SINGLE QUEUE ATTENDED BY ALTERNATIVE SERVERS WITH CHANGEOVER TIMES

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A single queue is serviced by two servers alternately, in alternate busy periods. A changeover time is required whenever the servers are replaced. In this model, the changeover time of a server is initiated as soon as one server completes the services of the units waiting before him namely at the end of his busy period. It may be noted that the empty state of the system is never reached. A comparison of this model with the model with empty state has been done.

Many queuing models involving changeover times and the interaction of queues have been studied in the literature by Gaver[1], Takacs[2], Skinner[3], Leibowitz[4] and Tanner[5]. Recently Scott[6] has considered a queuing model serviced by alternating servers working in shifts. An advantage in studying such a system is that if redundant modules are provided for service mechanism and if they work alternately, then there is a possibility of providing an efficient repairing or maintenance schedule for the module which are at rest without affecting the service provided by the system. Batra and Thiruvengadam[7] studied the problem of alternating servers with initial changeover times, i.e., when server's switchingover takes place, some initial time is spent in setting up the service mechanism and the server switchover takes place after recurrence of empty state of the system. In this paper, we have assumed that the changeover time of the server is initiated as soon as the server completes the service of the units that are waiting before him. Further, at the end of a changeover time if no unit is waiting for service for a server, then the other changeover time is again initiated for the other server. It may be noted here that the system will have no idle state. The study of this model helps us to compare the relative performance of the operational characteristics with the model in which the empty state of the system is introduced[7].

STATEMENT OF THE PROBLEM

Customers arrive in the system according to a Poisson process with intensity λ and join in a single counter queue, serviced by two servers namely server 1 and server 2. They work alternately with independent service rates, hence the service times of the units are governed by two probability distributions. Let $S_i(\theta)$ $(i=1, 2)$ be the probability density of service time distribution of units when serviced by the server i $(i=1, 2)$ respectively. Let $\eta_i(\theta)\, d\theta$ be the conditional probability that the service completion of a unit takes place between $(\theta, \theta+d\theta)$ given that it has not been completed upto time by the server i. Let $S'_1(\phi)$ and $S'_2(\phi)$ be the probability densities of changeover time distributions from server 2 to 1 and from server 1 to 2 respectively. Also we define $\psi_i(\phi)\, d\phi$ $(i=1,2)$ be the conditional probability of changeover time completion between ϕ and $\phi+d\phi$ given that it has not completed upto time ϕ for the density function $S'_i(\phi) (i=1, 2)$. Let us introduce the following definitions of the variables.

$M(t) =$ The number of customers waiting or being serviced at time t.

$N(t) =$ The number of customers already serviced upto time t.

$\tau_i (i = 1, 2) =$ The elapsed service time of a customer under service at time time t with the server i.

$\tau'_i (i = 1, 2) =$ The elapsed changeover time at time t when the changeover takes place from service i.

Using the method of supplementary variables[8] we define the following probabilities,

$$q_i(m, n, \theta, t) = P_r[M(t) = m, N(t) = n, \theta < \tau_i < \theta + d\theta]\ (i = 1, 2)$$

$$Q_i(m, n, \phi, t) = P_r[M(t) = m, N(t) = n, \phi < \tau'_i < \phi + d\phi]\ (i = 1, 2)$$

The above probabilities are mutually exclusive and exhaustive and provide a complete markovian characteristics of the system. We relate the state of the system at time $t + \triangle t$ to the state at time t, then the differential difference equations connecting these probabilities and governing the system are given by

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial \theta}+\left[\lambda+\eta_1(\theta)\right]\right\} q_1(m, n, \theta, t)=\lambda\, q_1(m-1, n, \theta, t) \qquad \text{for } m \geqslant 1 \tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial \theta}+\left[\lambda+\eta_2(\theta)\right]\right\} q_2(m, n, \theta, t)=\lambda\, q_2(m-1, n, \theta, t) \qquad \text{for } m \geqslant 1 \tag{2}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial \phi}+\left[\lambda+\psi_1(\phi)\right]\right\} Q_1(m, n, \phi, t)=\lambda\, Q_1(m-1, n, \phi, t) \qquad \text{for } m \geqslant 0 \tag{3}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial \phi}+\left[\lambda+\psi_2(\phi)\right]\right\} Q_2(m, n, \phi, t)=\lambda\, Q_2(m-1, n, \phi, t) \qquad \text{for } m \geqslant 0 \tag{4}$$

