PROFIT ANALYSIS IN SOME REDUNDANT SYSTEMS WITH REPAIR MAINTENANCE

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This paper deals with a modularly redundant system with many active units and a warm standby unit. The concepts of 'coverage' and 'manual recovery' have been incorporated. Probabilities that the system can recover automatically/manually at the time of failure of an active unit, are fixed. Failure time distributions of an active and standby units are exponential with different rates. However, distributions of time to repair a failed unit, recovery device, time to manual recovery are taken as general. It is assumed that the system earns a fixed amount for the duration it is operative and repair cost is incurred when a unit/RD is under repair. Expected profit of the system has been obtained by superimposing Howard's reward structure on the semi-Markov process generated by the system model. System performance (expected profit) has been studied for its behaviour. Several earlier well known models are included as special cases.

Expected profit is an extremely important parameter in economic evaluation of standby redundant systems. In fact, the environments under which modern complex business/industrial standby systems operate are critically economic sensitive. A review of the existing literature on standby systems reveals that economic aspects have not been analysed to the satisfactory extent. Most of the authors were interested in obtaining LS transform of the first passage distribution to system failure[1], availability of a system[2, 3].

Recently expected profit has been obtained for a two-dissimilar unit system[4] and has been suggested as the measure of maintenance effectiveness[5]. Optimal preventive maintenance policies that maximize expected profit rate in a two-unit standby system with degraded states has also been discussed by Mine Kawai[6]. Switch behaviour has also been incorporated in the evaluation of profit in a 2-unit warm standby redundant system.

The present paper deals with a system consisting of several units with a common warm standby. Concepts of 'coverage'[7] have also been incorporated. System performance (expected profit) has been related with other parameters e.g., failure rates of a unit, repair-time distribution of a failed unit, earning rate of the system, repair cost etc. The purpose of the paper is to discuss following aspects of standby redundant systems.

(i) To obtain analytic expression for the expected profit, the system will earn in steady-state if it is allowed to operate in an infinite time span.

(ii) To investigate the response of expected profit to changes in other system parameters viz., mean-time to failure, mean-time to repair, earning rate of the system etc.

(iii) To examine the impact of 'coverage' and 'manual recovery' on the economics of the system.

(iv) To study the effect of the warm standby on expected profit. The model discussed is quite general and includes several earlier well known models as special cases, some of them are shown in the end.

For the purpose of analysis, an income-structure[8] has been superimposed on the semi-Markov process generated by the system model.

SYSTEM MODEL

(i) There is a $(n+1)$ unit system; n units are required to operate in order to perform the necessary system task and one unit is put in the common warm standby. A warm standby can fail while as standby.

(ii) Failure-time distributions of operative and standby units are exponential whereas repair-time distribution is general.

(iii) There are following two devices :

(*a*) *Automatic Recovery Device* : It is used to switch the standby unit (if it is there) to operate at the time of failure of an operative unit.

(*b*) *Manual Recovery Device* : Some faults are not covered by ARD but a manual action may recover the system without performing the actual repair.

Probability of ARD operating successfully at the time of need is fixed. Probability that a fault can be recovered manually is also fixed. When ARD fails, it goes to repair immediately and the failed unit waits for repair because of a single repairman. Distribution of time to repair ARD is general. Further, time taken to recover the system manually is also random with general distribution.

(iv) Units and ARD are like new ones after each repair.

(v) The system earns (looses) a fixed amount per unit time in each state and transition rewards (costs) are involved whenever it changes its state.

(vi) All random variables defined to model the system and independent in statistical sense.

The system model allows different failure rates for an operative and standby units which is required in electronic and power systems.

By giving priority to repair ARD, system down-time will be reduced which will result in increased profit.

SYSTEM STATES AND TRANSITIONS

Define the following system states to identify the system at any time.

S_1: n units are operative and a unit is as warm standby,

S_2: a unit is under repair and the system is operational after successful recovery,

S_3: MRD is under repair,

S_4: ARD is under repair and the failed unit is waiting for repair,

S_5: one unit is under repair and another failed unit is waiting for repair.

