ON THE MODE OF VIBRATION OF A LAYERED FLAT-TOPPED CONICAL HILL (OF DECCAN TRAP TYPE) AND THE STRESSES GENERATED IN EACH LAYER—PART II

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In this paper the mode of vibration and the stresses in each layer of a 3-layered flat-topped conical hill have been calculated; and the solution is presented in terms of Bessel function. The condition of fracturing is also specified.

In an earlier paper[1] the authors discussed the mode of vibration and stresses generated in each layer in layered flat-topped conical hill, in the case when the elastic vibrational energy becomes nil at the top surface. In this work, these properties are determined in the case where the energy is not dissipated to a zero value at the top.

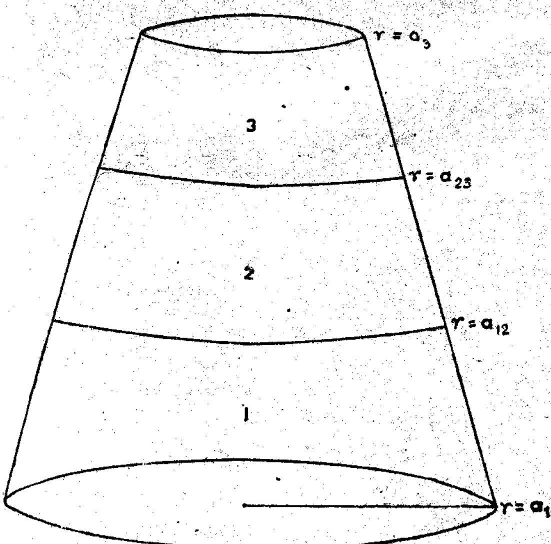

Fig. 1—Flat topped conical hill with three horizontal layers numbered 1, 2 and 3 and $r = a_{ij}$ for $i, j = 1, 2$ and 3 denote the distance of surface of separation of two consecutive layers i and j from the vertex of conical hill

We take up a 3-layered case instead of n-layered case as done previously. The procedure can, however, be easily extended to n-layered.

The case is presented in Fig. 1, where the layers are marked as 1, 2, and 3. A shock wave is generated by implosion (confined explosion) at the base of a conical hill, as drawn, with a base $r = a_1$. The vibration is radial and the co-ordinates are spherical (r, θ, ϕ). The moduli of rigidity, μ_1, μ_2 and μ_3 and the densities ρ_1, ρ_2 and ρ_3 of layers 1, 2 and 3 are constant.

FUNDAMENTAL EQUATIONS

Love's[2] stress equations of motion are used as previously[1]. Since it is a 3-layer case and the vibration is radial, the displacements.

$$(u_\theta)_1 = (u_\theta)_2 = (u_\theta)_3 = (u_\phi)_1 = (u_\phi)_2 = (u_\phi)_3 = 0$$

$$\left.\begin{aligned}(u_r)_1 &= R_1(r)e^{ipt}\\(u_r)_2 &= R_2(r)e^{ipt}\\(u_r)_3 &= R_3(r)e^{ipt}\end{aligned}\right\}\text{ where } R_1, R_2, R_3 \text{ are functions of } r \text{ only}$$

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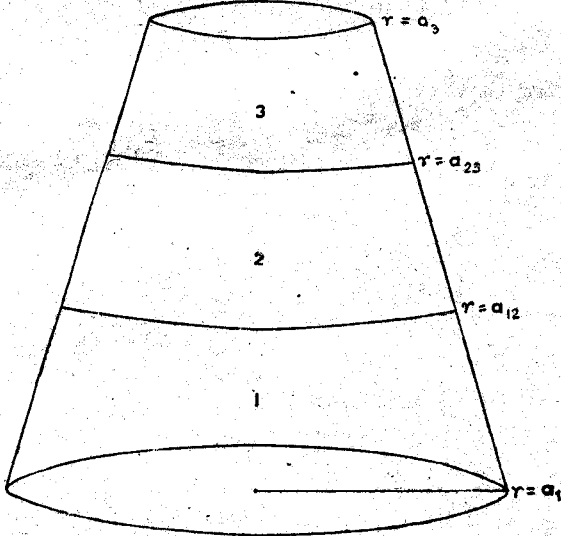

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The equations (1) and (2) of Datta & Mitra[1] can readily be obtained in this case by putting $N = 1, 2, 3$.

