WATER VAPOUR CONTENT OF THE ATMOSPHERE IN RELATION TO SURFACE HUMIDITY

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The theoretical relationship between precipitable water vapour in the atmosphere & surface humidity has been investigated. By introducing the concept of a lapse parameter α, a method has been devised for estimation of precipitable water vapour. Results have been compared for six Indian Stations for which upper air data were available.

Atmospheric water vapour plays an important role in all thermodynamic processes in the atmosphere, particularly in the absorption of radiation, both solar and terrestrial. Water vapour has absorption bands in the short infra-red spectrum of solar radiation, and much stronger absorption bands in the long infra-red spectrum of terrestrial radiation. Quantitative estimate of the total water vapour content of the atmosphere is essential for the study of atmospheric heat balance, specially at the earth-air interface. Besides its obvious importance in agricultural meteorology, it is also needed for the study of 'atmospheric turbidity', which is responsible for the variation in total attenuation of solar radiation during its passage through the atmosphere.

The water vapour content of the atmosphere between any two isobaric layers is, as a rule, expressed in terms of 'Precipitable water', which is simply the depth of liquid water which would be formed if the vapour contained in a vertical column of air of uniform cross section between the two pressure levels, were all condensed. The precipitable water W_1^2, between pressure levels, p_1 and p_2, is usually expressed in cm or in gm/cm^2, assuming density of water to be 1 gm/cm^3. Total precipitable water in the entire atmosphere above pressure level p_1 can be obtained with $p_2 = 0$.

Very little work seems to have been carried out so far, for reliable estimation of W in respect of Indian stations. Mukerjee[1] has estimated W by the 'Summation method', based on radiosonde data, from ground surface upto the highest level of observation, i.e. 600 mb. Anantha Krishna et al[2] have extrapolated the observed data upto 200 mb level. Apart from the high degree of uncertainty associated with the method of extrapolation used, the data used were not considered to be sufficiently reliable in view of poor performance at sub-zero temperatures, of humidity sensors used in those days in radiosonde equipment. Moreover, the highest level of observation being only 600 mb, estimation of total precipitable water based on such data cannot be relied upon. Robinson[3] has also stated that water content soundings are, in general, not as reliable as pressure and temperature soundings. Things have, however, improved since then, and data are now available upto 400 or 350 mb in respect of quite a number of Indian stations.

In the present paper, an attempt has been made to investigate the theoretical relationship between precipitable water and surface humidity, with a view to exploring the possibilities of using the latter for estimating the former. This was considered worth while because upper-air data are not available for a very large number of Indian stations.

THEORETICAL DERIVATION OF THE RELATIONSHIP BETWEEN PRECIPITABLE WATER AND SURFACE HUMIDITY

Both atmospheric pressure, p, and partial vapour pressure, e, are found to fall almost exponentially with height, although the latter falls at a relatively faster rate, so that more than 99% of the total water vapour content is confined within the troposphere. We have, therefore, defined a *lapse parameter*, α, by the following equation

$$\frac{de}{e} = \alpha \frac{dp}{p} \qquad (1)$$

which means that the relative change in vapour pressure is α times that in atmospheric pressure. α', thus represents the vertical moisture profile relative to total pressure.

Integration of eqn. (1) yields

$$\log e = \alpha \log p + c \quad (2)$$

or

$$e_2/e_1 = (p_2/p_1)^{\alpha} \quad (3)$$

where p_1 and p_2 are two specified pressure levels, and e_1 and e_2 are the corresponding vapour pressures.

Equation (2) shows that log e plotted against log p should give a straight line with slope 'α'. Fig. 1 gives such a plot in respect of six Indian stations, based on upper air data[4]. The stations and the months were so chosen as to cover a wide range of humidity and pressure levels. It will be seen that α is not a constant but varies between 2·90 and 3·83.

Now, precipitable water, dW, within a layer of thickness dh at a height h is given by

$$dW = \rho_\omega \, dh \quad (4)$$

where ρ_ω is the density of water vapour in that layer.