Initially, system starts in S_1. Upon failure of active unit, ARD is used to recover the system's task i.e., to switch the standby unit to operate; if ARD is successful, system enters S_2 but if ARD does not operate properly, system may be recovered manually in which it enters again S_2. But if MRD is not good system goes to S_3. Transitions between states are shown in Fig. 1. System is up in S_1, S_2 and it is down in S_3, S_4, S_5.

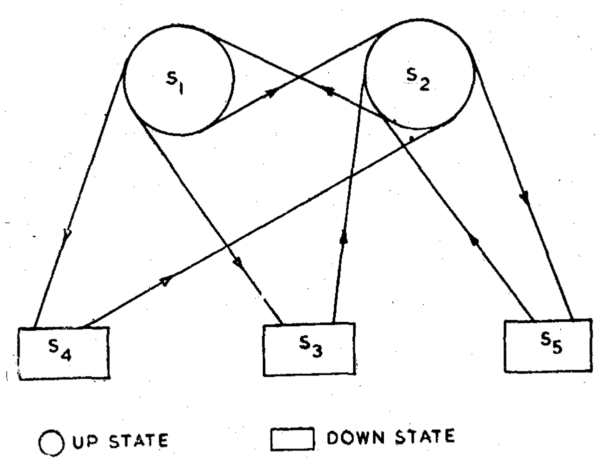

Fig. 1 Transition diagram for the model.

NOTATION

λ constant failure rates for an operative unit

λ_1 constant hazard rate for standby unit

ρ $\equiv \lambda_1/n\lambda$, normalised value of hazard rate

p probability that the system can recover automatically given that an active unit has failed

u probability that the system can be recovered manually but not automatically ($0 \leqslant u \leqslant \bar{p}$)

$f(t)$	p.d.f. of repair-time of a failed unit
$f_1(t)$	p.d.f. of repair-time for ARD
$f_2(t)$	p.d.f. for repair-time for MRD
ϕ	Laplace transform of $f(t)$ evaluated at $n\lambda$
m	expected time to repair a failed unit
m_1	mean-time to repair ARD
m_2	mean-time to repair MRD
M, M_1, M_2	$\equiv mn\lambda,\ m_1 n\lambda,\ m_2 n\lambda$
μ_i	mean unconditional sojourn time of the system in S_i
p_{ij}	one-step transition probability from S_i to S_j
P	transition probability matrix, $\equiv(p_{ij})$
I	identity matrix of order 5
D	$I-P$
di	subdeterminent of D, deleting ith row and ith column
π_i	probability that the embedded Markov chain is in S_i, $\equiv \dfrac{di}{\sum_i di}$
r_{ij}	transition reward for a transition from S_i to S_j
λ_i	earning rate per $1/n\lambda$ time of the system in S_i
g	expected profit per $1/n\lambda$ time in steady-state
—	implies the complement *e.g.*, $\bar{\phi}=1-\phi$.

ANALYSIS OF RESULTS

It has been shown in Howard (1964) that

$$g=\sum \pi_i \mu_i q_i \Big/ \sum \pi_i \mu_i \tag{1}$$

where

$$\mu_i q_i = \sum_i p_{ij} r_{ij} + y_i \mu_i n\lambda$$

It may be easy to see that the semi-Markov process generated by the system is irreducible. Elements of P are given by

$$p_{12}=p\int_0^\infty e^{-\lambda_1 t}\, n\lambda e^{-n\lambda t}\,dt+\int_0^\infty e^{-n\lambda t}\,\lambda_1 e^{-\lambda_1 t}\,dt=(p+\rho)/(1+\rho),$$

$$p_{14}=(1-p-u)\int_0^\infty e^{-\lambda_1 t}\, n\lambda e^{-n\lambda t}\,dt=(1-p-u)/(1+\rho),$$

$$p_{13}=u\int_0^\infty e^{-\lambda_1 t}\, n\lambda e^{-n\lambda t}\,dt=u/(1+\rho),$$

$$p_{25} = \int_0^\infty n\lambda e^{-n\lambda t}\, \bar{F}(t)\, dt = \bar{\phi},$$

$$p_{21} = \int_0^\infty e^{-n\lambda t}\, dF(t) = \phi,$$

$p_{32} = p_{42} = p_{52} = 1$, and

$p_{ij} = 0$ for other i and j.