BOUNDARY CONDITIONS

$$\left.\begin{array}{lll} (i) & \widehat{(rr)}_3 = 0 & \text{on } r = a_3 \\ (ii) & \widehat{(rr)}_1 = S\, e^{ipt} & \text{on } r = a_1 \\ & \text{where } S \text{ is a constant.} & \\ (iii)\ (a) & \widehat{(rr)}_2 = \widehat{(rr)}_3 & \\ \quad\ (b) & u_2 = u_3 & \text{on } r = a_{23} \\ (iv)\ (a) & \widehat{(rr)}_1 = \widehat{(rr)}_2 & \\ \quad\ (b) & u_1 = u_2 & \text{on } r = a_{12} \end{array}\right\} \qquad (1)$$

Applying (i) of the boundary conditions,

$$A_3 = -B_3 \frac{Y_{3/2}(\nu_3 a_3) - \nu_3 a_3 Y_{5/2}(\nu_3 a_3)}{J_{3/2}(\nu_3 a_3) - \nu_3 a_3 J_{5/2}(\nu_3 a_3)} \qquad (2)$$

$$= -B_3 \frac{Q_3}{P_3} \qquad (3)$$

where, Q_3 denotes the numerator and P_3 denotes the denomenator

Applying (ii) of the boundary conditions,

$$A_1 \left[J_{3/2}(\nu_1 a_1) - \nu_1 a_1 J_{5/2}(\nu_1 a_1) \right] + B_1 \left[Y_{3/2}(\nu_1 a_1) - \nu_1 a_1 Y_{5/2}(\nu_1 a_1) \right] = \frac{S\, a_1^{3/2}}{\mu_1} \qquad (4)$$

Applying (iii) (a) of the boundary conditions,

$$A_2 P_2^{(3)} P_3 + B_2 Q_2^{(3)} P_3 - \frac{\mu_3}{\mu_2} B_3 \left[P_3 Q_3^{(2)} - Q_3 P_3^{(2)} \right] = 0 \qquad (5)$$

where,

$$P_2^{(3)} = J_{3/2}(\nu_2 a_{23}) - \nu_2 a_{23} J_{5/2}(\nu_2 a_{23})$$

$$Q_2^{(3)} = Y_{3/2}(\nu_2 a_{23}) - \nu_2 a_{23} Y_{5/2}(\nu_2 a_{23})$$

$$P_3^{(2)} = J_{3/2}(\nu_3 a_{23}) - \nu_3 a_{23} J_{5/2}(\nu_3 a_{23})$$

$$Q_3^{(2)} = Y_{3/2}(\nu_3 a_{23}) - \nu_3 a_{23} Y_{5/2}(\nu_3 a_{23})$$

Applying (iii) (b) of the same, we get,

$$A_2 P_3 J_{3/2}(\nu_2 a_{23}) + B_2 P_3 Y_{3/2}(\nu_2 a_{23}) - B_3 \left[P_3 Y_{3/2}(\nu_3 a_{23}) - Q_3 J_{3/2}(\nu_3 a_{23}) \right] = 0 \qquad (6)$$

Eliminating the constants from (5) and (6) and using the equation (3) we get,

$$A_2 = \frac{B_3}{P_3} \frac{D_{A2}}{D_{B3}^{(3)}} \qquad (7)$$

$$B_2 = \frac{B_3}{P_3} \frac{D_{B2}}{D^{(3)}{}_{B3}} \qquad (8)$$

where,

$$D_{A_2} = \frac{\mu_3}{\mu_2} Y_{3/2}(\nu_2 a_{23}) \left[P_3 Q_3^{(3)} - Q_3 P_3^{(2)} \right] - Q_2^{(3)} \left[P_3 Y_{3/2}(\nu_3 a_{23}) - Q_3 J_{3/2}(\nu_3 a_{23}) \right]$$