The pressure drop dp across the layer is

$$dp = -\rho g \, dh = -(\rho_a + \rho_\omega) g \, dh$$

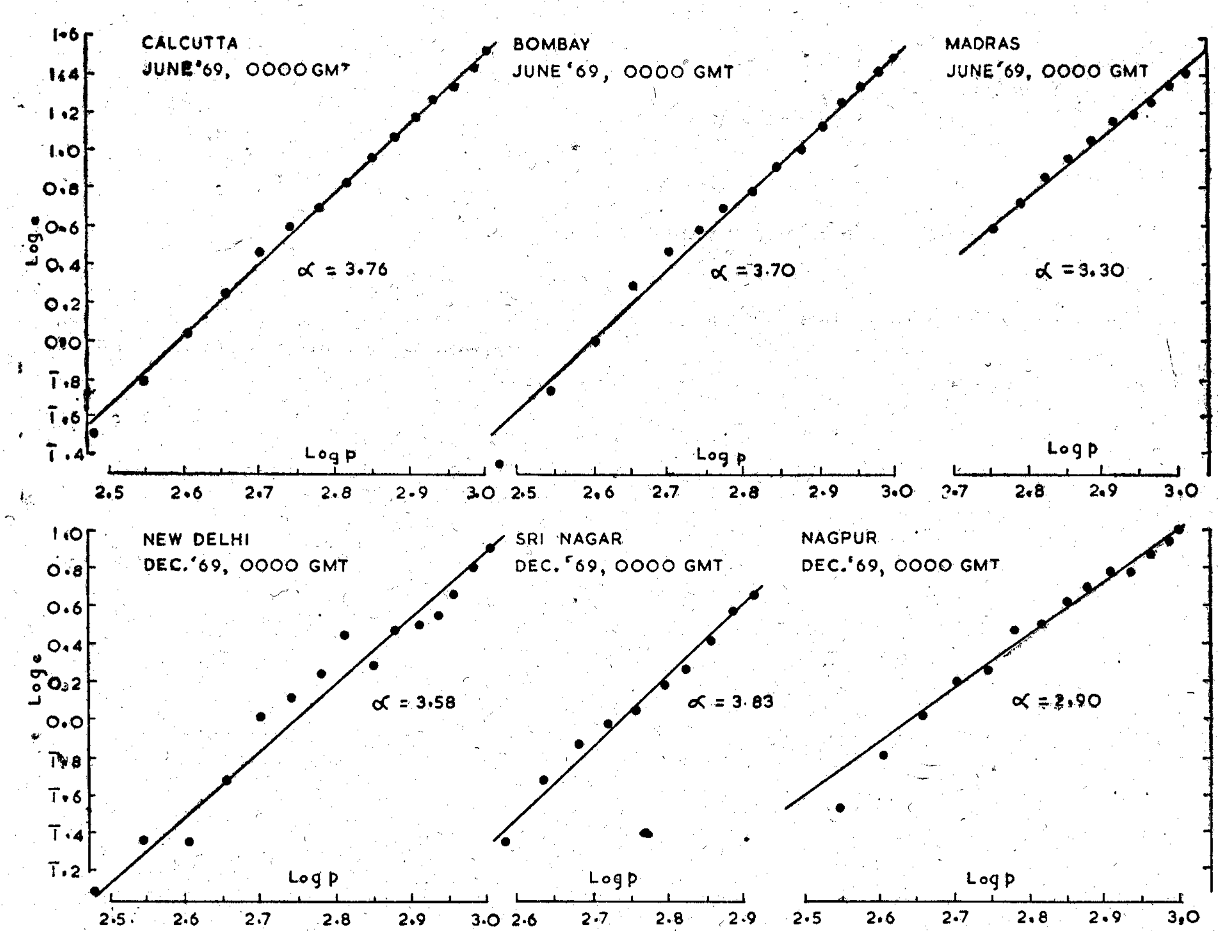

Fig. 1 Relationship between the logarithms of vapour pressure e_{mp} and barometric pressure P_{mb} for six Indian stations based on upper air data (slope = α).