Further, we can also find

$$d_1 = \phi\ ,\ d_2 = 1\ ,\ d_3 = u\phi/(1+\rho)\ ,\ d_4 = (1-p-u)\,\phi/(1+\rho),$$

$$d_5 = \bar{\phi}\ ,\ \mu_1 = \int_0^\infty e^{-n\lambda t}\, e^{-\lambda_1 t}\, dt = 1/n\lambda\,(1+\rho)\ ,$$

$$\mu_2 = \int_0^\infty e^{-n\lambda t}\, \bar{F}(t)\, dt = \bar{\phi}/n\lambda,\ \mu_3 = m_1,\ \mu_4 = m_2,$$

$$\mu_5 = (M - \bar{\phi})/n\lambda\bar{\phi}\ ,$$

Substituting above into (1) and simplifying, we get

$$g = W/X \tag{2}$$

where

$$W \equiv \Big[(p+\rho)\, r_{12} + u\,(r_{13} + r_{32} + y_3 M_1) + (1-p-u)\,(r_{14} + r_{42} + y_4 M_4^{6}) + + y_1 \Big]\phi + (1+\rho)\Big[(r_{25} + r_{52} + y_2 - y_5)\,\bar{\phi} + r_{21}\,\phi + y_5 M \Big]$$

$$X = (1+\rho)\, M + \Big[1 + uM_1 + (1-p-u)\, M_2 \Big]\,\phi$$

Particular Cases

(*i*) If $u = 0$, $n = 1$, then (3) reduces to

$$g = W/X \tag{3}$$

where

$$W \equiv \Big[(p+\rho)\, r_{12} + q\,(r_{14} + r_{42} + y_4 M_2) + y_1 \Big]\phi + + (1+\rho)\Big[(r_{25} + r_{52} + y_2 - y_5)\,\bar{\phi} + r_{21}\,\phi + y_5 M \Big]$$

$$X \equiv (C+\rho)\, M + (1 + qM_2)\,\phi$$

$$q = 1 - p$$

The above result is in agreement with equation (2) in Kumar[5] for the case when $f(t) = f_1(t)$.

Further let us consider the following cost structure :

R : earnings of the system per $1/\lambda$ times when system is operative

C : repair cost per $1/\lambda$ times for a failed unit when it is under repair

C_a : repair cost per $1/\lambda$ times for ARD to be repaired.

So, substituting $y_1 = R$, $y_2 = R - C$, $y_4 = -C_d$, $y_5 = -C$, $r_{ij} = 0$ for all i & j into (3), we get

$$g = W/X \tag{4}$$

where

$$W \equiv R(1 + \rho\bar{\phi}) - \left[(1 + \rho) MC + q\phi M_2 C_d \right]$$

$$X \equiv (1 + \rho) M + (1 + M_2 q) \phi$$

Obviously, g given by (4) is a non-decreasing function of R and a non-increasing function of C and C_d. In order to examine the effect of warm redundancy on expected profit for a cold standby case i.e., put $\rho = 0$ in (4) to get

$$g_0 = W_0/X_0 \tag{5}$$

where

$$W_0 \equiv R - (MC + qM_2 C_d)$$

$$X_0 \equiv M + (1 + qM_2)$$

So loss is expected profit due to failure of a unit while in standby is given by

$$L = g - g_0 = W/X \tag{6}$$

where

$$W \equiv \rho \left[R \left\{ M - \bar{\phi} (M + (1 + qM_2) \phi) \right\} + M \left\{ C (1 + qM_2) \phi - qM_2 C_d \phi \right\} \right]$$

$$X \equiv \left[M + (1 + qM_2) \phi \right] \left[(1 + \rho) M + (1 + qM_2) \phi \right]$$

It is evident from the above equation that loss vanishes if $\rho = 0$. Also, loss is a non-increasing function of C_d and is a non-decreasing function of C.

