THERMAL BUCKLING OF SOME HEATED PLATES PLACED ON ELASTIC FOUNDATION

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Thermal buckling of a heated equilateral triangular plate and a clamped elliptic plate placed on elastic foundation has been investigated. The boundary of the plate is transformed conformally onto the unit circle. The critical buckling temperature is obtained with the help of error function.

Thermal buckling of thin elastic plates is of much practical importance in modern engineering. Nowacki[1] has discussed the thermal buckling of a rectangular plate under different boundary conditions. Mansfield[2] has investigated the buckling and curling of a heated thin circular plate of constant thickness. Klosner & Forray[3] have studied the thermal buckling of simply supported plates under symmetrical temperature distribution.

Stability of thin elastic plates having exotic boundaries subjected to hydrostatic in-plane loading can easily be investigated with the help of approximate techniques such as collocation, finite difference, finite elements, etc. Laura & Shahady[4] have shown that it is convenient to conformally transform the given domain onto a simpler one, i.e., the unit circle and the boundary conditions can then be satisfied identically.

In this paper thermal buckling of a heated equilateral triangular plate and a clamped elliptic plate placed on elastic foundation has been investigated. The foundation is assumed to be of the Winkler type. The boundary has been transformed conformally onto the unit circle and solution has been obtained with the help of error function.

NOTATIONS

The following notations have been used in this paper :

B_n, B_m = constants ;

D = flexural rigidity of the plate $= \dfrac{Eh^3}{12\,(1-\nu^2)}$,

E = Young's modulus ;

h = plate thickness ;

$$N_T = \alpha E \int_{-\frac{h}{2}}^{\frac{h}{2}} T dz \;;$$

T = temperature ;

u, v = displacement in x and y direction respectively ;

W = deflection normal to the middle plane of the plate ;

K_1 = foundation reaction per unit area per unit deflection ;

α = coefficient of linear thermal expansion.

THEORY

Let us consider a plate of thickness, h, subjected to a temperature distribution T which is independent of x and y, but varies arbitrarily through the thickness, i.e.,

$$T = T\,(z)$$

The plate is subjected to no external load and motion of all supports in the plane of the plate is prevented. It justifies then, that under the above condition there are no displacements in the plane of the plate, i.e.,

$$u = v = 0$$

On the above propositions the differential equation for the displacement[5] is

$$D \nabla^4 W + \frac{N_T}{1 - \nu} \nabla^2 W = 0 \tag{1}$$

For a plate placed on elastic foundation having the foundation reaction, K_1, (1) becomes

$$D \nabla^4 W + \frac{N_T}{1 - \nu} \nabla^2 W + K_1 W = 0 \tag{2}$$

Eq. (2) may be written as

$$(\nabla^2 + P_1^2)(\nabla^2 + P_2^2) W = 0 \tag{3}$$

in which

$$P_1^2 P_2^2 = \frac{K_1}{D} \tag{4}$$

$$P_1^2 + P_2^2 = \frac{N_T}{D(1 - \nu)} \tag{5}$$

If $z = x + iy$, $\bar{z} = x - iy$ Eq. (3) changes into

$$\left(4 \frac{\partial^2}{\partial z \, \partial \bar{z}} + P_1^2 \right) \left(4 \frac{\partial^2}{\partial z \, \partial \bar{z}} + P_2^2 \right) W = 0 \tag{6}$$

Let $z = f(\xi)$ be the analytic function which maps the given shape in the ξ-plane onto a unit circle. Thus (6) transforms into complex co-ordinates as

$$\left(\nabla^2 + P_1^2 \left[\frac{dz}{d\xi} \right]^2 \right) \left(\nabla^2 + P_2^2 \left[\frac{dz}{d\xi} \right]^2 \right) W(\xi, \bar{\xi}) = 0 \tag{7}$$

Eq. (7) is written as

$$(\nabla^2 + \lambda_1^2)(\nabla^2 + \lambda_2^2) W(\xi, \bar{\xi}) = 0 \tag{8}$$

in which

$$\lambda_1^2 = P_1^2 (dz/d\xi)^2, \quad \lambda_2^2 = P_2^2 (dz/d\xi)^2$$

Let

$$W = \sum_{n=1}^{\infty} B_n \left[1 - (\xi \bar{\xi})^n \right] \tag{9}$$

Clearly the above form of W satisfies the edge condition $W = 0$ at $r = 1$. Putting (9) in (8) one gets the error function, $\epsilon_{r,\theta}$. Galerkin's procedure requires that the error function to be orthogonal over the domain, i.e.,

$$\int_C \epsilon_{r,\theta} (\xi, \bar{\xi}) W(\xi, \bar{\xi}) \, dC = 0. \quad (n = 1, 2, \ldots\ldots K) \tag{10}$$

This generates $(K \times K)$ determinantal equation. The lowest root of this gives the critical buckling temperature.