where ρ is the density of moist air and ρ_a, that of dry air at the same height and g is the acceleration due to gravity. Hence,

$$dh = -\frac{dp}{g(\rho_a + \rho_\omega)}$$

Substituting this value in eqn. (4), we obtain

$$dW = -\frac{1}{g}\left(\frac{\rho_\omega}{\rho_a + \rho_\omega}\right) dp \tag{5}$$

Since under practical circumstances, e is very small compared to p, even at ground level, the perfect gas equation may be reasonably applied to dry air as well as the vapour component. Thus we can write,

$$p - e = \rho_a R_a T \tag{6}$$

and

$$e = \rho_\omega R_\omega T \tag{7}$$

where $(p - e)$ is the partial pressure of dry air, T, the absolute temperature, and R_a and R_ω are the gas constants for dry air and water vapour respectively. Dividing eqn (6) by eqn (7), we obtain

$$\left(\frac{p - e}{e}\right) = \frac{\rho_a}{\rho_\omega} \cdot \frac{R_a}{R_\omega},$$

whence,

$$\left(\frac{\rho_\omega}{\rho_a + \rho_\omega}\right) = \frac{0.622e}{p - 0.378e} \tag{8}$$

This follows from the fact that $R_a/R_\omega = M_\omega M_a = 0{\cdot}622$

where M_ω is the molecular weight of water vapour (18·016) and M_a is the effective molecular weight of dry air (28·97).

From a scrutiny of climatological data for all Indian Stations[5], it is observed that 0·378 e at ground level never exceeds 1·7% of the total pressure even under extreme conditions, the value decreasing with height. No significant error will, therefore, be introduced by simplifying equation (8) to the form

$$\frac{\rho_\omega}{\rho_a + \rho_\omega} = \frac{0.622\,e}{p}$$

Substitution of this value in eqn. (5) gives,

$$dW = -\frac{0.622\,e}{g} \cdot \frac{dp}{p} \tag{9}$$

However, in meteorological practice, pressures are always expressed in millibars (1 $mb = 10^3$ dynes/cm^2). Since g varies very little within the troposphere, a constant value of 980 cm/sec^2 may be assumed without any significant error.

Equation (9), thus reduces to

$$dW = 0.635\,\frac{e\,(mb)}{p} \cdot dp \tag{10}$$

From (3), we have

$$e = e_1\,(p/p_1)^a \tag{11}$$

where e_1 and p_1 correspond to the initial level. Substitution of this value in (10), gives

$$dW = \frac{0\cdot635\, e_1\,(mb)}{p_1^{\alpha}} \cdot p^{\alpha-1}.\, dp$$

Integration of this equation between limits p_1 and p_2 gives the precipitable water, W_1^2, between these two levels, so that

$$W_1^2 = \frac{0.635\, e_1\,(mb)}{\alpha}\left[1-(p_2/p_1)^{\alpha}\right] \quad (12)$$

The total precipitable water, W, in the entire atmosphere above the ground surface, can be obtained by putting, $p_1 = p_0$ and $e_1 = e_0$ at the ground surface, and $p_2 = 0$.

Hence

$$W = \frac{0.635}{\alpha} \cdot e_0\,(mb) \quad cm \quad (13)$$

and, the precipitable water, W_p, from ground level upto any specified level, p, will be given by

$$W_p = \frac{0\cdot635\, e_0\,(mb)}{\alpha}\left[1-(p/p_0)^{\alpha}\right] \quad (14)$$

In order to avoid computational work, two nomograms have been prepared and presented in Figures 2 and 3. The former gives total precipitable water W for given values of ground vapour pressure, e_0, and lapse parameter, α. The latter gives the correction factor $[1-(p/p_0)^{\alpha}]$ for given values of p/p_0 and α in order to estimate W_p upto to the highest level of observation.

VALIDATION OF PROPOSED METHOD

The upper air data of six Indian stations, based on which Fig. 1 has been prepared, have been utilised for testing the validity of the proposed method of estimation of precipitable water. For each station, the lapse parameter, α, is given by the slope of the corresponding line in Fig. 1. Then, with the help of the two nomograms in Figs. 2 and 3, W_p was estimated upto the highest level of observation.