(ii) If $p = 1$, $u = 0$, $n = 1$, then (2) reduced to

$$g = W/X \tag{7}$$

where $$W \equiv \phi \left[(r_{12} + r_{21}) \phi + r_{21} + y_1 \right] + (1 + \rho) \left[(r_{25} + r_{52} + y_2 - y_5) \overline{\phi} + y_5 M \right]$$

$$X \equiv (1 + \rho) M + \phi$$

This agrees with (2) in Kumar[4] for $\phi_s = \phi_0$, $m_s = m_0$.

(iii) If $p = 1$, $u = 0$, $n = 1$, $\lambda_0 = n\lambda + \lambda_1$ and $\lambda' = n\lambda$ the model reduces to a 2-unit parallel redundant system[1]. In this case (2) reduces to

$$g = W/X \tag{8}$$

where

$$W \equiv (1 + \rho) \left[(r_{01} + r_{10}) \phi + (r_{12} + r_{21} + y_1) \overline{\phi} + (M - \overline{\phi}) y_2 \right] + y_0 \phi$$

$$X \equiv \phi + (1 + \rho) M$$

$$\rho = \lambda_1/\lambda'$$

In the above paper Nakagawa & Osaki[1] have included four earlier well known models as particular cases. So, those models can easily be derived as special cases of the present special case. Some of these models are given by Gaver[2] and Downton[9].

CONCLUSION

We have obtained expected profit for a modularly redundant system. Model contains several earlier well known models as special cases. Concepts of automatic and manual recovery incorporated in the model are quite useful parameters to system designers. Probabilities p and u are just design parameters and it is upto system designers to examine what constitutes these proportions in their cases.

'Coverage' is defined as the proportion of faults from which a system can recover automatically[7]. This proportion could really be controlled to the maximum possible extent. However, a line has to be imposed between recoverable and non-recoverable faults and the overall situation be examined either from the view point of objective functions or economics of the situation. Recoverable faults are usually connected with the software or the programming part of computer systems and non-recoverable faults are attributed to the hardware design portion. The concept of 'Black Box' explains the limits under which automatic coverage is economically feasible. It will not be out of place to mention that adaptive systems basically make no distinction between recoverable and non-recoverable failure states.

In order for a coverage to be complete and exhaustive two fundamental conditions in terms of concepts of 'Black Box' must be satisfied.

(i) The instrumental data must be complete and sufficient to define the situation completely.

(ii) The mathematical model must be capable of getting the solution.

As we go on moving towards the so called 'complete strategy', marginal cost increases rapidly and therefore a line separating one from the other (recoverable and non-recoverable) would solely depend upon objective functions.

Above discussion defines completely the concept of automatic recovery or 'coverage'. Hence 'coverage' may be defined as a 'strategy'. to recover from certain undesirable states within economic constraints and without supply of any data from the outside world.

The impact of automatic recovery, manual recovery, warm redundancy etc. on the overall economics of the system must be considered well in advance.

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REFERENCES

1. NAKAGAWA, T. & OSAKI, S., *Micr. and R.l.*, **14** (1975), 457.
2. GAVER, D.P., *IEEE Trans. Rel.*, **12** (1963), 30.
3. CHOW, D.K., *IEEE Trans. Rel.*, **R-24** (1975), 64.
4. ASHOK KUMAR, *ZAMM*, **57** (1977a), 124.
5. ASHOK KUMAR, *IEEE Trans. Tel.*, **R-25** (1976), 105.
6. MINE, H. & KAWAI, *IEEE Trans. Rel.*, **R-23** (1974), 81.
7. ARNOLD, T.F., *IEEE Trans. Comp.*, **C-22** (1973), 251.
8. HOWARD, R.A., *J. Ops. Res. Soc. Japan*, **6** (1964), 163.
9. DOWTON, P., *J. Roy. Stat. Soc., Ser B*, **28** (1966), 459.