LAMINAR SOURCE FLOW BETWEEN TWO PARALLEL ROTATING POROUS DISCS WITH DIFFERENT PERMEABILITY

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The steady laminar source flow between two infinite parallel porous discs rotating with equal speed in the same sense about a common axis has been investigated. The solution has been obtained by a double series expansion method. The effect of unequal porosity has been shown on radial and azimuthal velocity components and on radial shear stress.

Laminar source flow between two parallel discs rotating at the same speed has been investigated by Breitner & Pohlhausen[1], and by Kreith & Peube[2,3]. The latter obtained the solution by a series expansion method in powers of the radius vector. The solution is valid at a distance far away from the centre so that the entry effects are negligible. Pelech & Shapiro[4] obtained an approximate first order solution for one disc rotating and the other stationary as a by-product of an investigation of the deflection of a rotating magnetic recording device. Kreith & Viviand[5] investigated the laminar flow between two parallel coaxial discs rotating at different velocities, with a source at the centre. They solved the Navier-Stokes equations by double series expansion about a known solution at large radial distance, and obtained the velocity and pressure distributions. Rajvanshi[6] studied the laminar radial flow of second order fluid between two infinite parallel discs, one rotating and the other at rest, by adopting the method of Kreith & Peube.

Flow between porous boundaries is of practical as well as theoretical interest. Berman[7] studied the effect of porosity on two-dimensional flow in a channel. Elkouh[8] discussed the laminar source flow between parallel stationary porous discs, with equal suction or injection at the boundaries. The solution is in the form of a perturbation to the creeping flow solution. He obtained expressions for the velocity, pressure and shear stress and compared them with the corresponding results on the assumption of creeping flow. Khan[9] gave a solution for laminar source flow between two parallel coaxial porous discs rotating with same speed in the same sense. He has superposed injection at one disc and an equal suction on the other.

In the present study, steady laminar source flow of an incompressible viscous fluid between two rotating infinite porous discs with different permeability has been considered. The two discs have been taken rotating in the same sense with equal speed about a common axis. The double series expansion method similar to that of Kreith & Viviand[5] has been adopted. The effect of unequal suction and injection has been investigated on the flow variables. The effect of porosity on radial and azimuthal velocity components as also on radial shear stress has been exhibited graphically. It is found that in the central region, the radial velocity component is greater but the azimuthal velocity component is smaller in case of equal injection at both discs than those in case of equal suction.

EQUATIONS OF MOTION

In this analysis the common axis of rotation of the discs has been chosen as the z-axis, the origin being midway between them. The equations of motion in cylindrical polar coordinates $(\bar{r}, \bar{\theta}, \bar{z})$ for steady axi-symmetric flow are

$$\left.\begin{aligned} \bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\bar{v}^2}{\bar{r}} &= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{r}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2}\right), \\ \bar{u}\frac{\partial \bar{v}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\bar{u}\,\bar{v}}{\bar{r}} &= \nu\left(\frac{\partial^2 \bar{v}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{v}}{\partial \bar{r}} - \frac{\bar{v}}{\bar{r}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right), \\ \bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} &= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu\left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right), \end{aligned}\right\} \quad (1)$$

and the equation of continuity is

$$\frac{\partial}{\partial \bar{r}} (\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{z}} (\bar{r}\,\bar{w}) = 0. \tag{2}$$

The distance between the discs has been taken as $2a$. Both the discs have been assumed rotating with the same angular velocity ω in the same sense. The discs have been taken to be uniformly porous; the constant velocity of suction at the upper disc being $\bar{w}_1$ and that of injection at the lower $\bar{w}_2$. The boundary conditions are

$$\bar{u}(\bar{r}, \pm a) = 0, \qquad \bar{v}(\bar{r}, \pm a) = \bar{r}\,\omega, \qquad \bar{w}(\bar{r}, +a) = \bar{w}_1, \qquad \bar{w}(\bar{r}, -a) = \bar{w}_2,$$

and

$$\int_{-a}^{a} 2\pi \bar{r}\bar{u}\, d\bar{z} + \pi \bar{r}^2 \bar{w}_1 - \pi \bar{r}^2 \bar{w}_2 = Q, \tag{3}$$

where Q is the source strength.