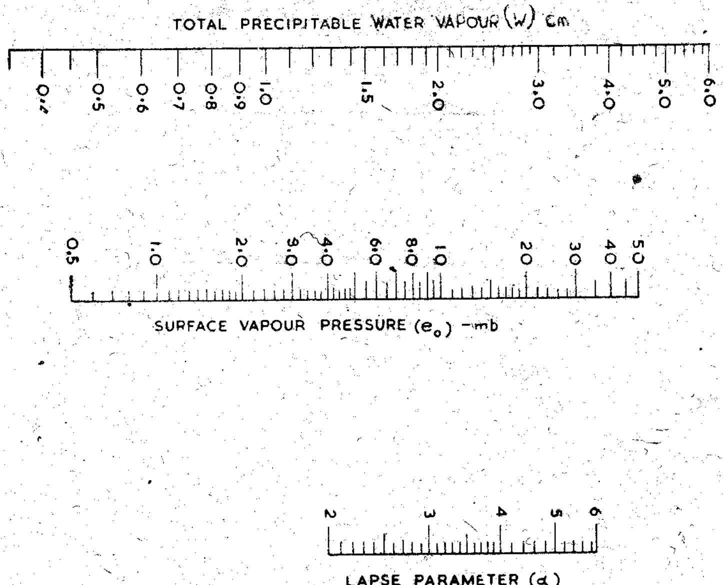

Fig. 2—Nomogram for estimation of total precipitable water vapour (W from surface vapour pressure e_0 and lapse parameter α).

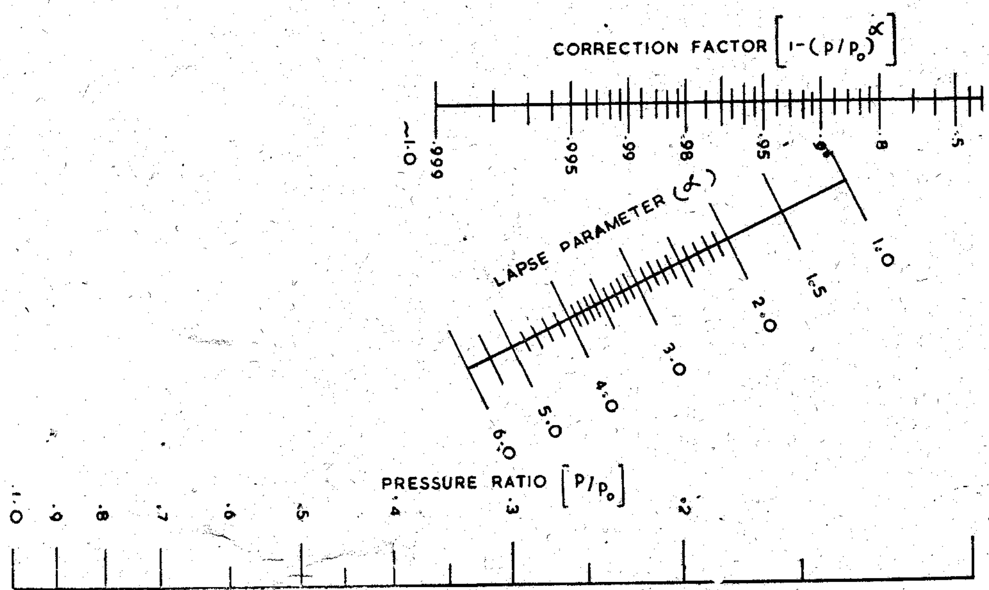

Fig. 3—Nomogram for evaluation of correction factor for precipitable water vapour W from ground pressure level p. to any pressure level p.