$$D_{B_2} = P_2^{(3)} \left[P_3 Y_{3/2}(\nu_3 a_{23}) - Q_3 J_{3/2}(\nu_3 a_{23}) \right] - \frac{\mu_3}{\mu_2} J_{3/2}(\nu_2 a_{23}) \left[P_3 Q_3^{(2)} - Q_3 P_3^{(2)} \right]$$

$$D_{B_3}^{(3)} = P_2^{(3)} Y_{3/2}(\nu_2 a_{23}) - Q_2^{(3)} J_{3/2}(\nu_2 a_{23})$$

Applying (*iv*) (*a*) of the boundary conditions, we obtain,

$$A_1 P_1^{(2)} P_3 D_{B_3}^{(3)} + B_1 Q_1^{(2)} P_3 D_{B_3}^{(3)} - \frac{\mu_2}{\mu_1} B_3 \left[P_2^{(1)} D_{A_2} + Q_2^{(1)} D_{B_2} \right] = 0 \qquad (9)$$

Applying (*iv*) (*b*) of the boundary conditions, we get,

$$A_1 P_3 D_{B_3}^{(3)} J_{3/2}(\nu_1 a_{12}) + B_1 P_3 D_{B_3}^{(3)} Y_{3/2}(\nu_1 a_{12}) - B_3 \left[D_{A_2} J_{3/2}(\nu_2 a_{12}) + D_{B_2} Y_{3/2}(\nu_2 a_{12}) \right] = 0 \quad (10)$$

Eliminating the constants from (9) & (10) and using equation (4) we obtain,

$$B_3 = \frac{Sa^{3/2}}{\mu_1 E} P_3 D_{B_3}^{(3)} D_{B_3}^{(2)}$$

$$A_1 = \frac{Sa_1^{3/2}}{\mu_1 E} D_{A_1}$$

$$B_1 = \frac{Sa_1^{3/2}}{\mu_1 E} D_{B_1}$$

where,

$$D_{B_3}^{(2)} = P_1^{(3)} Y_{3/2}(\nu_1 a_{12}) - Q_2^{(2)} J_{3/2}(\nu_1 a_{12})$$

$$D_{A_1} = \frac{\mu_2}{\mu_1} Y_{3/2}(\nu_1 a_{12}) \left[P_2^{(1)} D_{A_2} + Q_2^{(1)} D_{B_2} \right] - Q_1^{(2)} \left[D_{A_2} J_{3/2}(\nu_2 a_{12}) + D_{B_2} Y_{3/2}(\nu_2 a_{12}) \right]$$

$$D_{B_1} = P_1^{(2)} \left[D_{A_2} J_{3/2}(\nu_2 a_{12}) + D_{B_2} Y_{3/2}(\nu_2 a_{12}) \right] - \frac{\mu_2}{\mu_1} J_{3/2}(\nu_2 a_{12}) \left[P_2^{(1)} D_{A_2} + Q_2^{(1)} D_{B_2} \right]$$

$$E = \left[D_{A_2} J_{3/2}(\nu_2 a_{12}) + D_{B_2} Y_{3/2}(\nu_2 a_{12}) \right] \left[P_1^{(2)} Q_1 - Q_1^{(2)} P_1 \right] - \frac{\mu_2}{\mu_1} \left[P_2^{(1)} D_{A_2} + Q_2^{(1)} D_{B_2} \right] \cdot \left[Q_1 J_{3/2}(\nu_1 a_{12}) - P_1 Y_{3/2}(\nu_1 a_{12}) \right]$$

where, again,

$$P_1 = J_{3/2}(\nu_1 a_1) - \nu_1 a_1 J_{5/2}(\nu_1 a_1)$$
$$Q_1 = Y_{3/2}(\nu_1 a_1) - \nu_1 a_1 Y_{5/2}(\nu_1 a_1)$$
$$P_1^{(2)} = J_{3/2}(\nu_1 a_{12}) - \nu_1 a_{12} J_{5/2}(\nu_1 a_{12})$$
$$Q_1^{(2)} = Y_{3/2}(\nu_1 a_{12}) - \nu_1 a_{12} Y_{5/2}(\nu_1 a_{12})$$
$$P_2^{(1)} = J_{3/2}(\nu_2 a_{12}) - \nu_2 a_{12} J_{5/2}(\nu_2 a_{12})$$
$$Q_2^{(1)} = Y_{3/2}(\nu_2 a_{12}) - \nu_2 a_{12} Y_{5/2}(\nu_2 a_{12})$$