The following non-dimensional variables have been introduced :

$$r = \frac{\bar{r}}{a}, \quad z = \frac{\bar{z}}{a}, \quad u = \frac{\bar{u}}{\omega a}, \quad v = \frac{\bar{v}}{\omega a}, \quad w = \frac{\bar{w}}{\omega a}, \quad p = \frac{\bar{p}}{\nu \rho \omega}. \tag{4}$$

Equations (1) to (3) then take the form

$$\begin{aligned} R\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) &= -\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial z} - \frac{u}{r^2}, \\ R\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) &= \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}, \\ R\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) &= -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}, \end{aligned} \tag{5}$$

and

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0, \tag{6}$$

with

$$u(r, \pm 1) = 0, \qquad v(r, \pm 1) = r, \qquad w(r, +1) = w_1, \qquad w(r, -1) = w_2,$$

and

$$\int_{-1}^{1} u\, dz + \frac{1}{2} r(w_1 - w_2) = 2\frac{Re}{r}, \tag{7}$$

where $R = \dfrac{\omega a^2}{\nu}$ is the Reynolds number corresponding to rotation and $Re = Q/(4\pi a^3 \omega)$ is the Reynolds number corresponding to the source flow.

The equation of continuity suggests the use of the stream function Ψ, defined by

$$u = \frac{1}{r}\frac{\partial \Psi}{\partial z}, \qquad w = -\frac{1}{r}\frac{\partial \Psi}{\partial r}, \tag{8}$$

Following Kreith & Peube[2], series expansions in powers of r have been assumed for Ψ, v and p. Considering the radial symmetry, only even powers of r have been taken in series for Ψ and p, and only odd powers in the series for v as follows :

$$\left.\begin{aligned}\Psi &= \frac{1}{2}\, r^2 f_{-1}(z) + Re\, f_0(z) + \frac{Re^2}{r^2} f_1(z) + \frac{Re^3}{r^4} f_2(z) + \ldots\ldots\ldots, \\ v &= r\, g_{-1}(z) + \frac{Re}{r}\, g_0(z) + \frac{Re^2}{r^3}\, g_1(z) + \frac{Re^3}{r^5}\, g_2(z) + \ldots\ldots\ldots, \\ &\text{and} \\ p &= \frac{1}{4}\, r^2 h_{-1}(z) + h(z) + Re\, h_0(z) \ln r + \frac{Re^2}{r^2}\, h_1(z) + \frac{Re^3}{r^4}\, h_2(z) + \ldots\ldots\ldots,\end{aligned}\right\} \quad (9)$$

where f, g and h are functions of z only.

The solution is expected to be valid for small values of $Re^* = (R/r^2)$. The series assumed for Ψ together with (8) gives

$$\left.\begin{aligned} u &= \frac{1}{2}\, r f'_{-1} + \frac{Re}{r} f'_0 + \frac{Re^2}{r^3} f'_1 + \frac{Re^3}{r^5} f'_2 + \ldots\ldots\ldots, \\ &\text{and} \\ w &= -f_{-1} + \frac{2\, Re^2}{r^4} f_1 + \frac{4\, Re^3}{r^6} f_2 + \ldots\ldots\ldots, \end{aligned}\right\} \quad (10)$$

where prime denotes differentiation with respect to z. From (7), (9) and (10), the boundary conditions in terms of the functions f, g and h are

$$\left.\begin{aligned} &f_{-1} = -w_1,\; f_1 = f_2 = \ldots\ldots = 0 && \text{at } z = 1, \\ &f_{-1} = -w_2,\; f_1 = f_2 = \ldots\ldots = 0 && \text{at } z = -1, \\ &f'_{-1} = f'_0 = f'_1 = f'_2 = \ldots\ldots = 0 && \text{at } z = \pm 1, \\ &g_{-1} = 1,\; g_0 = g_1 = g_2 = \ldots\ldots = 0 && \text{at } z = \pm 1, \\ &\text{and} \\ &f_0(1) - f_0(-1) = 2. \end{aligned}\right\} \quad (11)$$

