ON THE PRESSURE RATIO IN AN H/L GUN DURING BURNING

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It has been shown that some conditions regarding the lower limit to the volume ratio of the two chambers dependent on the propellant property and the leakage parameter may exist in order that the pressure ratio is to be less than the critical pressure ratio in an H/L gun during burning. Also for progressive propellants constant pressure ratio may not exist for other than tubular propellant.

It is well known that during the motion of the shot inside the gun the pressure ratio $\omega = p_2/p_1$ is to be less than the critical pressure ratio ω^*, the value of which according to Corner[1] is 0·555. Recently Ray[2] in his paper had shown the conditions for the pressure ratio to be less than the critical for a tubular propellant ($\theta=0$). Here the author tries to discuss Ray's problem with the form function in the standard form

$$Z = (1-f)\,(1+\theta f)$$

we have given condition to the effect that the pressure ratio may remain less than the critical during the burning and this condition gives the lower limit to the volume ratio of the two chambers dependent on the propellant property and that of the leakage parameter Ψ.

We have also discussed the problem of the constant pressure ratio and it has been shown that the constant pressure ratio less than critical value may not be maintained for the propellant other than the tubular propellant.

FUNDAMENTAL EQUATIONS DURING BURNING

Assuming that the pressure ratio is less than the critical, the fundamental equations for the internal ballistics of H/L gun are

$$p_1\left[U_1 - \frac{c(1-Z)}{\delta} - cN\eta\right] = cN\lambda \tag{1}$$

$$p_2\left[U_2 + Ax - c(Z-N)\eta\right] = c(Z-N)\lambda \tag{2}$$

$$D\frac{df}{dt} = -\beta p_1 \tag{3}$$

$$Z = (1-f)\,(1+\theta f) \tag{4}$$

$$\frac{dN}{dt} = \frac{dZ}{dt} - \frac{\psi S p_1}{c\sqrt{\lambda}} \tag{5}$$

$$\omega_2 \frac{dv}{dt} = Ap_2 \tag{6}$$

From (5) and (3) one gets

$$\frac{dN}{dt} = \frac{dZ}{dt} + \Psi\frac{df}{dt}$$

where

$$\Psi = \frac{\psi SD}{\beta c\sqrt{\lambda}} \tag{7}$$

is the dimensionless leakage parameter whose value is of the order of 0·5.

Then we have on integration with the initial conditions $t = 0$,

$$Z = 0, \quad f = 1, \quad N = 0$$

$$N = Z - \Psi (1-f) \tag{8}$$

From (1), (4) and (8) we have

$$p_1 = \frac{c\lambda \{ (1-f)(1+\theta f) - \Psi (1-f) \}}{U_1 - c/\delta + c/\delta (1-f)(1+\theta f) - c\eta \{ Z - \Psi (1-f) \}} \tag{9}$$

From (3) and (9) we have

$$-D/\beta \; \frac{df}{dt} = \frac{c\lambda \{ (1-f)(1+\theta f) - \Psi (1-f) \}}{(U_1 - c/\delta) - c/\delta (1-f)(1+\theta f) - c\eta \{ (1-f)(1+\theta f) - \Psi (1-f) \}}$$

Let

$$1 - f = Z$$

Then

$$(1-f)(1+\theta f) = (1+\theta) Z - \theta Z^2$$

The above equation becomes

$$\frac{dZ}{dt} = \mu_1 \frac{Z - \dfrac{\theta}{1+\theta-\Psi} Z^2}{1 + \left\{ \dfrac{c\Psi/\delta - c(\eta - 1/\delta)(1+\theta-\Psi)}{U_1 - c/\delta} \right\} Z + \dfrac{c(\eta - 1/\delta)\theta}{U_1 - c/\delta} Z^2}$$

where

$$\mu_1 = \frac{\beta c\lambda}{D} \frac{1+\theta-\Psi}{U_1 - c/\delta}$$

Let $$Q = \frac{\theta}{1+\theta-\Psi}, \qquad H = \frac{c(\eta - 1/\delta)\theta}{U_1 - c/\delta} \tag{10}$$

and $$G = \frac{c\Psi/\delta - c(\eta - 1/\delta)(1+\theta-\Psi)}{U_1 - c/\delta}$$