The above equations are to be solved subject to the following boundary conditions

$$q_1(m, n, 0, t)=\int_0^\infty q_1(m+1, n-1, \theta, t)\, \eta_1(\theta)\, d\theta+\int_0^\infty Q_1(m, n, \phi, t)\, \psi_1(\phi)\, d\phi \tag{5}$$

$$q_2(m, n, 0, t)=\int_0^\infty q_2(m+1, n-1, \theta, t)\, \eta_2(\theta)\, d\theta+\int_0^\infty Q_2(m, n, \phi, t)\, \psi_2(\phi)\, d\phi \tag{6}$$

$$Q_1(m, n, 0, t)=\delta_{m0}\left[\int_0^\infty q_2(1, n-1, \theta, t)\, \eta_2(\theta)\, d\theta+\int_0^\infty Q_2(0, n, \phi, t)\, \psi_2(\phi)\, d\phi\right] \tag{7}$$

$$Q_2(m, n, 0, t)=\delta_{m0}\left[\int_0^\infty q_1(1, n-1, \theta, t)\, \eta_1(\theta)\, d\theta+\int_0^\infty Q_1(0, n, \phi, t)\, \psi_1(\phi)\, d\phi\right] \tag{8}$$

It may be noted that the range of values for the variable n in the above probabilities depends upon the initial conditions. Thus, for the initial condition $Q_1(m, n, \phi, o)=\delta_{m1}\, \delta_{no}\, \delta(\phi)$

The values of n for the equations (1) to (4) are $n \geqslant 0$, $n \geqslant 1$, $n \geqslant 0$ and $n \geqslant 1$ respectively.

SOLUTION OF THE PROBLEM

We define the generating functions as under:

$$F_1(\theta, x, y, t)=\sum_{m=1}^{\infty} x^m f_{1, m}(\theta, y, t)=\sum_{m=1}^{\infty} x^m \sum_{n=0}^{\infty} y^n q_1(m, n, \theta, t)$$

$$F_2(\theta, x, y, t)=\sum_{m=1}^{\infty} x^m f_{2, m}(\theta, y, t)=\sum_{m=1}^{\infty} x^m \sum_{n=1}^{\infty} y^n q_2(m, n, \theta, t)$$

$$G_1(\phi, x, y, t)=\sum_{m=0}^{\infty} x^m g_{1, m}(\phi, y, t)=\sum_{m=0}^{\infty} x^m \sum_{n=0}^{\infty} y^n Q_1(m, n, \phi, t)$$

$$G_2(\phi, x, y, t) = \sum_{m=0}^{\infty} x^m g_{2,m}(\phi, y, t) = \sum_{m=0}^{\infty} x^m \sum_{r=1}^{\infty} y^n Q_2(m, n, \phi, t)$$

With the help of these generating functions, after multiplying by the appropriate powers of y and x and summing over, we obtain the following equations :

$$\left\{\frac{\partial}{\partial t} + \frac{\partial}{\partial \theta} + \left[\lambda(1-x) + \eta_1(\theta)\right]\right\} F_1(\theta, x, y, t) = 0 \tag{9}$$

$$\left\{\frac{\partial}{\partial t} + \frac{\partial}{\partial \theta} + \left[\lambda(1-x) + \eta_2(\theta)\right]\right\} F_2(\theta, x, y, t) = 0 \tag{10}$$

$$\left\{\frac{\partial}{\partial t} + \frac{\partial}{\partial \phi} + \left[\lambda(1-x) + \psi_1(\phi)\right]\right\} G_1(\phi, x, y, t) = 0 \tag{11}$$

$$\left\{\frac{\partial}{\partial t} + \frac{\partial}{\partial \phi} + \left[\lambda(1-x) + \psi_2(\phi)\right]\right\} G_2(\phi, x, y, t) = 0 \tag{12}$$

We define the Laplace transform of a real valued function $f(t)$ by

$$\bar{f}(s) = \int_0^{\infty} e^{-st} f(t)\, dt \qquad \text{where } Re(s) \geqslant 0$$