It is later found convenient to assume $f_0(1) = 1$, so that

$$f_0(-1) = 1. \quad (12)$$

Substituting for u, v, w and p from (9) and (10) in the equations of motion (5), and equating coefficients of equal powers of r, an infinite set of system of coupled ordinary differential equations is obtained. The first four systems considered in this study are as follows :

System I

$$\left.\begin{aligned} f'''_{-1} &= h_{-1} + R\left[\frac{1}{2}\,(f'_{-1})^2 - f_{-1} f''_{-1} - 2\, g^2_{-1}\right], \\ g''_{-1} &= R\,(f'_{-1} g_{-1} - f_{-1} g'_{-1}), \\ h'_{-1} &= 0, \\ h' &= -f''_{-1} - R\, f_{-1} f'_{-1}. \end{aligned}\right\} \quad (13)$$

System II

$$\left.\begin{aligned} f_0''' &= h_0 - R\,(f_{-1} f_0'' + 2\, g_{-1} g_0), \\ g''_0 &= R\,(2\, f'_0 g_{-1} - f_{-1} g'_0), \\ h'_0 &= 0. \end{aligned}\right\} \quad (14)$$

System III

$$\left.\begin{aligned} f_1''' &= -2\, h_1 + R(-f'_{-1} f'_1 - f'^2_0 - f_{-1} f''_1 + f''_{-1} f_1 - g_0^2 - 2\, g_{-1} g_1), \\ g''_1 &= R\,(-f'_{-1} g_1 + 2 f'_1 g_{-1} + 2 f_1 g'_{-1} - f_{-1} g'_1), \\ h'_1 &= 0. \end{aligned}\right\} \quad (15)$$

System IV

$$
\left.
\begin{aligned}
f_2''' &= -4h_2 - \frac{8}{Re} f_1' + R\,(-4f_0' f_1' - 2f'_{-1} f_2' - f_{-1} f_2'' + 2f_0'' f_1 + \\
&\quad + 2f''_{-1} f_2 - 2g_{-1} g_2 - 2g_0 g_1), \\
g_2'' &= -\frac{8}{Re} g_1 + R\,(-2f'_{-1} g_2 - f_{-1} g_2' - 2f_0' g_1 + 2f_1 g_0' + \\
&\quad + 2f_2' g_{-1} + 4f_2 g'_{-1}), \\
h_2' &= \frac{2}{Re} f_1'' + \frac{R}{Re}(6f'_{-1} f_1 + 2f_{-1} f_1').
\end{aligned}
\right\} \qquad (16)
$$

SOLUTION OF EQUATIONS

The system of differential equations obtained above have been solved by assuming series expansions for f, g and h in powers of R. Keeping in view the negative powers of r already occuring in the coefficients and the fact that R itself has to be small to avoid boundary layer effects, terms containing powers of R upto two have been retained in the solution of first and second systems, and upto one in the solution of the third system.

System I

We note from third equation of (13) that h_{-1} is a constant. Let us assume

$$
\left.
\begin{aligned}
f_{-1} &= f_{-1,0} + R f_{-1,1} + R^2 f_{-1,2} + \ldots\ldots, \\
g_{-1} &= g_{-1,0} + R g_{-1,1} + R^2 g_{-1,2} + \ldots\ldots, \\
h_{-1} &= h_{-1,0} + R h_{-1,1} + R^2 h_{-1,2} + \ldots\ldots,
\end{aligned}
\right\} \qquad (17)
$$

From (11) and (17) the modified boundary conditions are

$$
\left.
\begin{aligned}
& f_{-1,0}(1) = -w_1,\; f_{-1,0}(-1) = -w_2, \\
& f_{-1,1} = f_{-1,2} = \ldots\ldots = 0 \qquad \text{at } z = \pm 1, \\
& f'_{-1,0} = f'_{-1,1} = f'_{-1,2} = \ldots\ldots = 0 \text{ at } z = \pm 1, \\
& g_{-1,0} = 1,\; g_{-1,1} = g_{-1,2} = \ldots\ldots = 0 \text{ at } z = \pm 1.
\end{aligned}
\right\} \qquad (18)
$$

