A FRESH APPROACH TO THE STUDY OF ATMOSPHERIC TURBIDITY

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The problem of assessment of atmospheric turbidity caused by aerosol particles, viz., dust, smoke, haze, and other atmospheric pollutants, apart from the effect of variable water vapour content of the atmosphere, has been studied afresh. The basic concept underlying Linke's turbidity factor, T, has been found to be theoretically sound, although its quantitative formulation suffers from one major defect, viz., its 'virtual variation' with air mass. This error has been traced to defective formulation of the quantitative expression for T. A 'Rational turbidity factor', Tr, has been proposed which is likely to overcome the limitations of Linke's turbidity factor, T. A nomogram has been developed for quick evaluation of Tr, and the effect of altitude has also been considered.

Atmospheric turbidity is one of the climatic elements whose importance has long been recognised. It represents the total effect of atmospheric matter in depleting solar radiation during its passage through the earth's atmosphere. Its knowledge is important for assessment of radiation and heat balance at the earth's surface, as well as for evaluation of the performance of solar energy devices. It has now assumed a role of special importance in view of the present emphasis on environmental sciences, particularly in respect of atmospheric pollutants in urban and industrial areas. It was, therefore considered worth-while to study the subject with a view to find a simple and practical method of estimating turbidity. From the knowledge of total turbidity, it should be possible to estimate the concentration of atmospheric pollutants by eliminating the effects of Rayleigh scattering and selective absorption by variable water vapour content of the atmosphere. The intensity of direct solar radiation as a function of altitude and humidity was considered in an earlier paper[1]. A simple method was subsequently developed[2,3] for estimation of precipitable water in the atmosphere from measurement of surface humidity. The present paper will be chiefly concerned with the problem of assessment of total atmospheric turbidity based on measurement of direct solar radiation at normal incidence comprising the entire spectrum.

QUANTITATIVE APPROACH TO THE STUDY OF TURBIDITY

Evolution of the concept of turbidity

The intensity of direct solar radiation at normal incidence I depends on its attenuation during its passage through the atmosphere, which is known with a reasonable degree of accuracy in respect of pure, dry air. This attenuation or depletion, increases with increasing turbidity caused by variable constituents of the atmosphere, viz., water vapour, dust, haze, smoke and other solid and liquid particles referred to as aerosol particles in general. It was thought, therefore, that a measure of this depletion of solar radiation intensity could, by itself, be taken as a measure of atmospheric turbidity.

Difficulties arose, however, because measurement of insolation is carried out at stations at different altitudes, and at different hours corresponding to different solar heights. Even for a pure, dry atmosphere, I is known to increase with the altitude of the place of observation and to decrease with increasing zenith-distance Z of the sun, signifying that depletion of solar radiation is due to the absolute air mass m, in the path of the solar radiation. On the other hand, a suitable measure of turbidity should be independent of air mass m, so far as it depends on solar height or zenith distance.

Subsequent attempts to overcome the above mentioned difficulties, led to the development of a number of measures of turbidity, chief among which are : Mean Extinction Coefficient a, Transmission Factor q, and Linke's Turbidity Factor T. These will be discussed below.

Mean Extinction Coefficient a

Transmission of monochromatic radiation through the atmosphere obeys Beer's law, so that

$$I_\lambda = I_{0\lambda} e^{-a_\lambda m} \tag{1}$$

where $I_{0\lambda}$ is the incident radiation intensity of wavelength λ, I_λ, is the transmitted radiation intensity and m is the air mass in the path of the radiation. It follows that

$$ln\ I_\lambda = ln\ I_{0\lambda} - a_\lambda m \qquad (2)$$

so that $ln\ I_\lambda$ plotted against m should yield a straight line whose slope is equal to a_λ (numerically), which is termed as the 'Extinction Coefficient'. This is shown in Fig. 1 for three different wave lengths for a Rayleigh atmosphere. The computed data have been extracted from Robinson[4]. It also follows that

$$a = ln\ (I_{0\lambda}/I_\lambda)/m \qquad (3)$$

In analogy with eqn (3), a Mean Extinction Coefficient a was defined as

$$a = ln\ (S_0/I)/m \qquad (4)$$

where S_0 is the solar constant, I is the intensity of direct solar radiation (for the entire spectrum) at normal incidence and m is the absolute air mass in the path of the solar radiation.