Applying the Laplace transform with the initial condition

$$Q_1(m, n, \phi, 0) = \delta_{m1}\, \delta_{n0}\, \delta(\phi) \tag{13}$$

to the equations (9) to (12) and integrating the resulting differential equations, we get

$$\bar{F}_1(\theta, x, y, s) = \bar{F}_1(0, x, y, s)\, e^{-[\lambda(1-x)+s]\theta}\, e^{-\int_0^{\theta}\eta_1(\theta)\,d\theta} \tag{14}$$

$$\bar{F}_2(\theta, x, y, s) = \bar{F}_2(0, x, y, s)\, e^{-[\lambda(1-x)+s]\theta}\, e^{-\int_0^{\theta}\eta_2(\theta)\,d\theta} \tag{15}$$

$$\bar{G}_1(\phi, x, y, s) = (x + \bar{G}_1(0, x, y, s)\; e^{-[\lambda(1-x)+s]\phi}\, e^{-\int_0^{\phi}\psi_1(\phi)\,d\phi} \tag{16}$$

$$G_2(\phi, x, y, s) = \bar{G}_2(0, x, y, s)\, e^{-[\lambda(1-x)+s]\phi}\, e^{-\int_0^{\phi}\psi_2(\phi)\,d\phi} \tag{17}$$

Now, we evaluate the values of the functions $\bar{F}_1(0, x, y, s)$, $\bar{F}_2(0, x, y, s)$, $\bar{G}_1(0, x, y, s)$ and $\bar{G}_2(0, x, y, s)$ as follows.

From the definition of generating functions, the boundary conditions (7) and (8) after the application of Laplace transform become

$$\begin{aligned}\bar{G}_1(0, x, y, s) &= \bar{g}_{1,0}(0, y, s)\\ &= y\int_0^{\infty}\bar{f}_{2,1}(\theta, y, s)\,\eta_2(\theta)\,d\theta + \int_0^{\infty}\bar{g}_{2,0}(\phi, y, s)\,\psi_2(\phi)\,d\phi\end{aligned} \tag{18}$$

$$\overline{G}_2(0, x, y, s) = \overline{g}_{2,0}(0, y, s)$$
$$= y\int_0^\infty \overline{f}_{1,1}(\theta, y, s)\, \eta_1(\theta)\, d\theta + \int_0^\infty \overline{g}_{1,0}(\phi, y, s)\, \psi_1(\phi)\, d\phi \tag{19}$$

Similarly we can easily get

$$\overline{F}_1(0, x, y, s) = y \sum_{m=1}^{\infty} x^m \int_0^\infty \overline{f}_{1,m+1}(\theta, y, s)\, \eta_1(\theta)\, d\theta + \sum_{m=1}^{\infty} x^m \int_0^\infty \overline{g}_{1,m}(\phi, y, s)\, \psi_1(\phi)\, d\phi \tag{20}$$

The equation (20) on using (18) after a little simplification becomes

$$\overline{F}_1(0, x, y, s) = \frac{y}{x}\int_0^\infty \overline{F}_1(\theta, x, y, s)\, \eta_1(\theta)\, d\theta + \int_0^\infty \overline{G}_1(\phi_1, x, y, s)\, \psi_1(\phi)\, d\phi - \overline{G}_2(0, x, y, s) \tag{21}$$

Substituting (14) and (16) in (21) one can easily obtain

$$\overline{F}_1(0, x, y, s) = \frac{[x + \overline{G}_1(0, x, y, s)]\, \overline{S}_1'[\lambda(1-x)+s] - \overline{G'}_2(0, x, y, s)}{1 - \frac{y}{x}\, \overline{S}_1[\lambda(1-x)+s]} \tag{22}$$

where $\overline{S}'_1(s)$ and $S_i(s)(i = 1, 2)$ are the Laplace transform of the probability density functions $S'_i(\phi)$ and $S_i(\theta)$ $(i = 1, 2)$ respectively.

Similarly we have

$$\overline{F}_2(0, x, y, s) = \frac{\overline{G}_2(0, x, y, s)\, \overline{S'_2}(\lambda(1-x)+s) - \overline{G_1}(0, x, y, s)}{1 - \frac{y}{x}\, S_2(\lambda(1-x)+s)} \tag{23}$$

With the help of boundary conditions (8) and equations (15) and (19) where $\overline{S}'_2\ (s)$ and $S_2\ (s)$ are defined earlier.