Then we have

$$\frac{dZ}{dt} = \mu_1 \frac{Z - QZ^2}{1 + GZ + HZ^2} \tag{11}$$

From (2), (6) and (8) we have

$$\omega_2 \frac{d^2x}{dt^2} = \frac{Ac\lambda\Psi Z}{U_2 + Ax - \eta c\Psi Z}$$

or $$\omega_2 \frac{dZ}{dt} \frac{d}{dZ} \left[\frac{dx}{dZ} \frac{dZ}{dt} \right] = \frac{Ac\lambda\Psi Z}{U_2 + Ax - \eta c\Psi Z}$$

Taking

$$X = (U_2 + Ax) \frac{\mu_1}{A} \left(\frac{\omega_2}{c\lambda\Psi} \right)^{\frac{1}{3}} \tag{12}$$

Then the above differential equation becomes,

$$\frac{d}{dZ}\left[\frac{Z-QZ^2}{1+GZ+HZ^2}\,\frac{dX}{dZ}\right]=\frac{Z}{X-\nu_1 Z}\;\frac{1+GZ+HZ^2}{Z-QZ^2} \tag{13}$$

where

$$\nu_1=\frac{\eta\mu_1}{A}\left(\frac{\omega_2 c\Psi}{\lambda}\right)^{\frac{1}{2}}$$

Initial condition for the above differential equation are

$$X=X_0=U_2\,\frac{\mu_1}{A}\left(\frac{\omega_2}{c\lambda\Psi}\right)^{\frac{1}{2}}\;,\quad Z\,\frac{dX}{dZ}=0\quad\text{when } Z=0$$

Since

$$\frac{dZ}{dt}=-\frac{df}{dt}=\frac{\beta}{D}\,p_1$$

we have from (11)

$$\left.\begin{aligned} p_1&=\frac{\mu_1 D}{\beta}\,\frac{Z-QZ^2}{1+GZ+HZ^2}\\ &\text{and}\\ p_2&=\frac{c\lambda\Psi Z}{U_2+Ax-c\eta\Psi Z}\end{aligned}\right\} \tag{14}$$

Hence

$$p_2=\frac{Z}{X-\nu_1 Z}\,\frac{\mu_1}{A}\left(\frac{c\Psi}{\omega_2}\right)^{\frac{1}{2}}$$

or

$$\frac{p_2}{p_1}=B\,\frac{Z(1+GZ+HZ^2)}{(X-\nu_1 Z)(Z-QZ^2)} \tag{15}$$

where

$$B=\frac{\beta}{AD}\,(c\lambda\Psi\omega_2)^{\frac{1}{2}} \tag{15a}$$

Now from (13) and (15) we have

$$\frac{d}{dZ}\left[\frac{Z-QZ^2}{1+GZ+HZ^2}\,\frac{dX}{dZ}\right]=\frac{\omega}{B} \tag{16}$$

For simplicity of calculation we take $\eta=1/\delta$, then from (10) Q is positive for θ positive, $H=0$ and $G=\dfrac{c\Psi\delta}{U_1-c/\delta}$ which is positive.

We analyse the problem for positive values of θ only for which Q is always positive. As for negative values of θ, Q changes from negative value to positive value and also for a particular negative value of θ, Q is not finite, the subsequent analysis is based on positive value of θ. Also ν_1 is positive when $\theta > 0$.