First three equations of (13), (17) and (18) give

$$
f_{-1,0} = -0.5\,(w_1 + w_2) - (w_1 - w_2)\,(0.75\,z - 0.25\,z^3), \qquad (19)
$$

$$
f_{-1,1} = \frac{1}{100}(w_1 - w_2)\Big[(w_1 - w_2)\,(0.8482\,z - 1.7411\,z^3 + 0.9375\,z^5 - 0.0446\,z^7) +
+ (w_1 + w_2)\,(3.125 - 6.250\,z^2 + 3.125\,z^4)\Big], \qquad (20)
$$

$$
\begin{aligned}
f_{-1,2} = \frac{1}{100}(w_1 - w_2)\Big[&1.0714\,z - 2.8571\,z^3 + 2.5000\,z^5 - 0.7143\,z^7 + (w_1^2 - w_2^2)\,. \\
&.\,(0.0577 + 0.1116\,z^2 - 0.4130\,z^4 + 0.2604\,z^6 - 0.0167\,z^8) + (w_1 + w_2)^2\,. \\
&.\,(0.3125\,z - 0.6250\,z^3 + 0.3125\,z^5) + (w_1 - w_2)^2\,(0.0159\,z + 0.0107\,z^3 - \\
&- 0.0759\,z^5 + 0.0564\,z^7 - 0.0074\,z^9 + 0.0003\,z^{11})\Big], \qquad (21)
\end{aligned}
$$

$$g_{-1,0} = 1, \tag{22}$$

$$g_{-1,1} = (w_1 - w_2)\left[0.3125 - 0.3750\, z^2 + 0.0625\, z^4\right], \tag{23}$$

$$g_{-1,2} = \frac{1}{100}(w_1 - w_2)\Big[(w_1 - w_2)(11.8415 - 11.2946\, z^2 - 0.8259\, z^4 + 0.3125\, z^6 - 0.0335\, z^8) + (w_1 + w_2)(9.5833\, z - 8.3333\, z^3 - 1.2500\, z^5)\Big], \tag{24}$$

$$h_{-1,0} = 1.5\,(w_1 - w_2), \tag{25}$$

$$h_{-1,1} = 2 - 0.385714\,(w_1 - w_2)^2, \tag{26}$$

$$h_{-1,2} = 1.078571\,(w_1 - w_2) + 0.587500\,(w_1 - w_2)(w_1 + w_2)^2 + 0{\cdot}007003\,(w_1 - w_2)^3. \tag{27}$$

Last equation of (13) and (19) to (21) give h upto second degree terms in R in the form

$$h = 0.75\,(w_1 - w_2)(1 - z^2) - \frac{R}{100}\Big[12.5\,(w_1 + w_2)^2 + 25\,(w_1^2 - w_2^2)\, z + (w_1 - w_2)^2 \cdot (0.8482 + 22.9018\, z^2 - 14.0625\, z^4 + 2.8125\, z^6)\Big] + \frac{R^2}{100}(w_1 - w_2)\Big[-1.0714 + 8.5714\, z^2 - 12.5000\, z^4 + 5\, z^6 + (w_1^2 - w_2^2)(2.5446\, z - 4.6875\, z^3 + 2.8125\, z^5 - 0.6697\, z^7) + 1.25\,(w_1 + w_2)^2 (1 - z^2) + (w_1 - w_2)^2 (-0.0159 + 0.6041\, z^2 - 1.1384\, z^4 + 0.7433\, z^6 - 0.2009\, z^8 + 0.0078\, z^{10})\Big] + \text{a constant.} \tag{28}$$

System II

Using the above technique, we obtain f_0, g_0 and h_0 upto second degree terms in R as

$$f_0 = 1.5\, z - 0.5\, z^3 - \frac{R}{100}\Big[6.25\,(w_1 + w_2)(1 - 2\, z^2 + z^4) + (w_1 - w_2)(3.0357\, z - 6{\cdot}4286\, z^3 + 3.7500\, z^5 - 0.3571\, z^7)\Big] + \frac{R^2}{100}\Big[-4.5238\, z + 9.2857\, z^3 - 5\, z^5 + 0.2381\, z^7 + (w_1^2 - w_2^2)(0.4074 - 1.1830\, z^2 + 1.1942\, z^4 - 0.4688\, z^6 + 0.0502\, z^8) - 0.625\,(w_1 + w_2)^2 (z - 2\, z^3 + z^5) + (w_1 - w_2)^2 (1.2594\, z - 2.8208\, z^3 + 1.9710\, z^5 - 0.5212\, z^7 + 0.1153\, z^9 - 0.0038\, z^{11})\Big]. \tag{29}$$

