# ON BALLISTICS SOLUTION OF H/L PRESSURE GUN FOR COMPOSITE CHARGES TAKING MOST GENERAL FORM FUNCTION

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The present paper gives the solution of the equations of internal ballistics of H/L gun for composite charges taking most general form function assuming constant pressure in both the chambers during the second stage of burning, i.e., after the burning of the first component charge.

A low peak pressure in the gun is desirable from the point of view of the projectile design as well as overall weight of the gun. By keeping low pressure in a gun the weight of the metal casing of the projectile can be decreased and more explosive can be carried in the projectile for a given total weight, thus a greater lethal effect can be produced.

The ignition difficulty of the propellant in low pressure weapons and the round to round variations of muzzle velocity were removed by the development of high-low pressure guns, by German Engineers in World War II. In these the propellants are made to burn in the main chamber under high pressure and the gases are passed on to the second chamber and the bore through nozzles. Thus by suitably choosing the nozzle area, the ignition and regularity can be improved while the projectile is exposed to much lower pressure.

The internal ballistics of high-low pressure gun was discussed by Aggarwal [1], Corner [2,3] and Kapur[4] for the general form function. Recently, Bhattacharyya[5] had discussed the attainment of constant pressures in both the chambers in an H/L gun with moderated charges during the second stage of burning. In the present paper, the author tries to solve the internal ballistics of H/L gun with composite charges having two components taking most general form function, pressures in both the first and second chamber in the H/L gun have been assumed to remain constant during the second stage of burning. The constant pressures being equal to the pressures at burnt during the first stage. The internal ballistics during the first stage of burning are assumed to be known and those for the second stage of burning are determined. The conditions for the attainment of constant pressures determine two relations between the four characteristics of the second propellant component of which two may be known from the physical properties of the propellant so that the other two may be calculated. The solution given by[6] can be derived as a particular case of the present solution. The solution for composite charges taking tabular form functions has also been derived.

## NOTATIONS

The following notations have been used in this paper.

$A$ = Bore-area,

$c$ = Mass of the propellant charge,

$D$ = Initial web-size,

$f$ = Fraction of the web-size remaining at time $t$,

$K$ = Volume of the first chamber containing the charge,

$K_0$ = Volume of the second chamber,

$P_1$ = Pressure of the gases in the first chamber taken to be uniform at the instant $t$,

$P_2$ = Pressure of the gases in the second chamber at the instant $t$,

$R$ = Gas constant per gram of the gas,

$S$ = Throat area of the venturi or nozzles connecting the two chambers,

$v$ = Shot velocity at the instant $t$,

$W$ = The effective shot weight,

$x$ = Shot travel at the instant $t$,

$z$ = Fraction of the charge burnt up to time,

$\beta$ = Rate of burning coefficient,

$\gamma$ = Ratio of the two specific heats of the propellant gases,

$\delta$ = Density of the propellant,

$\Psi$ = Leakage parameter which is a function of $\gamma$,

$cz$ = Mass of the charge burnt up to any instant $t$,

$cN$ = Amount of the gases in the first chamber at the instant $t$,

$c_1 z_1 + c_2 z_2$ = Amount of the two propellants burnt up to time $t$,

$(c_1 z_1 + c_2 z_2)N$ = Amount of the gases in the first chamber at the instant $t$,

## BALLISTICS EQUATIONS FOR COMPOSITE CHARGES

*First Stage of Burning*

The equation of state for gases in the first chamber

$$P_1 \left[ U_1 - \frac{c_1}{\delta_1} - \frac{c_2}{\delta_2} + \frac{c_1 z_1}{\delta_1} + \frac{c_2 z_2}{\delta_2} - (c_1 + c_2) N\eta \right] = (c_1 + c_2) NRT_1 \qquad (1)$$

The equation of state for the gases in the second chamber

$$P_2 \left[ U_2 + Ax - (c_1 z_1 + c_2 z_2)\, \eta + (c_1 + c_2)\, N\eta \right] = \left[ c_1 \phi_1 + c_2 \phi_2 - (c_1 + c_2)\, N \right] RT_2 \qquad (2)$$

The equation of continuity (when $\omega < \omega^*$)

$$(c_1 + c_2) \frac{dN}{dt} = c_1 \frac{dz_1}{dt} + c_2 \frac{dz_2}{dt} - \frac{\bar{\psi}\, SP_1}{\sqrt{RT_1}} \qquad (3)$$

where

$$\bar{\psi} = \left( \frac{2\gamma}{\gamma - 1} \right)^{\frac{1}{2}}$$ and $N$ is the fraction of the total charge turned into gas.