W_p was also independently computed from the same data by the conventional summation method[1]. According to this method, the local atmosphere is visualised as made up of n finite layers defined by isobaric levels, p_o (ground), $p_1, p_2 \ldots\ldots\ldots\ldots p_n$. Then, to a first approximation, the precipitable water, W_r, in the rth layer, according to equation (10) will be given by

$$W^r_{r-1} = 0\cdot635 \left[\overline{(e/p)}_r \, (p_{r-1} - p_r) \right] \tag{15}$$

where $\overline{(e/p)}_r$, the average value of e/p in the rth layer,

$$= \frac{1}{2} \left(\frac{e_{r-1}}{p_{r-1}} + \frac{e_r}{p_r} \right)$$

Hence total precipitable water, W_0^n, will be given by

$$W_0^n = 0\cdot318 \sum_{r=1}^{n} (e_{r-1}/p_{r-1} + e_r/p_r) (p_{r-1} - p_r) \tag{16}$$

Here, the last or the nth layer is bounded above by the highest level of observation, so that p_n is p, and W_0^n is W_p as given by eqn. (14).

Values of precipitable water estimated from surface vapour pressure according to the proposed method have been compared with those computed by the 'summation method' described above, in Fig. 4, from which one may be able to judge the closeness of agreement.

DISCUSSION

It will be appreciated that the introduction of the concept of lapse parameter, α, amounts to an idealisation of the real atmosphere. Actually, the vertical moisture profile has been found to depart considerably from the assumed picture in a number of cases. Even then, it has been found possible to estimate precipitable water from surface humidity with reasonable accuracy from the mean slope of the graph between log e and log p. It follows, therefore, that even if the lapse parameter varies from layer to layer, α, as defined in this paper, should serve the purpose of estimating precipitable water, being an integrated average for the whole atmosphere.

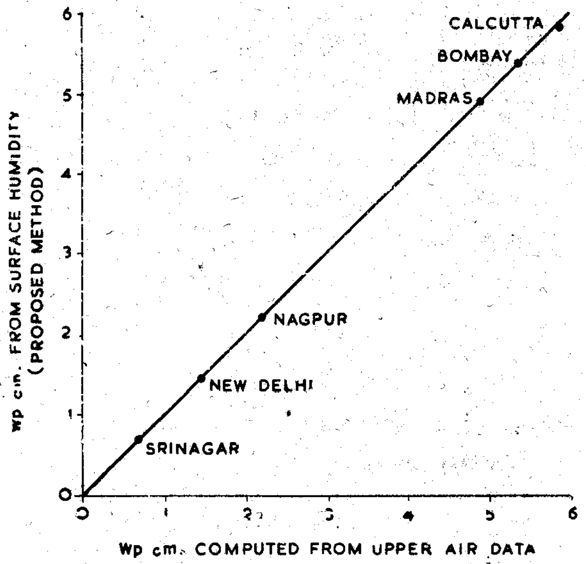

Fig. 4—Comparison between proposed method for estimation of precipitable water *Wp* Cm from surface humidity and computed value from upper air data by conventional summation method.

Attempts have been made in the past to estimate precipitable water from surface humidity. For example, Hann's formula, namely, $W=0\cdot25\ e_0^{m\,b}$ is still being used[3]. This formula is based on the extrapolation of the ground water vapour pressure $e_0^{m\,b}$ and gives the total water content of the atmosphere, W, in cm. of precipitable water vapour. This formula may be subject to systematic errors of more than 30% at same places. According to (13), Hann's formula assumes a constant value of 2·54 for α which is much lower that the values obtained from Fig. 1., and should, therefore, yield much higher values of W.

Charts are also available[6], from which precipitable water can be calculated as a function of dew point temperature at 1000 mb. These are based on the assumption of a pseudo-adiabatic lapse rate in the atmosphere. Values obtained from these charts are also much higher than those computed from upper air data.

Reitan[7] has proposed an empirical formula for prediction of precipitable water from surface dew point. This also suffers from similar drawbacks as discussed above in so far as it fails to consider the variation of α and yields much higher values than those obtained by actual computation from upper air data.

It appears, therefore, that the method proposed in this paper is likely to give more reliable information, particularly because, α has been found to vary between 2·90 and 3·83 for the six stations investigated so far.

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