LOVE WAVE PROPAGATION IN PORO-ELASTICITY

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It is observed that on similar reasons as in classical theory of elasticity, SH wave propagation in a semi-infinite poro-elastic body is not possible and is possible when there is a layer of another poro-elastic medium over it, i.e., Love waves. Two particular cases are considered in one of which phase velocity can be determined for a given wave length. In the same case, equation for phase velocity is of the same form as that of the classical theory of Elasticity.

In 1941, M. A. Biot[1] gave the governing equations of the solid phase and liquid phase of a poro-elastic medium based on a continuum model when static loads are applied on the boundary of the body. Numerous Soil Mechanics problems are solved based on it. Later he extended the theory for the case of dynamic loads[2]. Using this, only a few problems are solved. Pulse propagation in a poro-elastic medium was studied by J. P. Jones[3] and wave propagation due to a point moving load by S. Paul[4].

In this paper the problem of Love wave propagation is studied in a poro-elastic medium. Just as in the classical theory of elasticity, assuming the earth to be a semi-infinite poro-elastic medium, SH wave propagation cannot be studied because of a mathematical difficulty. A few general remarks can be made in this context. Later treating the earth to be consisting of a poro-elastic medium with a layer of different poro-elastic material over it, is studied, *i.e.*, Love waves. In one of the simplest cases, *i.e.*, when dissipative nature and mass coupling effects of the poro-elastic media are neglected, the equation for phase velocity bears a similarity with that of classical theory of elasticity. Hence no need of numerical work is felt, in this context.

FORMULATION AND SOLUTION OF THE PROBLEM

In the solution of the problem of SH wave propagation in a semi-infinite elastic medium, say earth the difficulty arises out of the stress free nature of the free surface. This makes the displacement to be zero identically. So Love proposed to treat the earth, to be consisting of a semi-infinite medium with a layer of another elastic material over it and wave mainly to oscillate in the layer. But recently, another picture came to light. Purushothama[5], K. M. Rao & B. K. Rao[6] considering same problem in magneto-elastic and micropolar semi-infinite media respectively showed that such motion exists. Here it is to be observed that when some side effects due to magnetic nature or micropolar nature are considered, the original difficulty can be circumvented. This is because the secondary effects produce additional shear stress(es).

In the case of a poro-elastic medium, the side effect of pores is considered. But within the limitations considered upto[2], this side effect does not contribute to additional shear stress(es). Hence in this case, as in the case of classical theory of elasticity, SH wave propagation can not be considere in a semi-infinite medium. Hence in the following the problem of Love wave propagation is considered a semi-infinite poro-elastic medium. However it may be pointed out that when visco-poro-elastic semi- ite medium is considered, the SH wave propagation may be possible in a semi-infinite medium. the

Let the origin of a rectangular cartesian co-ordinate system be at the interface, x-axis in the direction of propagation of the wave and y—axis into the semi-infinite medium. z—axis is taken appropriately so that oxyz is a right handed system. Let H be the thickness of the layer so that the free surface is given by $y=-H$.

In this case, the non-vanishing displacement component w and W of solid and liquid respectively are given by

$$w = B e^{\alpha y + ik(x - ct)}, \quad W = B_1 e^{\alpha y + ik(x - ct)} .$$

Substituting in the equation of motion

$$N \nabla^2 \bar{u} + (A + N) \text{ grad } e + Q \text{ grad } \epsilon = \frac{\partial^2}{\partial t^2} (\rho_{11} \bar{u} + \rho_{12} \bar{U}) + b \frac{\partial}{\partial t} (\bar{u} - \bar{U})$$

$$\text{grad} (Qe + R\epsilon) = \frac{\partial^2}{\partial t^2} (\rho_{11} \bar{u} + \rho_{12} \bar{U}) - b \frac{\partial^2}{\partial t} (\bar{u} - \bar{U})$$

where $\bar{u}$ and $\bar{U}$ are the displacement vector of a solid particle and a liquid particle respectively, one gets

$$r^2 - \delta^2 = k^2 - c^2 k^2 \frac{(c^2 k^2 \rho_{22} \rho_3^2 + b^2 \rho^2)}{N (c^2 k^2 \rho_{22}^2 + b^2)}$$

$$2 r \delta = \frac{c^3 k^3 b (\rho_3^2 - \rho \rho_{22})}{N (c^2 k^2 \rho^2_{22}{}^2 + b^2)}$$

$$B_1 = \frac{bi - \rho_{12} k c}{bi + \rho_{22} k c} \cdot B$$

where

$$\alpha = r + i\delta , \quad \rho_3^2 = \rho_{11} \rho_{22} - \rho_{12}^2 , \quad \rho = \rho_{11} + 2\rho_{12} + \rho_{22} .$$

Solving these two, we get two values for r, $\pm r_1$ say, and two values for δ, $\pm \delta_1$ say.