The law of burning is

$$Di \frac{df_i}{dt} = -\beta_i P_1 \qquad (i = 1, 2) \qquad (4)$$

The most general form function is

$$z_i = \phi_i (f_i) \qquad (i = 1, 2) \qquad (5)$$

The equation of energy for the first chamber :

$$\frac{d}{dt} \left[ (c_1 + c_2) NT_1 \right] = T_0 \left( c_1 \frac{dz_1}{dt} + c_2 \frac{dz_2}{dt} \right) - \gamma T_1 \frac{d}{dt} \left[ c_1 z_1 + c_2 z_2 - (c_1 + c_2) N \right] \qquad (6)$$

The equation of energy for the second chamber :

$$\frac{d}{dt}\left[\left\{c_1 z_1 + c_2 z_2 (c_1 + c_2) N\right\} T_2\right] = \gamma\, T_1 \frac{d}{dt}\left[c_1 z_1 + c_2 z_2 - (c_1 + c_2) N\right] \tag{7}$$

The equation of motion of the shot is

$$W \frac{dv}{dt} = AP_2 \tag{8}$$

These equations have been solved with initial conditions

$$P_1 = P_2 = T_1 = T_2 = N = 0 = v \text{ when } f_1 = f_2 = 1 \text{ ; i. e. } z_1 = z_2 = 0$$

We suppose that this solution gives

$$P_1 = P_{1B1}\,,\ P_2 = P_{2B1}\,,\ T_1 = T_{1B1}\,,\ T_2 = T_{2B1}\,,\ N = N_{B1}\,,\ v = v_{B1} \text{ and } x = x_{B1}$$

When the first component is completely burnt i.e., when $f_1 = 0$. From (4) the value of $f_2$ at burnt of the first component,

$$f_{2B1} = 1 - \frac{D_1/\beta_1}{D_2/\beta_2} = \left(1 - \frac{1}{\alpha_0}\right)$$

where

$$\alpha_0 = \frac{D_2/\beta_2}{D_1/\beta_1} \tag{9}$$

*Second Stage of Burning*

The equation of state for the gases in the first chamber

$$P_1\left[U_1 - \frac{c_2}{\delta_2} + \frac{c_2}{\delta_2} z_2 - (c_1 + c_2) N\eta\right] = (c_1 + c_2) NRT_1 \tag{10}$$

The equation of state for the gases in the second chamber

$$P_2\left[U_2 + Ax - (c_1 + c_2 z_2)\eta N + (c_1 + c_2) N\eta\right] = \left[c_1 + c_2 z_2 - (c_1 + c_2) N\right] RT_2 \tag{11}$$

The equation of continuity (when $\omega < \omega^*$)

$$(c_1 + c_2)\frac{dN}{dt} = c_2 \frac{dz_2}{dt} - \frac{\psi S P_1}{RT_1} \tag{12}$$

The equation of burning is

$$D_2 \frac{df_2}{dt} = -\beta_2 P_1 \tag{13}$$

The most general form function is

$$z_2 = \phi_2 (f_2) \tag{14}$$

The equation of energy for the first chamber is

$$\frac{a}{dt}\left[(c_1 + c_2) NT_1\right] = T_0 c_2 \frac{dz_2}{dt} - \gamma\, T_1 \frac{d}{dt}\left[c_2 z_2 - (c_1 + c_2) N\right] \tag{15}$$

The equation of energy for the second chamber is:

$$\frac{d}{dt}\left[\left\{c_1 + c_2 z_2 - (c_1 + c_2) N\right\} T_2\right] = \gamma T_1 \frac{d}{dt}\left[c_2 z_2 - (c_1 + c_2) N\right] \tag{16}$$

The equation of motion is

$$W \frac{dv}{dt} = AP_2 \tag{17}$$

SOLUTION OF THE EQUATIONS WITH DIFFERENT STAGES OF BURNING

We suppose that the solutions of the above equations are possible with

$$P_1 = P_{1B1}, \; P_2 = P_{2B1} \tag{18}$$

and find the conditions so that this solution may give

$$x = x_{B1}, \; v = v_{B1}, \; N = N_{B1}, \; T_1 = T_{1B1}$$

and

$$T_2 = T_{2B1} \quad \text{at } f_2 = f_{2B1}$$

The equations (10) to (17) may remain consistent for the solution $P_1 = P_{1B1}$ and $P_2 = P_{2B1}$