Thus we can determine the displacements and stresses at three layers. The displacements are measured as :

$$(u_r)_1 = \frac{S\, a_1^{3/2}}{\mu_1 E}\, r^{-1} \Big[D_{A_1} J_{3/2} (\nu_1\ r_1) + D_{B_1} Y_{3/2} (\nu_1\ r_1) \Big] e^{ipt}$$

$$(u_r)_2 = \frac{S\, a_1^{3/2}}{\mu_1 E}\, D_{B_3}{}^{(2)}\, r^{-1} \Big[D_{A_2} J_{3/2} (\nu_2\ r_2) + D_{B_2} Y_{3/2} (\nu_2\ r_2) \Big] e^{ipt}$$

$$(u_r)_3 = \frac{S\, a_1^{3/2}}{\mu_1 E}\, D_{B_3}{}^{(2)} D_{B_3}{}^{(3)}\, r^{-1} \Big[Q_3 J_{3/2} (\nu_3\ r_3) + P_3 Y_{3/2} (\nu_3\ r_3) \Big] e^{ipt}$$

The stresses at the boundary of each layers are:

$$\widehat{(rr)}_{r=a_{12}} = \frac{\mu_2}{\mu_1}\, \frac{S\, a_1^{3/2}}{a_{12}^{3/2}}\, D_{B_3}{}^{(3)} \Big[D_{A_2} \Big\{ J_{3/2} (\nu_2\ a_{12}) - \nu_2\, a_{12}\, J_{5/2} (\nu_2\ a_{12}) \Big\} +$$

$$+ D_{B_2} \Big\{ Y_{3/2} (\nu_2\ a_{12}) - \nu_2\, a_{12}\, Y_{5/2} (\nu_2\ a_{12}) \Big\}\Big] e^{ipt}$$

$$\widehat{(rr)}_{r=a_{23}} = \frac{\mu_3}{\mu_1}\, \frac{S\, a_1^{3/2}}{a_{23}^{3/2}}\, D_{B_3}{}^{(2)} D_{B_3}{}^{(3)} \Big[Q_3 \Big\{ J_{3/2} (\nu_3\ a_{23}) - \nu_3\, a_{23}\, J_{5/2} (\nu_3\ a_{23}) \Big\} +$$

$$+ P_3 \Big\{ Y_{3/2} (\nu_3\ a_{23}) - \nu_3\, a_{23}\, Y_{5/2} (\nu_3\ a_{23}) \Big\}\Big] e^{ipt}$$

DISCUSSION

In the 3-layer case we have deduced the amount of displacement and the nature of stresses in each of the layers while the vibration on the top layer exists. Again it is also noted that if the frequency equations (5) and (9) do not satisfy each other rupture would follow. This case can simply be extended to a n-layer case in a manner as presented.

CONCLUDING REMARKS

The mode of vibration in layered media has been the subject of considerable interest to many theoretical geophysicist.

The authors have evaluated the mode of vibration at different layers and also on the top surface when a shock wave is produced at the base of a Deccan trap hill by a confined explosion of the type that was created at Pokhran where the overlying rocks were horizontally stratified sedimentary beds of sandstone and shale only. Here the case deals with hills standing above the ground level.

In presuit of peaceful use of nuclear energy more underground explosions may be made and this theoretical, investigation would bear considerable guideline to evaluate the nature of vibrations at the surface, seismic shooting for structures, exploration of mineral deposits, generation of cracks and the consequent damages etc. on the structures above the Deccan trap country and which not only covers a vast territory of Peninsular India but also hides an enormous amount of mineral deposits of different varieties.

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