$$g_0 = -0.25\, R\,(5 - 6\, z^2 + z^4) + \frac{R^2}{100}\Big[(w_1 - w_2)(-46.9420 + 43.8393\, z^3 + 4.7768\, z^4 - 1.8750\, z^6 + 0.2009\, z^8) - (w_1 + w_2)(28{\cdot}3333\, z - 33.3333\, z^3 + 5\, z^5)\Big], \tag{30}$$

and

$$h_0 = -3 + 0.385714\, R\,(w_1 - w_2) - R^2\Big[1.942857 + 0.5\,(w_1 + w_2)^2 + 0.169245 \cdot (w_1 - w_2)^2\Big]. \tag{31}$$

System III

In a similar manner f_1, g_1 and h_1 upto first degree term in R are

$$f_1 = \frac{R}{100}(5.3571\, z - 11.7857\, z^3 + 7.5000\, z^5 - 1.0714\, z^7), \tag{32}$$

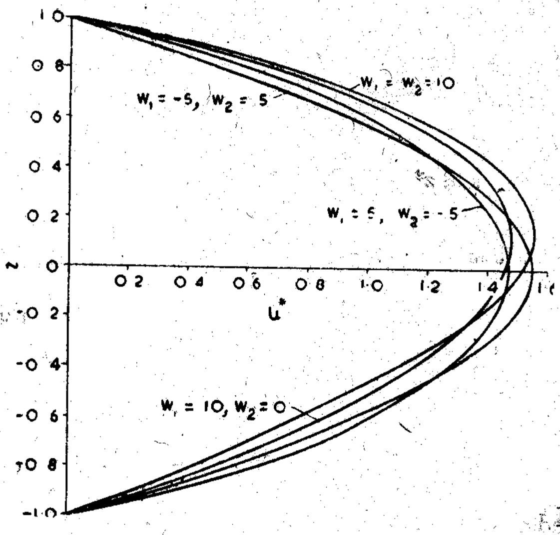

Fig. 1—u^* against z, (R=0·1, Re^*=2).

$$g_1 = 0, \tag{33}$$

$$h_1 = -0{\cdot}771429\, R. \tag{34}$$

System IV

In this system only terms containing R^0 have been retained. For this we assume

$$f_2 = f_{2,\,0} + \frac{r\, F_{2,\,0}}{Re} + \text{powers of } R, \tag{35}$$

and so on.

The boundary conditions are

$$f_{2,\,0} = F_{2,\,0} = 0,\ f'_{2,\,0} = F'_{2,\,0} = 0,$$
$$g_{2,\,0} = G_{2,\,0} = 0, \text{ at } z = \pm 1. \tag{36}$$

Hence (16), (35) and (36) give

$$f_2 = g_2 = h_2 = 0. \tag{37}$$

VELOCITY DISTRIBUTION

The non-dimensional radial velocity is given by (10), in which the functions f_{-1}, f_0, f_1 and f_2 are given by (19), (20), (21), (29), (32) and (37). If $<u>$ be the average radial velocity, we have

$$<u> = \frac{1}{2}\int_{-1}^{1} u\, dz = \frac{Re}{r} - \frac{1}{4}\, r\,(w_1 - w_2).$$

Let u^* be defined as $\dfrac{u}{<u>}$. Then

$$u^* = \frac{0.5 f'_{-1} + Re^*\, f_0' + Re^{*2} f'_1 + Re^{*3} f'_2 + \ldots\ldots}{Re^* - 0.25\,(w_1 - w_2)} \tag{38}$$

where $Re^* = (Re/r^2)$ is the "reduced Reynolds number" for the source flow.