APPLICATIONS

(*I*) Let us consider an equilateral triangular plate of side, $2a$, and placed on an elastic foundation. To solve the differential (8) let us put

$$W = W_1 + W_2 \tag{11}$$

From (8) one gets

$$(\nabla^2 + \lambda_1^2)\, W_1 = 0 \tag{12}$$

$$(\nabla^2 + \lambda_2^2)\, W_2 = 0 \tag{13}$$

For the edge condition $W = 0$ along the boundary, let

$$W_2 \approx \sum_{n=1}^{K} B_n \left[1 - (\xi\, \bar{\xi})^n \right] = \sum_{n=1}^{K} B_n \, (1 - r^{2n}) \tag{14}$$

It is sufficient to solve either (12) or (13). The mapping function

$$z = 1 \cdot 1352\, a \left[\xi + \frac{1}{6}\, \xi^4 + \frac{5}{63}\, \xi^7 + \frac{4}{81}\, \xi^{10} \right] \tag{15}$$

maps an equilateral triangular plate a unit circle in the ξ-plane.

With this mapping function putting (14) in (13) and remembering $\xi = re^{i\theta}$ one gets the required error function. After evaluating the integral given by (10) and taking K=2, the following determinant is obtained.

$$\begin{vmatrix} \dfrac{\lambda_2^2}{6} - 1 & \dfrac{5\lambda_2^2}{24} - \dfrac{4}{3} \\ \dfrac{5\lambda_2^2}{24} - \dfrac{4}{3} & \dfrac{4\lambda_2^2}{15} - 2 \end{vmatrix} = 0 \tag{16}$$

Solving (16) for the lowest root, one gets the critical buckling temperature.

$$(N_T)_{Cr} = D\,(1 - \nu) \left[\frac{5 \cdot 8}{(1 \cdot 1352\, a)^2} - \frac{K_1\, (1 \cdot 1352\, a)^2}{5 \cdot 8\, D} \right] \tag{17}$$

(*II*) Let us consider an elliptic plate having centre at the origin. Let h be the thickness of the plate. For clamped edge boundary condition let us take W in the following form.

$$W = \sum_{n=1}^{K} B_n \left[1 - (\xi\, \bar{\xi})^n \right]^2 \tag{18}$$

Clearly

$$W = \frac{\partial W}{\partial r} = 0 \qquad \text{at } r = 1$$

For the ellipse

$$\left(\frac{x^2}{4/3} + \frac{y^2}{4/5} \right) = 1$$

mapping function

$$z = 0 \cdot 99\, b\, (\xi + 0 \cdot 12\, \xi^3 + 0 \cdot 03\, \xi^5 + 0 \cdot 01\, \xi^7) \tag{19}$$

maps the above ellipse a unit circle in the ξ-plane. With this mapping function putting (18) in (8) and remembering $\xi = re^{i\theta}$ one gets the required error function. After evaluating the integral given by (10) and taking $K = 2$, the following determinant is obtained.

$$\begin{vmatrix} \dfrac{32}{3} - \dfrac{2}{3}\left(\lambda_1^2 + \lambda_2^2\right) + \dfrac{\lambda_1^2 \lambda_2^2}{10} & \dfrac{256}{15} - \dfrac{4}{5}\left(\lambda_1^2 + \lambda_2^2\right) + \dfrac{33}{70}\,\lambda_1^2 \lambda_2^2 \\ \dfrac{256}{15} - \dfrac{4}{5}\left(\lambda_1^2 + \lambda_2^2\right) + \dfrac{29}{576}\,\lambda_1^2 \lambda_2^2 & \dfrac{5632}{105} - \dfrac{4}{3}\left(\lambda_1^2 + \lambda_2^2\right) + \dfrac{127}{315}\,\lambda_1^2 \lambda_2^2 \end{vmatrix} = 0 \tag{20}$$

Solving (20) for the lowest root, the critical buckling temperature is obtained as

$$(N_T)_{Cr} = D\,(1-\nu)\left[\frac{44\cdot 6}{b^2} + 0\cdot 094\, b^2\,\frac{K_1}{D} - \right.$$
$$\left. -\left\{\left(\frac{29\cdot 2}{b^2} + 0\cdot 094\, b^2\,\frac{K_1}{D}\right)^2 + 0\cdot 106\, b^4\left(\frac{K_1}{D}\right)^2\right\}^{\frac{1}{2}}\right] \qquad (21)$$

CONCLUSION

Solutions obtained in this study are only approximate, because only the first term of the mapping function is considered and K is taken to be 2. More accurate results are obtained by considering the remaining terms of the mapping function and taking K more than 2. Solution of the eigenvalue problem governing the stability of the thin elastic plates having various configurations, such as regular polygonal shape, circular boundary with flat sides, epitrochoidal boundary etc. is easily accomplished with the help of the complex variable theory applied in this study.

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