be considered, the expressions for C are

$$C = (2n+1)\left[\frac{2n^3 + k(n^3 - n^2)}{4(2n-1)} + (4C_{fa})^{\frac{1}{3}}\,(l/d)\,\frac{(1-a)}{(n+1-a)}\right] d^2/l^3 \qquad (l, d) \text{ given (a)}$$

$$C = \frac{\pi^3 d^5}{4 s^3}\,\frac{(2n+1)\,[2n^3 + k(n^3 - n^2)]}{(n+1)^3\,(2n-1)} + \frac{4C_{fa}(1-a)(2n+1)}{d\,(n+1-a)} \qquad (s, d) \text{ given (b)}$$

and

$$C = (s^3/4\pi^2 l^5)\,\frac{(2n+1)(n+1)^2\,[2n^3 + k(n^3 - n^2)]}{(2n-1)} + 4C_{fa}\,\frac{\pi l (2n+1)(1-a)}{s(n+1)(n+1-a)} \qquad (s, l) \text{ given (c)} \qquad (5)$$

In order that C be minimum

$$dC/dn = 0 \qquad (6)$$

SOLUTION OF THE PROBLEM

For Prescribed Diameter and Length

From 5(a) and (6), we have

$$\frac{24n^4 - 8n^3 - 6n^2 + k(12n^4 - 12n^3 + n^2 + 2n)}{(4n-2)^2} + \frac{\alpha^3\,(1-a)}{(n+1-a)} = 0 \qquad (7)$$

where

$$\alpha = (4C_{fa})^{\frac{1}{3}}\,(l/d)$$

Knowing the values of n for given values of α and a, missile shapes of minimum ballistic factor can be obtained and from 5(a) the corresponding values of I $\left(\text{defined as } \frac{l^3}{d^2}\,C\right)$ can be calculated as shown in Table 1. Fig. 1 and Fig. 2 give the relation $I(\alpha)$ for $k = 0$ and $k = 1$ respectively, when $a = 0$, ·2, ·5 and $\alpha = 0$, 1, 2.

TABLE 1

VALUE OF n AND I FOR GIVEN VALUES OF α AND a FOR THE CASES $k=0$, $k=1$

$k=0$ (Newton Law)

	$\alpha=0$	$\alpha=1$			$\alpha=2$		
		$a=0$	$a=0{\cdot}2$	$a=0{\cdot}5$	$a=0$	$a=0{\cdot}2$	$a=0{\cdot}5$
n	0·6937	0·6780	0·6776	0·6775	0·6179	0·6162	0·6160
I	1·0286	2·4354	2·3066	2·0314	12·1738	11·2122	9·1245

$k=1$ (Newton—Busemann Law)

	$\alpha=0$	$\alpha=1$			$\alpha=2$		
		$a=0$	$a=0{\cdot}2$	$a=0{\cdot}5$	$a=0$	$a=0{\cdot}2$	$a=0{\cdot}5$
n	0·6372	0·6248	0·6244	0·6243	0·5822	0·5805	0·5799
I	0·7670	2·1537	2·0323	1·7693	11·7767	10·8569	8·8404

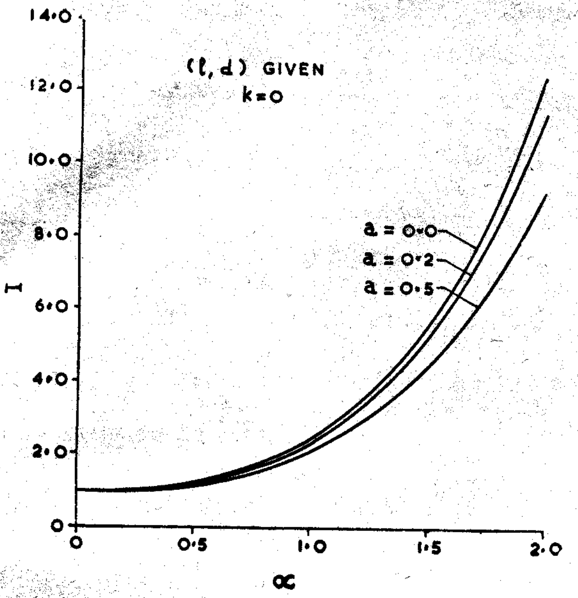

Fig. 1— α Versus I.