Since the extinction coefficient a_λ is a function of λ, the mean extinction coefficient, a, should vary with air-mass m. In other words $ln\ I$ plotted against m should not yield a straight line. This is shown in Fig. 2 for a pure, dry atmosphere. The data for the plotted points have been taken from Table 1.

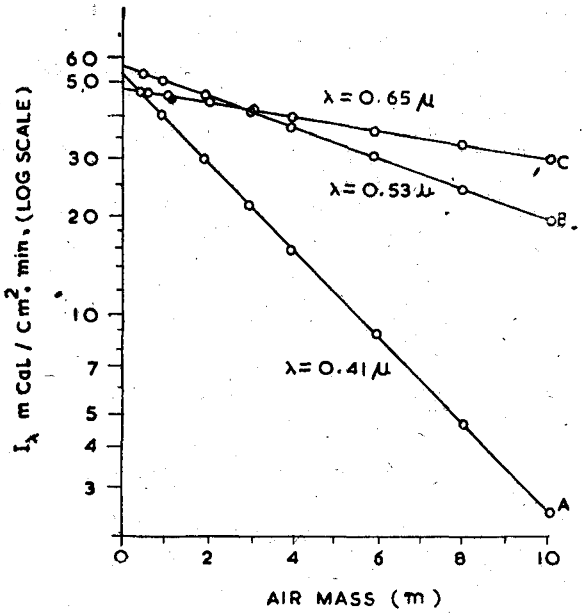

Fig. 1—Linear relationship between log I_λ and m for monochromatic solar radiation of three different wave lengths (Band width 0·02μ) for rayleigh atmosphere. Points represent computed values (Robinson[3]).

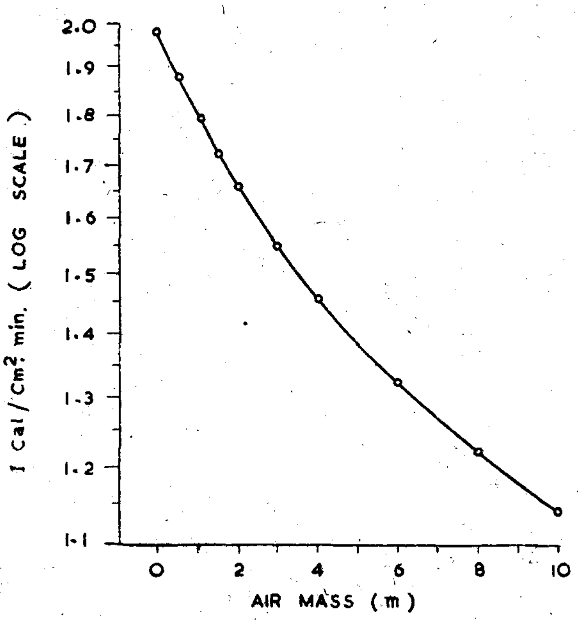

Fig. 2—Non-linear relationship between log I and m for total direct solar radiation. Points represent computed values (Robinson[3]).

Evidently, the mean extinction coefficient a is not a suitable measure of atmospheric turbidity.

Transmission Factor q

From eqn (4), we have

$$I = S_0 e^{-am} = S_0 q^m \qquad (5)$$

where $q = e^{-a}$, and is known as the Transmission Factor. This also suffers from the same defects as the mean extinction coefficient. Both a and q depend on optical air mass with regard to total direct solar radiation comprising the entire spectrum. Beer's law which is strictly valid for monochromatic radiation, is far from true in the case of solar radiation covering a wide range of wave lengths, *Linke's Turbidity Factor T*

It was Linke [5] who suggested a better method for estimation of turbidity. Considering scattering and absorption by pure, dry air as the basic atmospheric effect, Linke proposed to refer the mean extinction coefficient observed in an actual turbid atmosphere to the basic effect. In other words, he defined a turbidity factor T, which indicates how many atmospheres of pure, dry air produce the same depletion in direct solar radiation as the given turbid atmosphere.