Now it may be noted that denominators of expressions (22) and (23) has each one root x inside the circle $|x| = 1$. Let us suppose that $f_i(y, s)$ $(i=1, 2)$ be the root of the equation in x

$$x = \overline{S}_i[\lambda(1-x) + s] \tag{24}$$

lies inside $|x|=1$. Noting that $\overline{G_1}(0, x, y, s)$ and $G_2(0, x, y, s)$ are the functions that are independent of x and $F_1(0, x, y, s)$ and $F_2(0, x, y, s)$ are analytic functions of x inside $|x|=1$ the numerator of (23) and (24) should also vanish at $f_1(y, s)$ and $f_2(y, s)$ respectively. Therefore, we have

$$\overline{G}_1(0, x, y, s) = \frac{f_1(y, s)\, \overline{S}'_1[\lambda\{1-f_1(y, s)\} + s]\, \overline{S}'_2[\lambda\{1-f_2(y, s)\} + s]}{1-\overline{S}'_1[\lambda\{1-f_1(y, s)\} + s]\, \overline{S}'_2[\lambda\{1-f_2(y, s)\} + s]} \tag{25}$$

and

$$\overline{G}_2(0, x, y, s) = \frac{f_1(y, s)\, \overline{S}'_1[\lambda\{1-f_1(y, s)\} + s]}{1-\overline{S}'_1[\lambda\{1-f_1(y, s)\} + s]\, \overline{S}'_2[\lambda\{1-f_2(y, s)\} + s]} \tag{26}$$

Defining the generating function of the queue length probabilities as

$$\pi(x, y, t) = \int_0^\infty F_1(x, y, \theta, t)\, d\theta + \int_0^\infty F_2(x, y, \theta, t)\, d\theta + \int_0^\infty G_1(x, y, \phi, t)\, d\phi + \int_0^\infty G_2(x, y, \phi, t)\, d\phi \tag{27}$$

The Laplace transform of (27) can now be written as where $\bar{G}_1(0,x,y,s)$ and $\bar{G}_2(0,x,y,s)$ are defined by (25) and (26)

$$\bar{\pi}(x,y,s) = \frac{[\{x + \bar{G}_1(0,x,y,s)\}\,\bar{S}'_1\{\lambda(1-x)+s\} - \bar{G}_2(0,x,y,s)]}{\left[1 - \frac{y}{x}\,\bar{S}_1\{\lambda(1-x)+s\}\right]} \times \frac{[1-\bar{S}_1\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} +$$

$$+ \frac{[\bar{G}_2(0,x,y,s)\,\bar{S}'_2\{\lambda(1-x)+s\} - \bar{G}_1(0,x,y,s)]}{\left[1 - \frac{y}{x}\bar{S}_2\{\lambda(1-x)+s\}\right]} \times \frac{[1-\bar{S}_2\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} +$$

$$+ (x + \bar{G}_1(0,x,y,s)) \times \frac{[1-\bar{S}'_1\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} + \bar{G}_2(0,x,y,s) \times \frac{[1-\bar{S}'_2\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} \qquad (28)$$

The Laplace transform of the generating function of queue length distribution irrespective of the number of services completed is given by

$$\bar{\pi}(x,1,s) = \frac{[\{x + \bar{G}_1(0,x,1,s)\}\,\bar{S}'_1\{\lambda(1-x)+s\} - \bar{G}_2(0,x,1,s)]}{\left[1 - \frac{1}{x}\,\bar{S}_1(\lambda(1-x)+s)\right]} \times \frac{[1-\bar{S}_1\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} +$$

$$+ \frac{[\bar{G}_2(0,x,1,s)\,\bar{S}'_2\{\lambda(1-x)+s\} - \bar{G}_1(0,x,1,s)]}{\left[1 - \frac{1}{x}\,\bar{S}_2\{\lambda(1-x)+s\}\right]} \times \frac{[1-\bar{S}_2\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} +$$

$$+ \{x + \bar{G}_1(0,x,1,s)\} \times \frac{[1-\bar{S}'_1\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} +$$

$$+ \bar{G}_2(0,x,1,s) \times \frac{[1-\bar{S}'_2\{\lambda(1-x)+s\}]}{\lambda(1-x)+s} \qquad (29)$$