Condition for ω to be Less Than ω^*

If ω is less than critical during the period of burning then all the relations in the preceding section are true. Then from (16) we have

$$\frac{d}{dZ}\left[\frac{Z-QZ^2}{1+GZ}\,\frac{dX}{dZ}\right]\leqslant\frac{\omega^*}{B}$$

Integrating twice subject to the initial conditions we obtain,

$$X\leqslant X_0-\frac{\omega^*}{B}\left[\frac{G}{Q}\,Z+\frac{G+Q}{Q^2}\,\log(1-QZ)\right] \tag{17}$$

Again using suffix B to denote burnt values of variables from (13)

$$\frac{d}{dZ}\left[\frac{Z - QZ^2}{1 + GZ}\;\frac{dX}{dZ}\right] = \frac{1 + GZ}{(X - \nu_1 Z)(1 - QZ)} > \frac{1}{X_B}$$

Integrating the above inequality subject to initial conditions one gets

$$\frac{dX}{dZ} > \frac{1 + GZ}{1 - QZ} \cdot \frac{1}{X_B} > \frac{1}{X_B}$$

Which on integration subject to initial conditions yields

$$X > X_0 + \frac{Z}{X_B} \tag{18}$$

From (17), considering the burnt values we get,

$$X_B \leqslant X_0 - \frac{\omega^*}{B}\left[\frac{G}{Q} + \frac{G + Q}{Q^2}\log(1 - Q)\right]$$

which with (18) yields

$$X > X_0 + \frac{Z}{X_0 - \frac{\omega^*}{B}\left[\frac{G}{Q} + \frac{G+Q}{Q^2}\log(1 - Q)\right]} \tag{19}$$

Thus from (15) and (19)

$$\omega = B\,\frac{1 + GZ}{(X - \nu_1 Z)(1 - QZ)} < B\,\frac{1 + GZ}{(1 - QZ)(X_0 + dZ)}$$

where

$$d = \frac{1}{X_0 - \frac{\omega^*}{B}\left\{\frac{G}{Q} + \frac{G + Q}{Q^2}\log(1 - Q)\right\}} - \nu' \tag{20}$$

or

$$\omega < \omega_1(Z)$$

where

$$\omega_1(Z) = B\,\frac{1 + GZ}{(1 - QZ)(X_0 + dZ)} \tag{21}$$

now $\omega_1(Z)$ will be maximum at

$$G(1 - QZ)(X_0 + dZ) - (1 + GZ)\{-Q(X_0 + dZ) + d(1 - QZ)\} = 0$$

i.e.

$$Z = -1 \pm \sqrt{1 - \frac{G}{Q}\left\{\frac{X_0}{d}(G + Q) - 1\right\}}$$

Taking the positive sign, the value of Z is Z_0

where

$$Z_0 = -1 + \sqrt{1 - \frac{G}{Q}\left\{\frac{X_0}{d}(G + Q) - 1\right\}} \tag{22}$$

The condition for maximum yields

$$d(1 - GZ_0) < 0$$

and the corresponding greatest value being

$$B\,\frac{1 + GZ_0}{(1 - QZ_0)(X_0 + dZ_0)}$$

So if we have

$$\left.\begin{array}{ll} & B\,\dfrac{1+GZ_0}{(1-QZ_0)\,(X_0+dZ_0)} < \omega^* \\ \text{when} & 0 < Z_0 < 1 \\ \text{and} & d\,(1-GZ_0) < 0 \end{array}\right\} \tag{23}$$

The pressure ratio ω will remain less than critical during burning. We will now write the inequalities (23) in different form. For this we take the following dimensionless quantities.

$$\alpha = \frac{U_1 - c/\delta}{c/\delta} \tag{24}$$

$$K = U_2/U_1 \tag{25}$$

$$\frac{U_2}{c/\delta} = K\,(\alpha+1) \tag{26}$$

$$M = \frac{A^2 D^2}{\beta^2 c \lambda \omega_2} \tag{27}$$

In that case (taking $\eta = 1/\delta$) from (10)

$$G = \Psi/\alpha \tag{28}$$

$$Q = \frac{\theta}{1+\theta-\Psi} \tag{29}$$

From (14), (15a), (24) and (25)

$$\frac{B}{X_0} = \frac{\Psi}{1+\theta-\Psi} \cdot \frac{\alpha}{K(\alpha+1)} \tag{30}$$