It has been noted that the mean extinction coefficient a is a function of air mass m. We, therefore, denote by $a(m)$, the mean extinction coefficient of the local atmosphere for air mass m and by T, the turbidity factor while $a_p\,(m)$ is the mean extinction coefficient for pure, dry air for air mass m.

According to Linke, we should have

$$I = S_0 e^{-a(m)m} \; S_0 e^{-a_p(m)mT} \qquad (6)$$

since an air mass m of turbid air is equivalent to mT air mass of pure, dry air. Hence, we obtain

$$T = ln\;(S_0/I)\; ma_p\;(m)$$

or

$$T = 2{\cdot}303 \;\log\;(S_0/I/ma_p\;(m).$$

or

$$T = P(m)\;\log\;(S_0/I) \qquad (7)$$

where

$$P(m) = 2{\cdot}303/ma_p\;(m).$$

Here S_0 is the solar constant, assumed to be 1·98 cal/cm^2 min, and I is the intensity of direct solar radiation at normal incidence at the place of observation. Numerical values of the factor $P(m)$ as a function of air mass m for computation of turbidity factor T for total radiation, can be estimated from equation (7) by putting T=1 and using the values of I_p computed for different air masses in respect of pure, dry atmosphere. Thus for T=1, eqn (7) leads to

$$P(m) = 1/\log\,(S_0/I_p) \qquad (8)$$

and

$$\log I_p = \log S_0 - 1/P(m) \qquad (9)$$

for pure, dry atmosphere. Computed value of $P(m)$ with S_0=1·98 cal/cm^2 min have been tabulated by Robinson[6] for air mass m varying from 0·5 to 10. Eqn (9) has been utilised to find I_p, for the same air masses for pure, dry air. Values of $P(m)$ and I_p for selected values of m are shown in Table 1.

TABLE 1

VALUES OF $P\,(m)$ AND I_p FOR PURE, DRY ATMOSPHERE AS RELATED TO AIR MASS m

Air mass (m)	$P(m)$	I_p cal/cm min (Pure, dry air)
0·5	43·9	1·879
1·0	23·2	1·793
1·5	16·4	1·721
2	12·86	1·655
3	9·33	1·547
4	7·55	1·460
6	5·72	1·324
8	4·77	1·222
10	4·19	1·143

From the observed value of I and $P(m)$ given by Table 1, Linke's Turbidity factory T, can be estimated from eqn (7) with $S_0=1\cdot98$ cal/cm^{2}min. Computational work can be avoided with the help of the chart shown in Fig. 3. It should be noted, however, that the observed I must be reduced to the mean sun-earth distance by multiplying it by the factor $(R_d/R_o)^2$, where R_d is the sun-earth distance on the day of observation, and R_o is the annual mean[6].

