APPLICATION OF AN OPTIMISATION TECHNIQUE TO AN AIR FORCE PROBLEM

S.K. Sahiar, P.P. Sangal & B.L. Sethi

Office of Scientific Adviser to Chief of the Air Staff, New Delhi

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This paper deals with an optimisation technique for determining the relative effectiveness of different types of bombs being dropped over a certain target area. The measure of effectiveness is the total effort required for each type of bomb to lethalise a given fraction of the total target area and the one with the least total effort is taken to be the most cost-effective solution. The total effort takes into account the number of bombs dropped, their costs and also the costs of the sorties performed to carry out the mission. A mathematical model has been formulated and discussed in detail.

A variety of bombs are available today for carrying out air-to-ground close support missions. In this context, it often becomes important to determine the relative effectiveness of different types of bombs against a range of small targets encountered on the battle field and immediate tactical area. This paper describes the technique which can be used to arrive at the relative effectiveness of different types of bombs.

MEASURE OF EFFECTIVENESS

For a quantitative assessment of the effectiveness of each type of bomb, a suitable measure of effectiveness has to be defined. The measure of effectiveness chosen here is in terms of the total effort required to lethalise a given fraction of the total target area and the one with the least total effort is taken to be the most cost-effective solution. It is evident that the total effort expended would depend mainly upon the following three factors :

(i) Number of bombs dropped over the target area for lethalising a given fraction of that area;

(ii) Number of sorties required and cost thereof ; and

(iii) Cost of a bomb.

The mathematical relation for the number of bombs required to lethalise a given fraction of the target area is based on the following assumptions :

(a) The targets such as troops, soft vehicles, gun emplacements, are uniformally distributed over the total area which is to be lethalised.

(b) Mean area of effectiveness of a bomb is small as ompared to the total target area to be lethalised It is on this assumption that the mathematical model discussed below will hold good.

MATHEMATICAL RELATION FOR DETERMINING THE NUMBER OF BOMBS

The first requirement of the optimisation process involved is to arrive at the number of bombs required (for each type of bomb) to lethalise a given fraction of the total target area. If the effect of overlapping of the bombs is to be taken into account and the mean area of effectiveness of a bomb is small as compared to the total target area, then

$$MD = \log_e \left(\frac{1}{1-F} \right) \qquad (1)$$

where F is the fraction of the total target area lethalised,

M is the mean area of effectiveness of a bomb, and

D is the density of bombs per unit of area.

With the help of (1), knowing the value of F and M, one can find out the value of D—the density of bombs required per unit of area to damage a fraction F of the total target area. Knowing D, one can find out the total number of bombs required to cause that degree of damage.

METHOD OF ANALYSIS

(*a*) *Number of Bombs/Weapons* : Let there be n different types of weapons or bombs under study denoted by $W_1\ W_2 \ldots\ldots\ldots W_n$. The number of bombs of each type required for lethalising a given fraction F of the target area, as obtained with the help of equation (1), is arranged as shown in Table 1.

TABLE 1

NUMBER OF WEAPONS REQUIRED TO LETHALISE A GIVEN FRACTION 'F' OF TARGET AREA

Types of weapons (bombs)	$F=25\%$	$F=50\%$	$F=60\%$	$F=75\%$
W_1	N_{11}	N_{12}	N_{13}	N_{14}
W_2	N_{21}	N_{22}	N_{23}	N_{24}
W_3	N_{31}	N_{32}	N_{33}	N_{34}
..	..	..	..	..
W_n	N_{n_1}	N_{n_2}	N_{n_3}	N_{n_4}

(*b*) *Cost of total number of sorties performed*: To find out a cost-effective solution to the problem, the other two important parameters are the cost of a bomb and the cost of performing an operational flight of one aircraft, called a sortie. The cost of a bomb is rather easy to find out but the cost of a sortie of an aircraft has to be carefully determined. The latter will depend upon the type of aircraft, mode of attack, radius of action and a number of other factors. To determine the total cost involved in performing the sorties, we find out the number of sorties required to drop the quantity of bombs as determined in Table 1. The cost of performing a single sortie will then be determined and multiplied by the total number of sorties performed. The results are arranged as shown in Table 2.

TABLE 2

COST OF SORTIES REQUIRED TO LETHALISE A GIVEN FRACTION 'F' OF TARGET AREA

Types of weapons (bombs)	$F=25\%$	$F=50\%$	$F=60\%$	$F=75\%$
W_1	S_{11}	S_{12}	S_{13}	S_{14}
W_2	S_{21}	S_{22}	S_{23}	S_{24}
W_3	S_{31}	S_{32}	S_{33}	S_{34}
..	..	..	..	..
W_n	S_{n_1}	S_{n_2}	S_{n_3}	S_{n_4}

(*c*) *Cost of the total number of bombs*: The cost of a single bomb is determined and multiplied by the total number of bombs dropped. The resulting costs are arranged as shown in Table 3.

TABLE 3

COST OF BOMBS REQUIRED TO LETHALISE A GIVEN FRACTION 'F' OF TARGET AREA

Type of Weapons (bombs)	$F=25\%$	$F=50\%$	$F=60\%$	$F=75\%$
W_1	C_{11}	C_{12}	C_{13}	C_{14}
W_2	C_{21}	C_{22}	C_{23}	C_{24}
W_3	C_{31}	C_{32}	C_{33}	C_{34}
..	..	..	..	..
W_n	C_{n_1}	C_{n_2}	C_{n_3}	C_{n_4}

A COST EFFECTIVE SOLUTION/CONCLUSION

The cots arrived at in Tables 2 and 3 are based on the number of bombs required to lethalise a given fraction F of the target area. For a given fraction of the target area to be lethalised, the two costs are added up for each type of bomb separately and the one with the least cost is taken as the optimum solution on cost effectiveness considerations.

A word of caution is necessary here. In applying a technique of analysis of this type, the assumptions made in arriving at the formula for determining the number of bombs should be carefully examined and the mean area of effectiveness has to be correctly determined.