With (18), (13) and (17), we have

$$\frac{dv}{df_2} = -\left(\frac{A}{\omega}\right) \frac{D_2}{\beta_2} \frac{P_{2B1}}{P_{1B1}} \tag{19}$$

but by (13) and (18) becomes

$$\frac{d^2x}{df_2^2} = \frac{AD_2^2}{\beta_2^2 \, \omega \, P_{1B1}} \frac{P_{2B1}}{P_{1B1}} \tag{20}$$

Integrating (19) with the condition $v = v_{B1}$ at $f_2 = f_{2B1}$, we have

$$v = v_{B1} + \frac{A}{W} \frac{D_2}{\beta_2} \frac{P_{2B1}}{P_{1B1}} (f_{2B1} - f_2) \tag{21}$$

Now we put the condition that (10) and (11) with (18) gives

$$x = x_{B1}, \; T_1 = T_{1B1}, \; T_2 = T_{2B1}$$

and $N = N_{B1}$ at $f_2 = f_{2B1}$ and further (10) and (11) are consistent with (20).

Now $x = x_{B1}$, $T_2 = T_{2B1}$, $N = N_{B1}$ and $z_2 = x_{2N1}$ will satisfy (11) if

$$P_{2B1}\left[U_2 + Ax_{B1} - (c_1 + c_2 z_{2B1})\eta + (c_1 + c_2) N_{B1} \eta\right] = \left[c_1 + c_2 z_{2B1} - (c_1 + c_2) N_{B1}\right] RT_{2B1} \tag{22}$$

which is true, since (22) is obtained from (2) by considering values when the first component burns out.

Again $N = N_{B1}$, $T_1 = T_{1B1}$ and $z_2 = z_{2B1}$ will satisfy (10) if

$$P_{1B1}\left[U_1 - \frac{c_2}{\delta_2} + \frac{c_2}{\delta_2} z_{2B1} - (c_1 + c_2) N_{B1} \eta\right] = (c_1 + c_2) N_{B1} RT_{1B1} \tag{23}$$

which is true since (23) is obtained from (1) by considering values when the first component burns out.

With the help of (13), equations (12), (15) and (16) can be written as

$$(c_1 + c_2)\frac{dN}{df_2} = c_2 \frac{dz_2}{df_2} + \frac{D_2}{\beta_2} \frac{\bar{\psi} S}{\sqrt{RT_1}} \tag{24}$$

$$\frac{d}{df_2}\left[(c_1 + c_2) N T_1\right] = T_0 c_2 \frac{dz_2}{df_2} - \gamma T_1 \frac{d}{df_2}\left[c_2 z_2 - (c_1 + c_2) N\right] \tag{25}$$

and

$$\frac{d}{df_2}\left[\left\{c_1 + c_2 z_2 - (c_1 + c_2) N\right\} T_2\right] = \gamma T_1 \frac{d}{df_2}\left[c_2 z_2 - (c_1 + c_2) N\right] \tag{26}$$

Integrating (6), (7), (25) and (26), we get

$$\left\{c_1 + c_2 z_2 - (c_1 + c_2) N\right\} T_2 + (c_1 + c_2) N T_1 = T_0 (c_1 + c_2 z_2) \tag{27}$$

$$P_{1B1}\left[U_1 - \frac{c_2}{\delta_2} + \frac{c_2}{\delta_2} z_2 - (c_1 + c_2) N\eta\right] + P_{2B1}\left[U_1 + Ax - (c_1 + c_2 z_2)\eta + \right.$$
$$\left. + (c_1 + c_2) N\eta\right] = RT_0 (c_1 + c_2 z_2) \tag{28}$$

Differentiating (28) and (14) with respect to $f_2$, with the help of (13), (18) and (24), we get

$$\frac{-AP_{2B1}\, vD_2}{\beta_2 P_{1B1}} + \eta \frac{P_{2B1}\bar{\psi} SD_2}{\beta_2 RT_1} + \frac{c_2}{\delta_2} P_{1B1}\phi'_2(f_2) -$$
$$-\eta P_{1B1}\left\{c_2 \phi_2(f_2) + \frac{SD_2}{\beta_2\sqrt{RT_1}}\right\} = RT_0 c_2 \phi'_2(f_2) \tag{29}$$