The steady state distribution of the queue length distribution can be obtained by taking the limit $s \to 0$ $s\bar{\pi}(x,1,s) = \pi(x,1)$

$$\pi(x,1) = K\left[\frac{\bar{S}_1\{\lambda(1-x)\}\,[\bar{S}'_1\{\lambda(1-x)+s\} - 1]}{x - \bar{S}_1\{\lambda(1-x)+s\}} + \frac{\bar{S}_2\{\lambda(1-x)\}\,[\bar{S}'_2\{\lambda(1-x)+s\} - 1]}{x - \bar{S}_2\{\lambda(1-x)+s\}}\right] \qquad (30)$$

where

$$K = \left[\frac{\psi_1}{1-\lambda\eta_1} + \frac{\psi_2}{1-\lambda\eta_2}\right] \qquad (31)$$

Therefore, the mean number of units in the system is given by

$$\underset{x\to 1}{\text{Limit}}\ \frac{d\pi(x,1)}{dx} = \frac{K\lambda}{2}\left[\frac{\lambda\eta_1'\mu_1' + (1-\lambda\eta_1)(2\eta_1\eta_1' + \mu'_1)}{(1-\lambda\eta_1)^2} + \right.$$

$$\left. + \frac{\lambda\eta_2'\mu_2 + (1-\lambda\eta_2)(2\eta_2\eta'_2 + \mu'_2)}{(1-\lambda\eta_2)^2}\right] \qquad (32)$$

where

$$\eta_i = \int_0^\infty x\,S_i(x)\,dx$$

$$\eta_i' = \int_0^\infty x \, S_i'(x) \, dx$$

$$\mu_i = \int_0^\infty x^2 \, S_i(x) \, dx$$

$$\mu_i' = \int_0^\infty x^2 \, S_i'(x) \, dx \quad (i = 1, 2) \tag{33}$$

The system of equations (1) to (6) can be solved for any other given initial conditions.

DISCUSSION OF RESULTS

In the earlier paper[6] it was assumed that when first server is replaced by second server a changeover time is required and the empty state of the system also exists. The expression for the probability that no unit is present in the system, i.e., the proportion of time the system is idle is given by

$$H = 2\,H(1) = 2\left[\frac{1+\lambda\,\eta_1'}{1-\lambda\,\eta_1} + \frac{1+\lambda\,\eta_2'}{1-\lambda\,\eta_2}\right]^{-1} \tag{34}$$

The expression for the probability for the proportion of time the system switchover takes place without servicing a unit is given by

$$P = \lim_{\substack{x\to 0\\ s\to 0}} \left[\int_0^\infty s\,\overline{G}_1(\phi, x, 1, s)\,\psi_1(\phi)\,d\phi + \int_0^\infty s\,\overline{G}_2(\phi, x, 1, s)\,\psi_2(\phi)\,d\phi\right] \tag{35}$$

After substituting the values and simplifying

$$P = \left[\frac{\eta_1'}{1-\lambda\,\eta_1} + \frac{\eta_2'}{1-\lambda\,\eta_2}\right]^{-1} [\overline{S}_1'(\lambda) + \overline{S}_2'(\lambda)] \tag{36}$$

Assuming $\overline{S}_1'(\lambda) = \dfrac{\eta_1'}{\lambda + \eta_1'}$ and $\overline{S}_2'(\lambda) = \dfrac{\eta_2'}{\lambda + \eta_2'}$

i.e., the changeover time distribution is exponential

$$P = \left[\frac{\eta_1'}{1-\lambda\,\eta_1} + \frac{\eta_2'}{1-\lambda\,\eta_2}\right]^{-1}\left[\frac{\eta_1'}{\lambda+\eta_1'} + \frac{\eta_2'}{\lambda+\eta_2'}\right] \tag{37}$$