From (10), (14), (13a), (25) and (26)

$$\frac{X_0}{\nu_1} = \frac{K(\alpha+1)}{\Psi} \tag{31}$$

Also

$$\frac{B}{\nu_1} = \frac{\alpha}{1+\theta-\Psi} \tag{32}$$

and

$$\nu_1 X_0 = \frac{K(\alpha+1)}{M\alpha^2}\,(1+\theta-\Psi)^2 \tag{33}$$

From

$$Z_0 \geqslant 0$$

we have

$$\frac{X_0}{d}\,(G+Q) < 1$$

$$\frac{d}{\nu_1} > (G+Q)\,\frac{X_0}{\nu_1}$$

Or from (20), (31), (32) and (33)

$$\frac{1}{\frac{K(\alpha+1)}{M\alpha^2}(1+\theta-\Psi)^2-\omega^*\frac{1+\theta-\Psi}{\alpha}\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}}-1 \geqslant (G+Q)\ \frac{K(\alpha+1)}{\Psi} \tag{34}$$

Again $Z_0 \geqslant 1$ gives

$$1-\frac{G}{Q}\left\{\frac{X_0}{d}(G+Q)-1\right\}<4$$

or by (20), (31), (32) and (33)

$$\frac{1}{\frac{K(\alpha+1)}{M\alpha^2}(1+\theta-\Psi)^2-\omega^*\frac{1+\theta-\Psi}{\alpha}\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}}-1 \leqslant \frac{G(G+Q)}{G-3Q}\ \frac{K(\alpha+1)}{\Psi} \tag{35}$$

Now $d(1-GZ_0)<0$ gives

$$1-GZ_0>0 \quad \text{and} \quad d<0$$

Now $d<0$ gives

$$\frac{1}{X_0-\frac{\omega^*}{B}\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}}-\nu'<0$$

or

$$K>\frac{M\alpha^2}{(\alpha+1)(1+\theta-\Psi)^2}\left[1+\frac{1+\theta-\Psi}{\alpha}\ \omega^*\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}\right] \tag{36}$$

and $1-GZ_0>0$ gives

$$\frac{d}{X_0}<\frac{G+Q}{1+\frac{Q}{G}\left\{1-\left(\frac{1+G}{G}\right)^2\right\}} \tag{37}$$

or

$$\frac{1}{\frac{K(\alpha+1)}{M\alpha^2}(1+\theta-\Psi)^2-\omega^*\frac{1+\theta-\Psi}{\alpha}\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}}-1 < \frac{G+Q}{1+\frac{Q}{G}\left\{1-\left(\frac{1+G}{G}\right)^2\right\}}\ \frac{K(\alpha+1)}{\Psi} \tag{38}$$

Writing

$$\frac{M\alpha^2}{(\alpha+1)(1+\theta-\Psi)^2}=\alpha_0$$

$$-\frac{M\alpha\omega^*}{(\alpha+1)(1+\theta-\Psi)}\left\{\frac{G}{Q}+\frac{G+Q}{Q^2}\log(1-Q)\right\}=\beta_0$$

$$(G+Q)\frac{\alpha+1}{\psi} = \gamma_0, \quad \frac{G(G+Q)}{G-3Q}\,\frac{\alpha+1}{\Psi} = \delta_0$$

and

$$\frac{G+Q}{1+\frac{Q}{G}\left\{1-\left(\frac{1+G}{G}\right)^2\right\}}\,\frac{\alpha+1}{\Psi} = m_0$$

we have the result that the pressure ratio will be less than critical during burning if we have,

$$\frac{\alpha_0}{K+\beta_0} \geqslant 1+\gamma_0 K \tag{39}$$

$$\frac{\alpha_0}{K+\beta_0} \leqslant 1+\delta_0 K \tag{40}$$

$$K > \alpha_0\left[1-\frac{(\alpha+1)(1+\theta-\Psi)^2}{M\alpha^2}\,\beta_0\right] \tag{41}$$

$$\frac{\alpha_0}{K+\beta_0} < 1+m_0 K \tag{42}$$

and

$$B\,\frac{1+GZ_0}{(1-QZ_0)(X_0+dZ_0)} < \omega^* \tag{43}$$

These conditions give lower limit to the value ratio of the two chambers of an H/L gun when M, α and Ψ are known. It may be noted that the conditions suffer from the following defect. There may be cases of subcritical pressure ratio in which neither of the conditions (39) to (43) holds. Some numerical results have been tabulated for different values of θ. For $\alpha = 1$, $\Psi = 0{\cdot}5$, $\theta = 1$ we see that for $M=1$, 2, 3, 4 the lower limits of K are respectively 0·4125, 0·3920, 0·3570, 0·3210, and with these lower limits and corresponding values of M, α and Ψ we calculated the value of ω as $Z \to 0$ and the values are 0·492, 0·516, 0·532 and 0·552.