Let the displacement w in the layer be

$$w = \left(A_2 e^{-r_1 y} + A_3 e^{r_1 y} \right) e^{i \delta_1 y + ik(x - ct)}$$

and for the half space.

$$w = \left(A_1 e^{-r_2 y + i\delta_2 y + ik(x - ct)} \right)$$

where A_1, A_2, A_3 are arbitrary constants.

The displacement W for pore water is given by

$$W = e^{i \delta_1 y + ik(x - ct)} \frac{i b_1 - \rho'_{12} kc}{i b_1 + \rho'_{22} kc} \left(A_2 e^{-r_1 y} + A_3 e^{r_1 y} \right)$$

in the layer and $W = e^{i \delta_2 y + ik(x - ct) - r_2 y} \dfrac{ib_2 - \rho_{12}'' kc}{ib_2 + \rho_{22}'' kc} A_1$

for the half space.

The boundary conditions to be satisfied are :

(*i*) w must be continuous across the interface,

(*ii*) σ_{zy} is continuous across the interface and σ_{zy} at the free surface must be zero.

From the first condition, we get

$$A_2 + A_3 = A_1 \tag{1}$$

The second condition gives

$$N_1 \left[A_2 (- r_1 + i\, \delta_1) + A_3 (r_1 + i\, \delta_1) \right] = N_2 \left[A_1 (-r_2 + i\, \delta_2) \right] \tag{2}$$

The third condition gives

$$\left[A_2 (- r_1 + i\, \delta_1) \, e^{r_1 H} + A_3 (r_1 + i\, \delta_1) \, e^{- r_1 H} \right] = 0 \, . \tag{3}$$

For a non-trivial solution of (1), (2) and (3), we must have,

$$\begin{vmatrix} -1 & 1 & 1 \\ N_2 (r_2 - i\, \delta_2) & N_1 (- r_1 + i\, \delta_1) & N_1 (r_1 + i\, \delta_1) \\ 0 & (- r_1 + i\, \delta_1) \, e^{r_1 H} & (r_1 + i\, \delta_1) e^{-r_1 H} \end{vmatrix} = 0 \tag{4}$$

which reduces to

$$\tanh \, (r_1 H) = - \frac{N_2 (r_2 - i\, \delta_2) \, r_1}{[N_1 (r_1^2 + \delta_1^2) - i\, \delta_1 \, N_2 (r_2 - i\, \delta_2)]} \tag{5}$$

Separating real and imaginary parts, two equations in c and k are obtained.

Particular cases

(*1*) When dissipative nature of the media is negligible

In this case $b_1 = b_2 = 0$, then $\delta = 0$ and $r = \pm k \sqrt{1 - \frac{c^2 \rho_3^2}{N \rho_{22}}}$

One of the equations from (5) is identically satisfied and then second reduces to,

$$\tanh \, (r_1 H) = - \frac{N_2 \, r_2}{N_1 \, r_1} \, . \tag{6}$$

In this case there is only one equation (6) connecting c and k. Hence for a given value of c in (c_1, c_2) k can be determined.

where

$$c_1^2 = \frac{N_1 \, \rho_{22}'}{\rho_3'^2} \text{ and } c_2^2 = \frac{N_2 \, \rho_{22}''}{\rho_3''^2} \, . \tag{7}$$

This interval is bigger than its counterpart of classical theory of elasticity. Equation (7) is in the same form as that of the classical theory of the elasticity, but for replacing the constants (c_1 and c_2). So the graphs for c and k are not given below.

(*2*) Additionally mass coupling effects are negligible

In this case, $\rho_{12}'' = \rho_{12}' = 0$. As we can get c and k in the same way as in the first particular case, the same conclusions are valid here also.

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