Substituting

$$\lambda\,\eta_i = \rho_i$$

$$\lambda\,\eta_i' = \rho_i' \qquad (i = 1, 2)$$

expressions (34) and (37) become respectively

$$H = \left(\frac{1+\rho_1'}{1-\rho_1} + \frac{1+\rho_2'}{1-\rho_2}\right)^{-1} \tag{38}$$

$$P = \lambda\left(\frac{\rho_1'}{1-\rho_1} + \frac{\rho_2'}{1-\rho_2}\right)^{-1}\left(\frac{\rho_1'}{\rho_1'+\lambda^2} + \frac{\rho_2'}{\rho_2'+\lambda^2}\right) \tag{39}$$

In order to study the behaviour of P and H a programme in Fortran language was written and executed on IBM 1620 computer. The parameters λ_i, ρ_i and ρ_i' (i=1, 2) take values from 0.1 to 0.9 with rate of increment as 0.1.

CONCLUSION

The expression for P which represents system changeover time without servicing a unit depends upon the form of changeover time distribution whereas the expressions for H which represents idle time spent by the system without servicing a unit is independent of the form of changeover time distribution. After going through detailed numerical results obtained through computer the following conclusion is made. In the region under consideration, i.e., λ_i, ρ_i and ρ'_i $(i=1, 2)$ all take values from 0.1 to 0.9 it has been found that $P > H$ in greater part of this region. Therefore, the model[7] in which empty state of the system exists is more efficient than the model considered in this paper. This trend can be easily seen from the table 1.

TABLE 1

COMPARISON OF TWO MODLTS

	ρ_1/ρ_1		0·1	0·2	0·3	0·4	0·5	0·N	0·7	0·8	0·9
$\lambda=0{\cdot}3$	0·1	H	0·3364	0·3200	0·3010	0·2790	0·2531	0·2222	0·1846	0·1379	0·0784
		P	0·6591	0·6444	0·6265	0·6041	0·5754	0·5370	0·4833	0·4027	0·2685
$\rho_2=0{\cdot}2$	0·2	H	0·3243	0·3076	0·2886	0·2666	0·2409	0·2105	0·1739	0·1290	0·0727
		P	0·6255	0·6023	0·5750	0·5421	0·5019	0·4517	0·3872	0·3011	0·1807
$\rho'_2=0{\cdot}4$	0·3	H	0·4090	0·3809	0·3500	0·3157	0·2777	0·2352	0·1875	0·1333	0·0714
		P	0.7826	0.6956	0.6086	0.5217	0.4347	0.3478	0.2608	0.1739	0.0869
$\lambda=0{\cdot}5$	0·4	H	0·1867	0·1801	0·1724	0·1630	0·1516	0·1369	0·1181	0·0925	0·0561
		P	0·3114	0·3039	0·2947	0·2833	0·2688	0·2496	0·2230	0·1839	0·1205
$\rho_2=0{\cdot}5$	0·5	H	0·1829	0·1762	0·1682	0·1587	0·1470	0·1324	0·1136	0·0884	0·0531
		P	0·3076	0·2988	0·2882	0·2751	0·2587	0·2375	0·2090	0·1685	0·1065
$\rho'_2=0{\cdot}9$	0·6	H	0·1792	0·1724	0·1643	0·1546	0·1428	0·1282	0·1094	0·0847	0·0585
		P	0·3017	0·2918	0·2800	0·2658	0·2480	0·2255	0·1158	0·1550	0·0954
$\lambda=0{\cdot}4$	0·7	H	0·1757	0·1687	0·1605	0·1507	0·1388	0·1242	0·1056	0·0813	0·0480
		P	0·2580	0·2486	0·2375	0·2242	0·2078	0·1873	0·1609	0·1255	0·0755
$\rho_2=0{\cdot}5$	0·8	H	0·1724	0·1652	0·1569	0·1470	0·1351	0·1204	0·1020	0·0781	0·0458
		P	0·2502	0·2403	0·2286	0·2147	0·1979	0·1770	0·1506	0·1160	0·0686
$\rho'_2=0{\cdot}9$	0·9	H	0·1691	0·1619	0·1535	0·1435	0·1315	0·1169	0·0986	0·0751	0·0438
		P	0·2425	0·2322	0·2201	0·2058	0·1886	0·1677	0·1415	0·1078	0·0628

P—represents the proportion of time the system switchover takes place without servicing a unit.

H—represents the proportion of time the system remains idle.

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