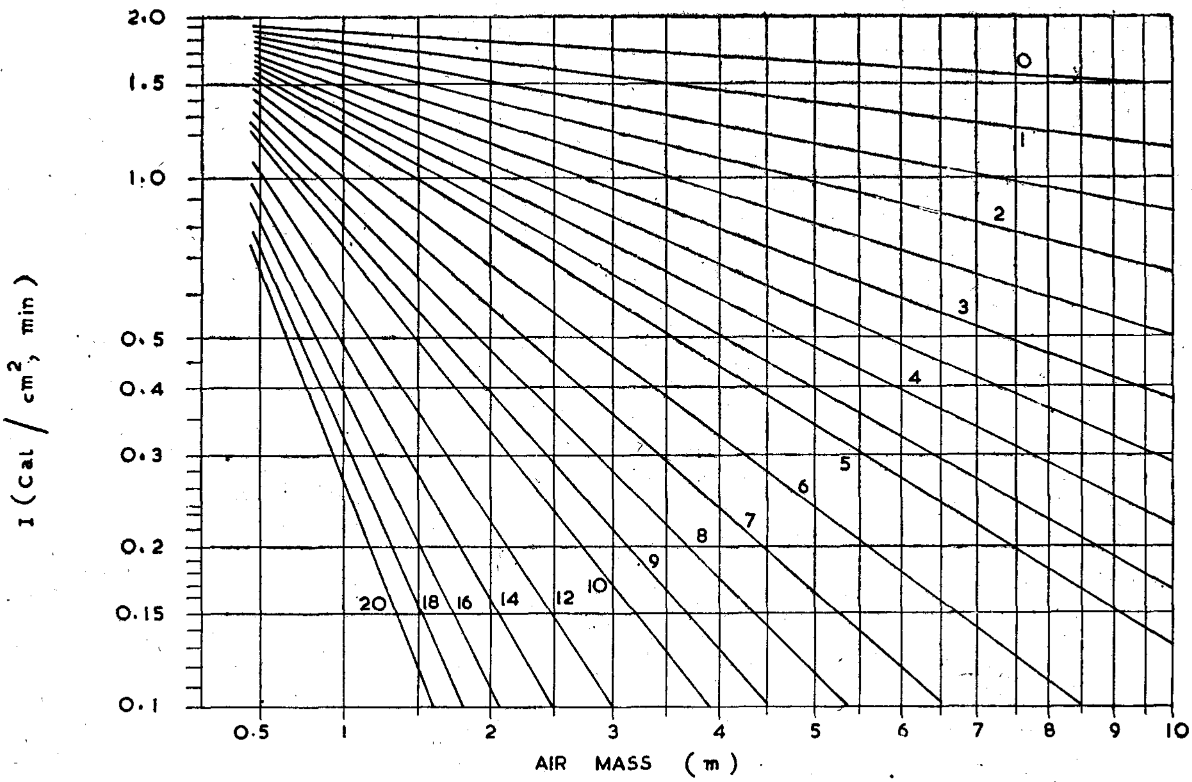

Fig. 3—Chart for computation of linke's turbidity factor, T from given values of intensity of direct solar radiation at normal incidence, I, and absolute air mass, m.

Linke's turbidity factor, T, as defined by equation (7) and shown in Fig. 3, proved to be a distinct improvement on earlier measures of turbidity. It was, however, found to suffer from one great disadvantage[6]. Even under constant turbidity conditions, T was found to exhibit a diurnal variation referred to as 'virtual variation'. In other words, T is not strictly independent of air mass m. This was ascribed by Linke[7] to dependence of the extinction coefficient on the wave length of radiation.

PROPOSED APPROACH

A critical study of Linke's turbidity factor, T, leads to the conclusion that there is nothing materially wrong with the basic definition of T according to which T is the number of atmospheres of pure, dry air which produce the same total depletion of direct solar radiation as the given turbid atmosphere.

We have traced the error to the formulation of the quantitative expression for T given by eqn (7), derived from eqn (6) which states that

$$I = S_0 e^{-a(m)\, m} = S_0 e^{-a_p(m)\, mT}$$

where $a(m)$ is the mean extinction coefficient for the given air of turbidity factor, T, while $a_p(m)$ is the mean extinction coefficient for pure dry air. Since $a_p(m)$ is a function of air mass m and mT is the equivalent air mass of pure, dry atmosphere, it follows that $a_p(m)$ in the above equation should be replaced by $a_p(mT)$. Equation (6) should, therefore, be modified as

$$I = S_0 e^{-a(m)m} = S_0 e^{-a_p(mT_r)\, mT_r} \qquad (10)$$

The turbidity factor, T_r, defined by the above equation has been termed as the 'rational turbidity factor', in order to distinguish it from Linke's turbidity factor, T. It follows, therefore, that

$$T_r \quad \frac{ln(S_0/I)}{{}^{m}a_p\,({}^{m}T_r)} = \frac{2\cdot303}{{}^{m}a_p\,({}^{m}T_r)} \log\,(S_0/I) \tag{11}$$