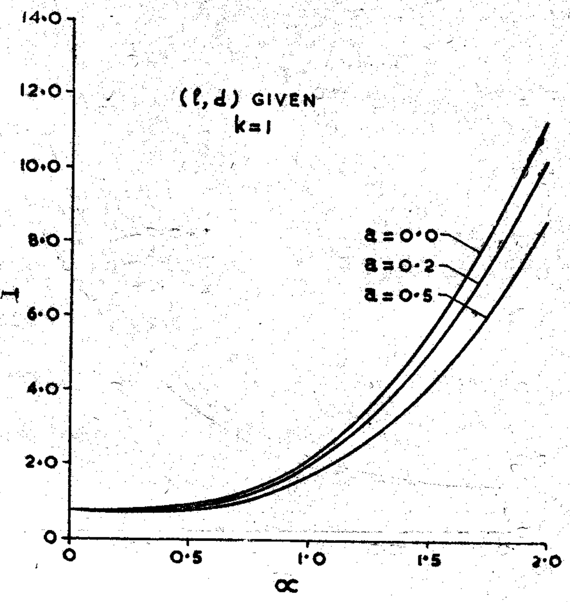

Fig. 2— α Versus I.

For given Surface Area and Diameter

From 5(b) and (6), we obtain

$$\frac{8\,n^4 - 4\,n^3 - 3\,n^2 + k\,(6\,n^4 - 4\,n^3 + n)}{(2\,n - 1)^2} + \frac{2\,\alpha^3\,(n+1)\,(1-a)\,(1-2a)}{(n+1-a)^2} = 0 \qquad (8)$$

where

$$\alpha = (4\,C_{fa})^{\frac{1}{3}} \quad \frac{s\,(n+1)}{\pi\,d^2}$$

The values of n can be calculated from (8) for different values of α and a and therefore corresponding values of I can be obtained as shown in Table 2. The corresponding shape is then known from (4).

TABLE 2

VALUE OF n AND I FOR GIVEN VALUE OF α AND a FOR THE CASES $k=0$, $k=1$

$k=0$ (Newton Law)

	$\alpha=0$	$\alpha=1$			$\alpha=2$		
		$a=0$	$a=0\cdot2$	$a=0\cdot5$	$a=0$	$a=0\cdot2$	$a=0\cdot5$
n	0·9114	0·8402	0·8663	0·9114	0·6558	0·6992	0·9114
I	1·2987	2·6250	2·5244	2·2987	12·2147	11·2676	9·2987

$k=1$ (Newton—Busemann Law)

	$\alpha=0$	$\alpha=1$			$\alpha=2$		
		$a=0$	$a=0\cdot2$	$a=0\cdot5$	$a=0$	$a=0\cdot2$	$a=0\cdot5$
n	0·7309	0·6939	0·7067	0·7309	0·5993	0·6222	0·7309
I	0·8492	2·2114	2·0978	1·8492	11·7909	10·8702	8·8492

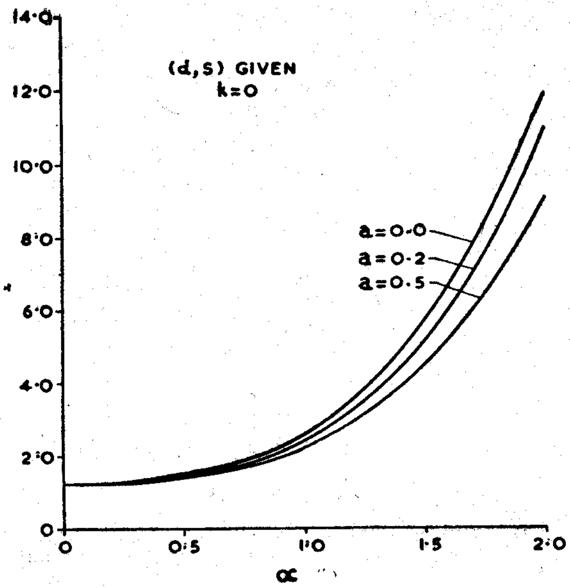

Fig. 3—a Versus I.

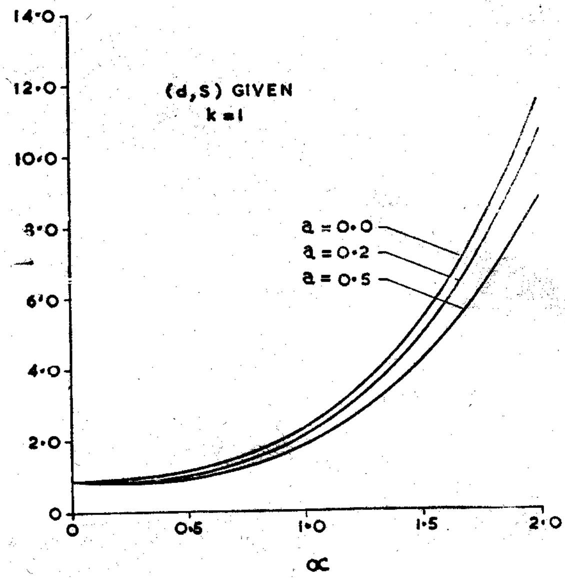

Fig. 4—a Versus I.

Fig. 3 & 4 compare the values of $I(\alpha)$ in the two cases $k=0$ and $k=1$ respectively when $a=0$, 0.2 and 0·5.