Now $v = v_{B1}$, $T_1 = T_{1B1}$ and $f_2 = f_{2B1}$ will satisfy (29) if

$$Av_{B1}\frac{D_2}{\beta_2}\frac{P_{2B1}}{P_{1B1}} + \left[\phi^1_2(f_2)\right]_{f_2 = f_{2B1}}\left[RT_0 c_2 + c_2 P_{1B1}\left(\eta - \frac{1}{\delta_2}\right)\right] =$$
$$= -\frac{D_2}{\beta_2}\frac{\psi\eta S}{\sqrt{RT_{1B1}}}(P_{1B1} - P_{2B1}) \tag{30}$$

Introducing the following dimensionless constants

$$\left.\begin{aligned} &\frac{c_2}{c_1} = \beta_0, \quad \frac{D_2/\beta_2}{D_1/\beta_1} = \alpha_0, \quad \psi = \frac{\bar{\psi} SD_1}{\beta_1 c_1 RT_0}, \\ &\eta_{B1} = \frac{v_{B1}/AD_1}{c_1\beta_1 RT_0}, \quad \frac{P_{2B1}}{P_{1B1}} = \omega_{B1}, \quad \frac{T_{1B1}}{T_0} = T'_0 \\ &\eta\frac{P_{1B1}}{RT_0} = v_0, \left(\eta - \frac{1}{\delta_2}\right)\frac{P_{1B1}}{RT_0} = \delta_0 \end{aligned}\right\} \tag{31}$$

we get

$$\left[\phi'_2(f_2)\right]_{f_2 = f_{2B1}} = -\frac{\alpha_0}{\beta_0}\frac{\gamma_0\psi(1 - \omega_{B1})/\sqrt{T'_0} + \eta_{B1}\omega_{B1}}{1 + \delta_0} \tag{32}$$

For the consistency of (20) and (28), differentiating (28) twice w.r.t. $f_2$ and with the help of the equation (14), (23) and (20) we get

$$\frac{A^2_2\, D^2_2\, P^2_{2B1}}{\beta^2_2\, \omega\, P^2_{1B1}} - \frac{\eta P_{2B1}\, D_2\, \psi\, S}{2\beta_2\sqrt{R}\, T_1^{3/2}}\, \frac{dT_1}{df_2} + \phi''_2(f_2)\, \frac{c_2}{\delta_2}\, P_{1B1} - \eta P_{1B1} \cdot \left\{ \phi''_2(f_2) c_2 - \frac{D_2 \overline{\psi} S}{2\beta_2\sqrt{R}\, T_1^{3/2}}\, \frac{dT_1}{df_2} \right\} = \phi''_2(f_2)\, RT_0\, c_2$$

or

$$\phi''_2(f_2)\, c_2 \left[ RT_0 + \left( \eta - \frac{1}{\delta_2} \right) P_{1B1} \right] = \frac{A^2\, D^2_2\, P^2_{2B1}}{\beta^2_2\, \omega\, P^2_{1B1}} + \frac{\eta D_2 \overline{\psi} S}{2\beta_2\sqrt{R}\, T^{3/2}_1} (P_{1B1} - P_{2B1})\, \frac{dT_1}{df_2} \tag{33}$$

Also from (23) and (24), we have

$$\frac{dT_1}{df_2} = \frac{1}{(c_1 + c_2)\, N} \left[ c_2 (T_0 - T_1)\, \phi'_2(f_2) + \frac{D_2}{\beta_2}\, \frac{\overline{\psi} S (\gamma - 1)}{\sqrt{R}}\, (T_1)^{1/2} \right] \tag{34}$$

For $T_1 = T_{1B1}$, $N = N_{B1}$ and $f_2 = f_{2B1}$ will satisfy (33) and (34) we get

$$\phi''_2(f_2)\, c_2 \left[ RT_0 + \left( \eta - \frac{1}{\delta_2} \right) P_{1B1} \right] = \frac{A^2\, D^2_2\, P^2_{2B1}}{\beta^2_2\, \omega\, P^2_{1B1}} + \frac{\eta D_2\, \overline{\psi}\, S\, (P_{1B1} - P_{2B1})}{2\beta_2\sqrt{R}\, T^{3/2}_{1B1}\, (c_1 + c_2)\, N_{B1}} \cdot \left[ \frac{D_2}{\beta_2}\, \frac{\overline{\psi} S (\gamma - 1)}{\sqrt{R}} \left( T_{B1} \right)^{\frac{1}{2}} + c_2 (T_0 - T_{1B1}) \left\{ \phi'_2(f_2) \right\}_{f_2 = f_{2B1}} \right] \tag{35}$$