The main difficulty in applying the above equation for practical computation of T_r arises because of the fact that data on direct solar radiation in a pure, dry atmosphere are not available beyond an optical air mass of 10, while the equivalent pure air mass, mT_r, of a given turbid atmosphere may be many times the above value. For example, with $m = 5$ and $T_r = 30$, the equivalent pure air mass is 150. Extrapolation of solar-radiation data upto such large values of air mass with any reasonable degree of accuracy, has not yet been possible because of the non-linearity between $\log I_p$ and m as shown in Fig. 2. No simple formula has yet been developed which could enable extrapolation much beyond $m = 10$. Our foremost task was, therefore, to obtain a simple as well as reliable formula correlating I_p with m, which could be transformed into a linear equation, thereby enabling extrapolation to much larger values of m with reasonable accuracy.

Correlation of I_p with m

The concavity upwards of the curve in Fig. 2 suggests that linlearity may be achieved by plotting $\log I_p$ against m^p, where p is *a* suitable positive fraction $(0<p<1)$, such that

$$\log I_p = A—B\; m^p, \tag{12}$$

where A, B and p are three undetermined constants. This equation may be put in the form

$$(A— \log I_p) = B\; m^p$$

or $$\log\,(A—\log I_p) = \log B + p \log m, \tag{13}$$

so that $\log\ (A—\log I_p)$ plotted against $\log m$ should yield a straight line whose slope is equal to p.

As a first approximation, A was taken to be $\log S_0$, S_0 being the solar constant equal to 2 cal/cm^2 min. The graph thus obtained was found to be slightly concave upwards signifying that A should be somewhat greater than log 2. After a few trials, with progressively larger value of A, almost perfect linearity (within the limitations of graphical method) was achieved with $A = 0\cdot318$, as may be seen from Fig. 4. The slope p of the line is found to be practically equal to 0·6. With p=0·6, values of A and B were more accurately calculated with the help of the least square method, since according to eqn (12) $\log I_p$ is linearly related to m^p. The constants of the regression eqn (12) alongwith σ, the standard deviation of the scatter about the regression were calculated.

In order to arrive at the best value of p the same method was repeated with several values of p in the range 0·56 to 0·62. The corresponding values of the constants A and B and σ are shown in Table 2.

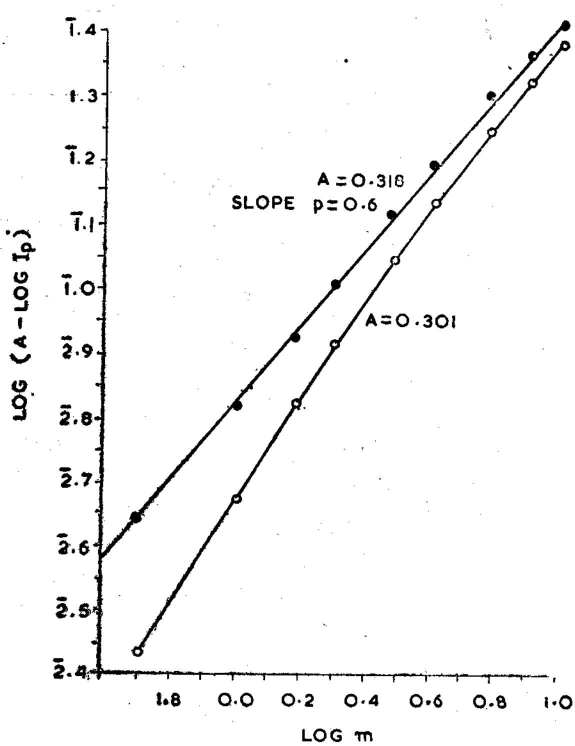

Fig. 4—Graphical evaluation of p in terms of equation (13) after adjustment of A so as to yield a linear relationship. (Data from Table 1).