For $\alpha=1$, $\Psi=0{\cdot}5$, $\theta=0{\cdot}5$ and for $M=1$, 2,3,4, the lower limits of K are 0·6250, 0·5120, 0·4450 and 0·4700 and the corresponding values of ω are 0·416, 0·489, 0·555 and 0·532.

For $\alpha=1$, $\Psi=0{\cdot}5$ and $\theta=0{\cdot}1$ and for $M=1$,2,3,4, the lower limits of K are 0·751, 1·273, 1·142 and the values of ω are 0·555, 0·489, 0·547.

These results indicate that there is at least one stage during burning when the pressure ratio is not much less than the critical.

Case of Constant Pressure Ratio

If possible let the pressure ratio ω be constant at a value less than or equal to the critical during burning. Hence we have

$$\omega_0 = \frac{1+GZ+HZ^2}{(X-\nu_1 Z)(1-QZ)} \tag{44}$$

where ω_0 is the constant pressure ratio. This value will be the initial value of ω

$$\omega_0 = \frac{B}{X_0}$$

$$\frac{B}{X_0} = \frac{(1+GZ+HZ^2)}{(X-\nu_1 Z)(1-QZ)}$$

Hence

$$X = \nu Z + X_0\,\frac{1+GZ+HZ^2}{1-QZ} \tag{45}$$

Again

$$\frac{d}{dZ}\left(\frac{Z-QZ^2}{1+GZ+HZ^2}\frac{dX}{dZ}\right)=\frac{\omega_0}{B}=\frac{1}{X_0}$$

Integrating with the initial conditions

$$Z=0,\quad Z\frac{dX}{dZ}=0$$

we have

$$X=X_0-\frac{1}{X_0}\left[\frac{H}{2Q}Z^2+\frac{GQ+H}{Q^2}Z+\frac{Q^2+GQ+H}{Q^3}\log(1-QZ)\right] \tag{46}$$

In order that (45) and (46) may be identical

$$\nu_1 Z(1-QZ)+X_0(1+GZ+HZ^2)=X_0(1-QZ)-\frac{1}{X_0}(1-QZ).$$
$$\cdot\left[\frac{H}{2Q}Z^2+\frac{GQ+H}{Q^2}Z+\frac{Q^2+GQ+H}{Q^3}\log(1-QZ)\right]$$

The conditions are

$$H=0$$
$$Q(G+Q)=0$$
$$\nu_1 Q=-\frac{G}{X_0}$$

Now $H=0$ gives $Q=0$ as we take in this case $\eta\neq 1/\delta$ and the case of constant pressure ratio for $\theta=0$ has already discussed by Ray in his paper. Again if we take $\theta\neq 0$ and $\eta=1/\delta$ then, the second condition gives $G+Q=0$ which gives θ must be negative as G is always positive for $\eta=1/\delta$. Then in this case the last two conditions are identical i.e. $\nu_1 Q=-G/X_0$. Hence for non-zero positive values of θ, the constant pressure ratio may not be maintained throughout the burning.

Again $G+Q=0$ gives

$$\frac{\theta}{1+\theta-\Psi}=-\frac{\Psi}{\alpha}$$

and $\nu_1 X_0=-\dfrac{G}{Q}$ gives

$$\frac{K(\alpha+1)}{M\alpha^2}(1+\theta-\Psi)^2=-\frac{\Psi}{\alpha\theta}(1+\theta-\Psi)$$

But these relations gives absurd results for negative values of θ. Hence the above relations will not hold good. Hence for progressive propellants constant pressure ratio may not exist. The constant pressure ratio only for tubular propellant was discussed by Ray.

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