Curves showing variation of u^* against z have been plotted in Fig. 1 for R=0·1 and Re^*=2 in four cases:

(*i*) $w_1 = 5$, $w_2 = -5$, *i.e.*, equal suction at both the discs,

(*ii*) $w_1 = -5$, $w_2 = 5$, *i.e.*, equal injection at both the discs,

(*iii*) $w_1 = w_2 = 10$, *i.e.*, suction at one disc and equal injection at the other, and

(*iv*) $w_1 = 10$, $w_2 = 0$, *i.e.*, suction at one disc only, the other being non-porous.

The effect of injection is seen to decrease the magnitude of velocity gradient near the disc and to increase the maximum velocity in the central region. Suction has opposite effect. This observation agrees with that of Elkouh[8] for the particular case of stationary discs with equal suction or injection. The layer of maximum radial velocity is seen pulled towards the disc with suction and pushed away from the disc with injection. It may be noted that effects of source and injection are complementary, in so far as both result in increased outflow in radial direction. On the other hand, a strong suction may necessitate a back flow towards the axis for its sustenance.

The non-dimensional azimuthal velocity component v is given by (9), in which values of the functions g_{-1}, g_0, g_1 and g_2 are now known. Using the values of $v = r$ on the discs, we define

$$v^* = \frac{v}{v_{disc}} = g_{-1} + Re^*\, g_0 + Re^{*2}\, g_1 + Re^{*3}\, g_2 + \ldots\ldots \tag{39}$$

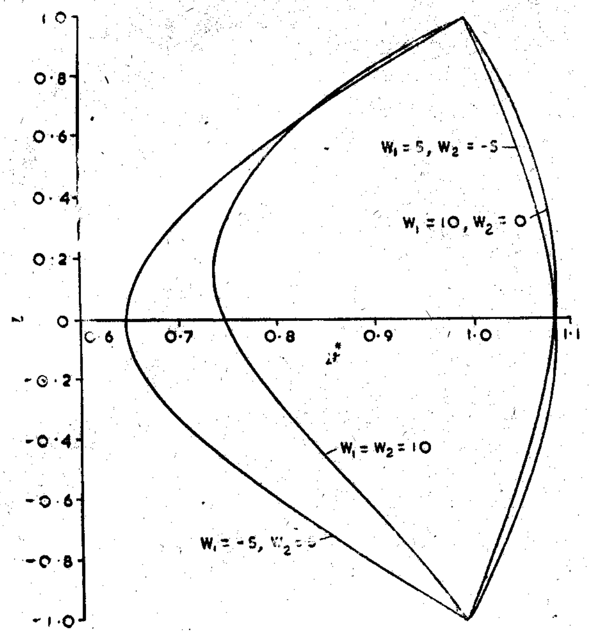

Fig. 2—v^* against z, (R=0·1, Re^*=2).

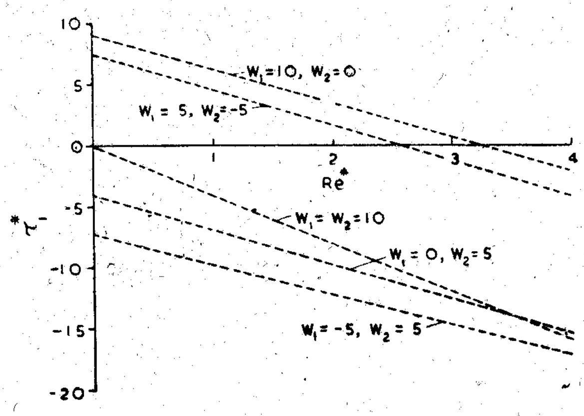

Fig. 3—τ_1^* against Re^*, (R=0·1).

Variation of v^* against z have been shown in Fig. 2 for the same particular cases as considered above. The azimuthal velocity in the central region is found to be less than that on the discs in case of injection at both the discs, but slightly more in case of suction. The layer of maximum deviation in the azimuthal velocity (i.e., the layer of greatest or the least velocity as the case may be) is seen pulled towards the disc with suction and pushed away from the disc with injection. The case of suction at one disc and equal injection at the other is interesting, in which azimuthal velocity even near the disc with suction is less than that on the disc itself. In the absence of source, and $w_1 = w_2$, we note from (39) that the fluid rotates as a solid mass with $v^* = 1$ everywhere. Further, we have already observed that the effect of source is similar to that of injection. Therefore, in the absence of the source v^* falls short of the corresponding values in its presence. The layer in which the deviation from unity is maximum (*i.e.*, the layer of minimum velocity) is pulled nearer the disc with suction in this case too.