Table 2

Constants of regression equation (12) and σ for different values of p

	p						
	0·56	0·57	0·58	0·59	0·60	0·61	0·62
A	0·32725	0·32491	0·32265	0·32048	0·31837	0·31633	0·31435
B	0·074600	0·072375	0·070233	0·068170	0·066181	0·064266	0·062413
σ	0·001619	0·001609	0·001618	0·001654	0·001757	0·001914	0·002066
	(0·373%)	(0·369%)	(0·373%)	(0·380%)	(0·405%)	(0·440%)	(0·480%)

In Fig. 5, σ has been plotted against p. It will be seen that the value of p yielding the least value of σ is 0·57. Further refinement is not likely to yield better results as will be apparent from the curve in Fig. 5.

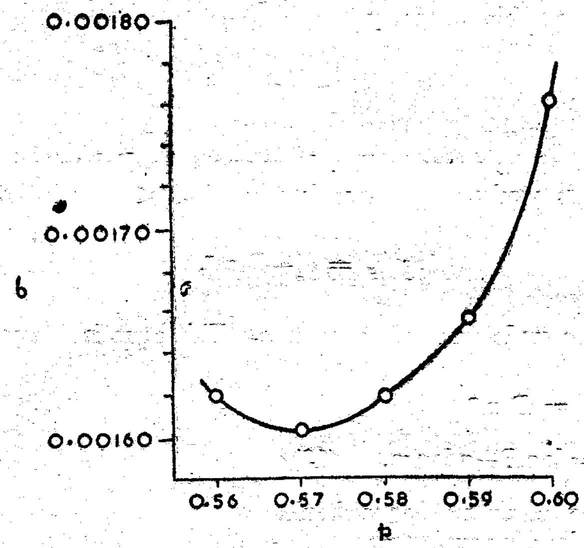

Fig. 5—Estimation of the best value of the exponent, p which yields the minimum value of σ.

The regression equation finally accepted may, therefore, be written as

$$\log I_p = 0{\cdot}32491 - 0{\cdot}072375 m^{0.57} \tag{14}$$

Computation of the Rational turbidity factor, T_r

With the help of eqn (14), we are now in a position to compute the Rational turbidity factor, T_r, which has been defined as the number of atmospheres of pure, dry air which produce the same total depletion of direct solar radiation as the given turbid atmosphere.

If I is the observed intensity of direct Solar radiation at normal incidence in a turbid atmosphere T_r of absolute air mass m, then it follows from the definition of T_r that eqn (14) can be used by replacing I_p by I and m by (mT_r), so that

$$\log I = 0{\cdot}32491 - 0{\cdot}072375\,(mT_r)^{0.57} \tag{15}$$

Whence, we obtain

$$T_r = \frac{1}{m}\left[\frac{0{\cdot}32491 - \log I}{0{\cdot}072375}\right]^{1/0{\cdot}57} \tag{16}$$

An alternative expression for T_r has been provided by eqn (11), but it will be much simpler to use eqn (16) for the purpose since it requires the knowledge of only two parameters, viz., I and m.

In order to avoid tedious computational work, a nomogram has been developed for quick evaluation of T_r, and the same is shown in Fig. 6.

DISCUSSION

Despite the sound physical concept underlying Linke's turbidity factor, T, it has fallen out of use because of its so called virtual variation with air mass, which was ascribed by Linke to the spectral dependence of the 'extinction Coefficient', a_λ. The real cause of this virtual variation has been explained in the foregoing, and the proposed 'Rational turbidity factor', T_r, is likely to overcome this limitation.

Let us now look into the nature of this virtual variation of T, which has been defined by eqn (7) namely

$$T = P(m) \log (S_0/I)$$

where $P(m)$ as a function of air mass m may be obtained from Table 1. The value of S_0 was taken by Linke as 1·98 cal/cm² min. After substituting this value of S_0 and that of log I from eqn (15) in the above eqn (7), we finally obtain

$$T = P(m) \, [0{\cdot}072375 \, (mT_r)^{0{\cdot}57} - 0{\cdot}02824] \qquad (17)$$

It follows from the above equation that T does depend on m for any given value of T_r. The results of calculation are shown in Fig. 7, in which values of T have been plotted against m for different fixed levels of atmospheric turbidity expressed in terms of T_r.