For given Surface Area and Length

Using 5(c) and (6) we obtained

$$\frac{40\,n^5+16\,n^4-18\,n^3-6\,n^2+k\,(20\,n^5-8\,n^4-13\,n^3+5\,n^2+2n)}{(2\,n-1)^2}-\frac{4\alpha^3\,(2n^2+2n+a)\,(1-a)}{(n+1-a)^2}=0 \quad (9)$$

where

$$\alpha=(4\,C_{fa})^{\frac{1}{3}}\;\frac{\pi\,l^2}{s\,(n+1)}$$

With the help of (9) and (6) relationship between $n(\alpha)$ and $I(\alpha)$ has been obtained as shown in Table 3. Optimum shape profile is then known from (4). Fig. 5 and Fig. 6 show corresponding relationship $I(\alpha)$ for $a=0\cdot2$ and $0\cdot5$ for the cases $k=0$, $k=1$.

TABLE 3

VALUES OF n AND I FOR $\alpha=0, 1, 2$ AND $a=0, 0\cdot2, 0\cdot5$ WHEN s, l ARE KNOWN

	$k=0$ (Newton Law)						
	$a=0$	$a=1$			$a=2$		
		$a=0$	$a=0\cdot2$	$a=0\cdot5$	$a=0$	$a=0\cdot2$	$a=0\cdot5$
n	0·6492	0·6633	0·6447	0·6552	1·000	1·0135	0·9968
I	1·0538	2·4382	2·3276	2·0468	13·5000	12·2169	9·4920

(TABLE 3—*Contd.*)

$k=1$ (Newton—Busemann Law)

	$a=0$	$a=1$			$a=2$		
		$a=0$	$a=0\cdot2$	$a=0\cdot5$	$a=0$	$a=0\cdot2$	$a=0\cdot5$
n	0·6109	0·6221	0·6240	0·6262	0·9086	0·9237	0·9280
I	0·7783	2·1538	2·0324	1·7686	13·0365	11·8828	9·2815

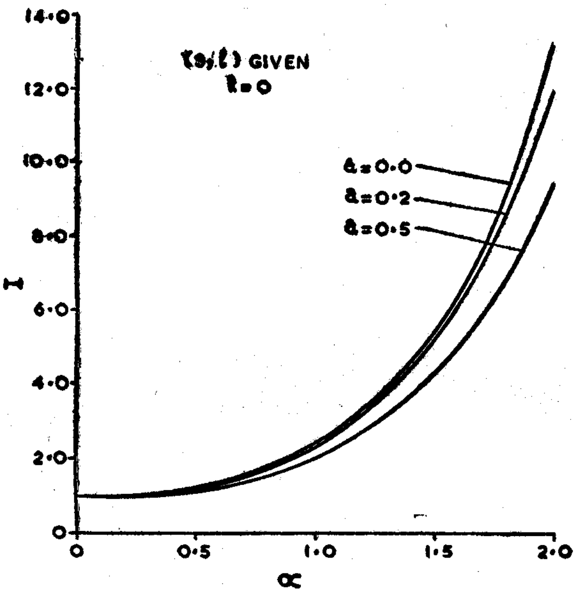

Fig. 5—α Versus I.

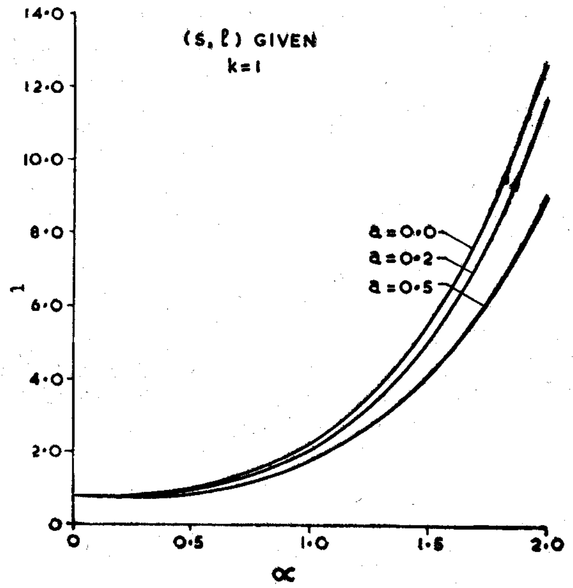

Fig. 6—α Versus I.

CONCLUSION

It can be observed that for given (l, d), given the values of the power law exponent decrease with the increasing values of a. For given (s, d), the values of n decrease with increasing values of a, but is independent of α when $a = 0\cdot5$. Also in the case when (s, l) is given, the values of n first decrease and then increase with increasing values of a. Finally in all the three cases the ballistic factor I decreases with increasing values of the constant a corresponding to the same value of α.

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