Introducing the ballistic parameter

$$M = \frac{A^2\, D^2_1}{\beta^2_1\, \omega\, c_1\, RT_0}$$

From (31), the equation (35) can be written in the non-dimensional form as :

$$\phi''_2(f_2) \cdot (1 + \delta_0) = M_1\, \frac{2\alpha^2_0\, \omega^2_{B1}}{\beta_0} + \frac{\gamma_0\, \psi\, \alpha_0\, (1 - \omega_{B1})}{2(1 + \beta_0)\sqrt{T'_0}\, N_{B1}} \cdot \left[ \left\{ - \phi'_2(f_2) \right\}_{f_2 = f_{2B1}} \left( 1 - \frac{1}{T'_0} \right) + \frac{\psi \alpha_0 (\gamma - 1)}{\beta_0 \sqrt{T_0'}} \right] \tag{36}$$

The consistency of (32) and (36) gives the condition that $P_1 = P_{1B1}$ and $P_2 = P_{2B1}$ may be the solution of (10) to (17). The connection between the four parameters $\alpha_0$, $\beta_0$, $\phi_2$ and $\delta_0$ defining the second propellant component is clear from the equations (32) and (36).

The properties and mass of the first component are assumed to be known. $\delta_0$ involves $\eta$ and $1/\delta_2$ and $\beta_0$ involves $c_2$ which are supposed to be known in the integration of equation for the first stage of burning. Thus (32) and (36) give the size and shape of the second propellant component provided $\alpha_0$ is positive.

If $\eta = 1/\delta_2 \approx 0$ then $\gamma_0 = \delta_0 = 0$, (32) and (36) reduce to,

$$\left[ \phi'_2(f_2) \right]_{f_2 = f_{2B1}} = - \frac{\alpha_0}{\beta_0}\, \eta_{B1}\, \omega_{B1}$$

and

$$\phi''_2(f_2) = M_1\, \frac{\alpha^2_0 \cdot \omega^2_{B1}}{\beta_0}$$

Let

$M_2 = \dfrac{A^2 D^2_2}{\beta^2_2 \, \omega \, c_2 \, RT_0} = \dfrac{M_1 \alpha^2_0}{\beta_0}$ be the central ballistic parameter corresponding for the second component charge.

Then

$$\phi''_2 (f_2) = M_2 \, \omega^2_{B1}$$

Thus within the practical range of values of $M_2$ and $\omega_{B1}$ , $\alpha_0$ may be positive. From (21)

$$v_{B2} = v_{B1} + \frac{A}{\omega} \; \frac{D_2}{\beta_2} \; \frac{P_{2B1}}{P_{1B1}} \; f_{2B1} \tag{37}$$

then

$$\frac{v_{B2}}{v_{B1}} = 1 + \frac{M_1 \alpha_0}{\eta_{B1}} \; \omega_{B1} \, f_{2B1}$$

which is the velocity ratio and its value is less than the velocity ratio for the moderated charges.

From (20), (13) and (18), we get

$$\frac{-dx}{df_2} \; \frac{\beta_2 P_{1B_1}}{D_2} = v_{B1} + \frac{A}{\omega} \; \frac{D_2}{\beta_2} \; \frac{P_{2B1}}{P_{1B1}} \; (f_{2B1} - f_2) \tag{38}$$

Integrating (38) taking $x = x_{B1}$ at $f_2 = f_{2B1}$

$$\frac{\beta_2 P_{1B_1}}{D_2} (x - x_{B1}) = v_{B1} (f_{2B1} - f_2) + \frac{AD_2 \, P_{2B1}}{2\omega \beta_2 \, P_{1B_1}} (f_{2B1} - f_2)^2 \tag{39}$$

Let $x = x_{B2}$ when the second component burns out, i. e. $f_2 = 0$.