The velocity component in the axial direction may be obtained by substituting the values of f_{-1}, f_1 and f_2 in the expression for w in (10).

Skin Friction

The radial shear stress in the non-dimensional form is

$$\tau_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}.$$

Since $\frac{\partial u}{\partial z} \gg \frac{\partial w}{\partial r}$, at $z = \pm 1$, it can be approximated by $\frac{\partial u}{\partial z}$ only. Then

$$\tau_{rz} = \frac{1}{2} r\, f''_{-1} + \frac{Re}{r} f''_0 + \frac{Re^2}{r^3} f''_1 + \frac{Re^3}{r^4} f''_2 + \cdots\cdots, \tag{40}$$

in which f_{-1}, f_0, f_1 and f_2 are known. We define

$$\tau^* = \frac{\tau_{rz}}{r} = \frac{1}{2} f''_{-1} + Re^* f''_0 + Re^{*2} f''_1 + Re^{*3} f''_2 + \cdots\cdots, \tag{41}$$

and denote its value at z=1 by τ_1^*. Fig. 3 shows variation of τ_1^* against Re^* for a fixed value of R=0·1 but different values of w_1 and w_2. All curves slope downwards, showing that slopes of radial velocity profiles at the discs decrease with an increase in source strength. In case suction at the disc z=1 is not accompanied by equal or greater injection at the other, τ_1^* is positive for smaller values of Re^*. This shows that as z decreases from 1, u decreases from zero, implying a back flow towards the axis.

The non-dimensional transverse shearing stress for the axisymmetric case is

$$\tau_{\theta z} = \frac{\partial v}{\partial z} = r\, g'_{-1} + \frac{Re}{r}\, g'_0 + \frac{Re^2}{r^3}\, g_1' + \frac{Re^3}{r^5}\, g'_2 + \ldots\ldots, \tag{42}$$

where g_{-1}, g_0, g_1 and g_2 are known.

Pressure Drop

Substituting from (25) to (28), (31), (34) and (37) in (9), the expression for p may be obtained, which is found undetermined to the extent of a constant of integration. Pressure drop in the radial direction is obtained by comparing magnitudes of pressure at two points in the same z-plane. If $p\,(r_0, z)$ be the pressure at a particular point, the pressure drop

$$p\,(r_0, z) - p\,(r, z) = (r_0^2 - r^2)\Big[\,0.375(w_1 - w_2) + R\Big\{0.5 - 0.096429\,(w_1 - w_2)^2\Big\} + R^2\,(w_1 - w_2)\cdot \cdot\Big\{0.269643 + 0.146875\,(w_1 + w_2)^2 + 0.001751\,(w_1 - w_2)^2\Big\}\Big] + Re\ ln\left(\frac{r}{r_0}\right)\Big[3 - 0.385714\,R\cdot \cdot(w_1 - w_2) + R^2\Big\{1\cdot 942857 + 0.5\,(w_1 + w_2)^2 + 0.169245\,(w_1 - w_2)^2\Big\}\Big] + 0.771429\,Re^2\cdot \cdot R\,(r^{-2} - r_0^{-2}). \tag{43}$$

It may be noted that the radial pressure drop is independent of the axial coordinate of the points.

DISCUSSION

In the above analysis, the Reynolds number corresponding to rotation ($R = a^2\,\omega/\nu$) has been assumed small to eliminate boundary layer effects. By assigning different values to w_1 and w_2, various cases of equal or unequal suction or injection can be deduced. The effect of source is to cause an increased outflow of fluid in the radial direction, which is similar to that of injection at the boundaries. Both these cause an increase in the radial velocity component and a decrease in the azimuthal velocity component, and a decrease in the slopes of both radial and azimuthal velocity profiles near the discs. The effect of suction is opposite. Suction pulls nearer the layer of maximum deviation in velocity from its value on the boundary, injection pushes it away. If the source is not sufficiently strong to match the suction, back flow in the radial direction towards the axis may occur. The radial pressure drop depends on the distance of the points from the axis, and not on the distance from the discs.

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