# SHOCK STRENGTH IN A THERMALLY RADIATIVE GAS

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The strength of the shock in a thermally radiative gas has been calculated for the steady flow of gas under the condition that the gradient of total pressure vanishes on the flow side of the shock.

Verma and Mishra[1] have derived jump relations for thermally radiative shocks in plasma. Using compatibility conditions of Thomas[2], propagation of thermally radiative shocks has been discussed by Ojha & Verma[3] under certain boundary conditions. The strength of the shock has been derived by many authors e.g., Kanwal[4], Pant & Mishra[5] in the presence of dissipative and non-dissipative mechanisms and in absence of thermal radiation. Our aim in this paper is to derive the density shock strength in a thermally radiative gas. For simplicity we have taken steady flow of gas and stationary normal shocks in course of derivation. We have assumed that the gradient of total pressure vanishes on the flow side of the shock so that $\frac{\partial p^*}{\partial^n} = 0$ and Thomas[2] compatibility conditions have been used.

## SHOCK CONDITIONS AND STRENGTH OF THE SHOCK

Equations of conservation of mass and momentum for steady optically thick radiative gas flow (Pai[6]) using Thomas[2] equations (7 & 9) yield

$$\rho\lambda_n + u_n \xi + A = 0 \tag{1}$$

$$\rho u_n \lambda_i + \mu n_i + g^{\alpha\beta} [p^*]_{,\alpha} x_{i,\beta} + B_i = 0 \tag{2}$$

where

$$\xi = [\rho, i]\, n_i\, ; \;\; \lambda_i = [u_i, j]\, n_j\, ; \;\; \mu = [p^*, i]\, n_i \tag{3}$$

$$A = u^\alpha \rho_{,\alpha} + g^{\alpha\beta} x_{i,\beta} u_{i,\alpha} \tag{4}$$

and

$$B_i = \rho u_j g^{\alpha\beta} u_{i,\alpha} x_{j,\beta} = \rho u^\alpha u_{i,\alpha} \tag{5}$$

The shock conditions across the shock sufrace

$$x_i = x_i\,(y^\alpha)\;\;(i = 1, 2, 3\,;\;\alpha = 1, 2)$$

are

$$[\rho\, u_n] = 0 \tag{6}$$

$$\rho_1 u_{1n} [u_i] = -\left[ \quad \frac{aT^4}{3} \right] n_i \tag{7}$$

$$\left[ \rho u_n \left( \tfrac{1}{2} u^2 + h + \frac{4aT^4}{3\rho} \right) \right] = [K\, T, ii] + [F_i\, n_i] \tag{8}$$

where

$$h = \frac{\gamma p}{\rho(\gamma - 1)} \tag{9}$$

and $x_i$ are cartesian co-ordinates, $y^a$ are curvilinear co-ordinates and a comma (,) indicates partial differentiation with respect to curvilinear co-ordinates. Assume that the shock strength $\phi$ is defined by

$$\phi = \frac{[\rho]}{\rho_1}$$

Then we have

$$[u_i] = -\frac{\phi}{(1+\phi)} u_{1n} n_i \tag{11}$$

$$\left[ p + \frac{aT^4}{3} \right] = \frac{\phi}{(1+\phi)} \rho_1 u_{1n}{}^2 \tag{12}$$

From equation of state, we have

$$[p, i] = R\ \{T\, [\rho, i] + \rho\, [T, i]\}$$

Thus

$$[p^*, i]\, n_i = R\, \{T\, [\rho, i] + \rho\, [T, i]\}\ n_i + \frac{4aT^3}{3} [T,_i] n_i$$

By using the boundary condition $\frac{\partial p^*}{\partial n} = 0$ as well as the equation (1), (2), (4) and (5) we get

$$[T, i]\, n_i = \frac{T}{\rho u_n (1 + 4R_p)} \left\{ \qquad \frac{\rho u^a}{u_n} u_{i,a}\, n_i \right\}$$

where

$$R_p = \frac{aT^4}{3p}$$

Also we can write (8) in the form

$$\left[ \rho u_n \left( \tfrac{1}{2} u^2 + h + 4 \frac{aT^4}{3\rho} \right) \right] = K^* [T, i] n_i$$

where

$$K^* = \left( K + \frac{4aDT^3}{3} \right)$$

and is called the effective condutivity due to radiation. Eliminating $[T,i]\, n_i$ between (15) and (16) we get

$$\left[ \rho u_n \left( \tfrac{1}{2}\, u^2 + h + \frac{4aT^4}{3\rho} \right) \right] = \frac{K^* T \left\{ u^a \rho_{,a} + \rho g^{a/\beta}\, u_{i,a}\, x_{i,\beta} - \dfrac{\rho u^a}{u_n}\, u_{i,a}\, n_i \right\}}{\rho u_n \,(1 + 4R_p)}$$

which is a first order differential equation in $\rho$ and $y^a$ that can be solved in principle. For simplicity consider the stationary normal shock for which, $u^a = 0$

$$u_{i,a} = [u_i]_{,a} = \frac{-\phi}{(1+\phi)}\, u_{1n}\, n_{i,a} - \left( \frac{\phi}{1+\phi} u_{1n} \right)_{,a} n_i \tag{19}$$

$$T = T_2 = \frac{[p] + p_1}{R\,(1+\phi)\, \rho_1} = \frac{\dfrac{1}{(1+R_p)} \left( \dfrac{\phi}{1+\phi} \right) \rho_1\, u_1{}^2{}_n + p_1}{(1+\phi)\, \rho_1\, R}$$

$$\left[ \frac{p}{\rho} \right] = \frac{1}{(1+R_p)}\, \frac{\phi}{(1+\phi)^2}\, u_1{}^2{}_n - \frac{\phi}{(1+\phi)}\, \frac{p_1}{\rho_1}$$

and

$$u_n = u_{2n} = \frac{u_{1n}}{(1+\phi)}$$

Also, we have

$$[\tfrac{1}{2}\, u^2] = \tfrac{1}{2} \{ [u_i]\, [u_i] + 2u_{1i}\, [u_i] \} = \frac{-\phi(2+\phi)}{2\,(1+\phi)^2}\, u^2{}_{1n}$$

Now making use of (18) to (23), we get

$$\phi = \frac{\left\{ \dfrac{K^* g^{a\beta}\, x_{i,\beta}\, n_{i,a}}{\rho_1\, u_{1n}\,(1 + 4\,R_p)}\, \dfrac{C_1{}^2}{NR} + \dfrac{N u_1{}^2{}_n}{1 + R_p} - u_1{}^2{}_n - C_1{}^2 \right\}}{\left\{ \dfrac{-K^* g^{a\beta}\, x_{i,\beta}\, n_{i,a}}{\rho_1\, u_{1n}\,(1 + 4R_p)} \left( \dfrac{u_1{}^2 n}{R\,(1 + R_p)} + \dfrac{C_1{}^2}{NR} \right) + (\tfrac{1}{2}\, u^2{}_{1n} + C_1{}^2 \right\}}$$

where

$$N = \frac{\gamma}{(\gamma - 1)} + 4R_p$$

$$C_1{}^2 = \frac{N p_1}{\rho_1}$$

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