It may be noted in this connection that with absolute air mass m, T_r as defined by eqn (14) and shown in Fig. 6 refers to near sea level conditions ($\simeq$ 1000 *mb*). In respect of stations at different altitudes, T_r should be understood to denote the number of standard (sea level) atmospheres of pure dry air which cause the same depletion of I as the local turbid atmosphere of the station of observation. In general, therefore, m in eqn (14) and Fig. 6 should be replaced by m_r, the relative air mass, which is practically equal to Sec z, where z is the zenith distance of the sun. T_r determined in this way is applicable to all altitudes, including sea level.

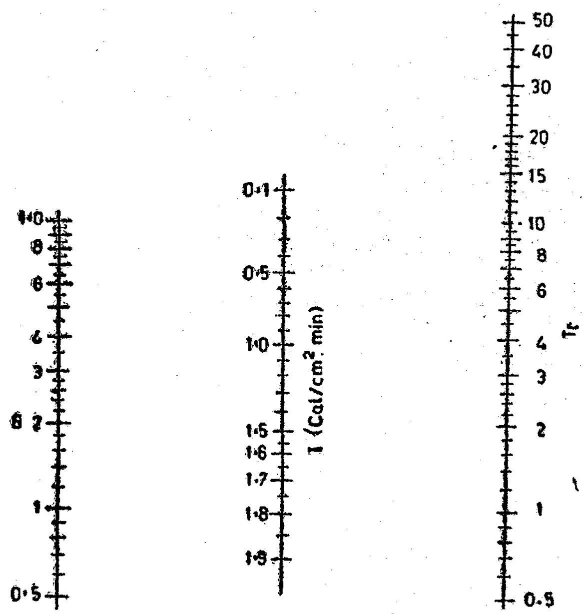

Fig. 6—Nomogram for computation of rationalised turbidity factor, Tr (A straight line through absolute air mass, m, and intensity of direct solar radiation at normal incidence, I, gives Tr on the appropriate line).

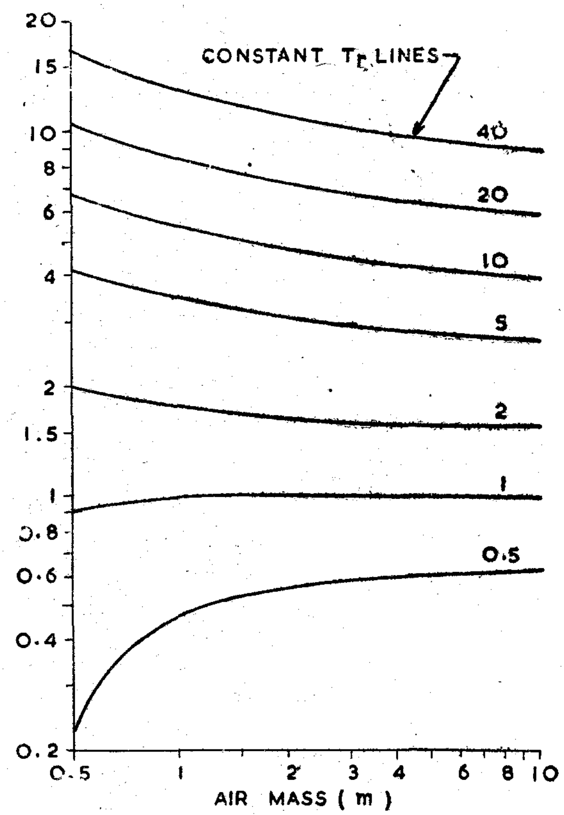

Fig. 7—Virtual variation of Linke's turbidity factor T with air mass, m, for fixed levels of atmospheric turbidity, Tr.

CONCLUSION

The proposed 'Rational turbidity factor' T_r is likely to overcome the limitations of Linke's turbidity factor T, particularly in respect of the virtual variation of T with air mass. The next step should be elimination of the effect of Rayleigh scattering and absorption of direct solar radiation by variable water vapour content of the local atmosphere, so as to arrive at a reasonable measure of turbidity caused by aerosols.

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