Then we get

$$x_{B2} - x_{B1} = \frac{D_2}{\beta_2} \; \frac{v_{B1}}{P_{1B1}} \; f_{2B_1} + \frac{AD^2_2 \, P_{2B1}}{2\omega\beta^2_2 \, P^2_{1B_1}} \; f^2_{2B1} \tag{40}$$

From (31) we get

$$\frac{x_{B2} - x_{B1}}{x_{B1}} = \frac{\eta c_1}{\gamma_0 \, Ax_{B1}} \left[ \alpha_0 \, \eta_{B1} f_{2B1} + \frac{M_1 \alpha^2_0}{2} \; \omega_{B1} \; f^2_{2B1} \right] \tag{41}$$

Introducing another dimensionless quantity

$$\zeta_{B1} = \frac{\eta c_1}{Ax_{B1}}$$

the travel ratio is

$$= \frac{\zeta_{B1} \, f_{2B_1}}{\gamma_0} \left[ \alpha_0 \, \eta_{B1} + \frac{M_1 \alpha^2_0}{2} \; \omega_{B1} f_{2B1} \right] \tag{42}$$

which is also less than the ratio corresponding to the moderated charges. Differentiating (10) w.r.t. to $f_2$ and using (18), (24), (25) and (31), we get

$$\phi'_2 (f_2) \cdot (1 + \delta_0) = - \psi \, \frac{\alpha_0}{\beta_0} \; \frac{\gamma_0}{\sqrt{T'}} + \gamma \sqrt{T'} \tag{43}$$

where $T = \frac{T_1}{T_0}$. Thus (43) determines $T'$ as a function of $f_2$. Equation (24) can be written as

$$\left(1 + \frac{1}{\beta_0}\right) \frac{dN}{df_2} = \frac{dz_2}{df_2} + \psi \frac{\alpha_0}{\beta_0} \frac{1}{\sqrt{T'}} \tag{44}$$

On putting the value of $T'$ from (43) and integrating (44), we get $N$ as a function of $f_2$.

$$N = \frac{\beta_0}{1+\beta_0} z_2 + \frac{\psi \alpha_0}{1+\beta_0} \int \frac{1}{\sqrt{T'}} df_2 + B \tag{45}$$

where $B$ is determined by the condition $N = N_{B1}$ and $f_2 = f_{2B1}$.

$T_1$ and $N$ are determined from (43) and (45) $T_2$ is determined as a function of $f_2$ from the relation (27).

## PARTICULAR CASES WITH DIFFERENT FORM FUNCTIONS

(*i*) Let us take the form function given by

$$\phi_1(f_1) = (1 - f_1)(1 + \theta_1 f_1)$$

$$\phi_2(f_2) = (1 - f_2)(1 + \theta_2 f_2)$$

$$\frac{\alpha_0}{\beta_0} \cdot \frac{\gamma_0 \psi (1 - \omega_{B1}) / \sqrt{T_0'} + \eta_{B1} \omega_{B1}}{1 + \delta_0} = 1 + \theta_2 + 2\theta_2 f_{2B1} \tag{46}$$

$$M_1 \frac{2\alpha_0^2 \omega^2_{B1}}{\beta_0} + \frac{\gamma_0 \psi \alpha_0 (1 - \omega_{B1})}{2(1+\beta_0)\sqrt{T'_0} N_B} \cdot \left[\left(1 - \theta_2 + 2\theta_2 f_{2B1}\right)\left(1 - \frac{1}{T'_0}\right) + \frac{\psi \alpha_0 (\gamma - 1)}{\beta_0 \sqrt{T'_0}}\right] = 2\theta_2 (1 + \delta_0 +) \tag{47}$$

which are the similar equations as given by Corner[3]

(*ii*) Further assuming the form function in tabular form given by

$$\phi_1(f_1) = (1 - f_1)$$

$$\phi_2(f_2) = (1 - f_2)$$

assuming $\theta_1 = 0$ and $\theta_2 = 0$, we have the following relations

$$\frac{\alpha_0}{\beta_0} \frac{\gamma_0 \psi (1 - \omega_{B1}) / \sqrt{T'_0} + \eta_{B1} \omega_{B1}}{1 + \delta_0} = 1 \tag{48}$$

$$M_1 \frac{2\alpha_0^2 \omega^2_{B1}}{\beta_0} + \frac{\gamma_0 \psi \alpha_0 (1 - \omega_{B1})}{2(1+\beta_0)\sqrt{T'_0} N_B} \left[\left(1 - \frac{1}{T'_0}\right) + \frac{\psi \alpha_0 (\gamma - 1)}{\beta_0 \sqrt{T_0'}}\right] = 0 \tag{49}$$

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