ON ATTAINMENT OF CONSTANT PRESSURE PHASE IN AN ORTHODOX GUN WITH RESISTANCE

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In this paper the conditions for the existence of constant pressure phase for certain period during burning in an orthodox gun with constant bore resistance have been obtained and then the internal ballistics is calculated.

The internal ballistics of an orthodox gun with constant bore resistance having single charge was discussed by Clemmow[1], Kapur[2] and Ray[3]. Kapur discussed the problem in a general non-isothermal model and Ray discussed the problem in an isothermal model and obtained and approximate solution which is valid for M around unity. He also integrated the equation numerically and obtained some numerical results which the author uses for numerical calculation. It is very difficult to solve for the internal ballistics of the gun having moderated charges with constant bore resistance. But it has been demonstrated that if a moderated charge of two components (of which first in known and the second component is also known except for the size and shape) burns in an orthodox gun the pressure can be kept constant during the period of burning of the second component. Generally, the solution determines two relations from which the size and shape of the second propellant component can be determined, the solution during the first stage of burning are assumed to be known.

FIRST STAGE OF BURNING

Ballistic equations when the first component burns with usual notations the basic equations are

$$F_1 C_1 Z_1 = P \left\{ K_0 + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} - C_1 Z_1 \left(b_1 - \frac{1}{\delta_1} \right) \right\} + \frac{1}{2} \left(\gamma - 1 \right) \omega_1 v^2 + + \left(\gamma - 1 \right) A P_R x \tag{1}$$

$$D_1 \frac{df_1}{dt} = - \beta_1 P \tag{2}$$

$$\omega_1 v \frac{dv}{dx} = AP - AP_R \tag{3}$$

where P is the pressure at the shot and AP_R is the constant resistance and

$$Z_1 = (1 - f_1)(1 + \theta_1 f_1) \tag{4}$$

These equations are to be solved with initial conditions,

$$x = 0, \; v = 0, \; P = P_0, \; Z_1 = Z_{1\,0} \text{ at } f_1 = f_{1\,0} \tag{5}$$

Now with the following usual substitutions

$$\left.\begin{aligned} &\xi = 1 + \frac{x}{l}, \quad && Al = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} \\ &\eta = \frac{v A_1 D_1}{F_1 C_1 \beta_1}, \quad && \zeta = P \frac{Al}{F_1 C_1} \\ &\zeta_R = \frac{P_R Al}{F_1 C_1}, \quad && M_1 = \frac{A^2 D_1^2}{F_1 C_1 \beta_1^2 \omega_1} \\ &\text{and} && \\ &B_1 = \left(b_1 - \frac{1}{\delta_1} \right) \frac{C_1}{Al} && \end{aligned}\right\} \tag{6}$$

The above ballistic equations reduces to

$$Z_1 = \zeta \left(\xi - B_1 Z_1\right) + \frac{1}{2}\left(\gamma - 1\right)\frac{\eta^2}{M_1} + \left(\gamma - 1\right)\zeta_R\left(\xi - 1\right) \tag{7}$$

$$\eta \frac{d\eta}{d\xi} = M_1 (\zeta - \zeta_R) \tag{8}$$

$$\eta \frac{df}{d\xi} = -\zeta \tag{9}$$

$$Z_1 = (1 - f_1)(1 + \theta_1 f_1) \tag{10}$$

now the initial conditions are

$$\xi = 1,\ \eta = 0,\ \zeta = \zeta_0 \text{ at } Z_1 = Z_{10} \text{ or at } f_1 = f_{10}$$

Kapur explained the method of solving the above equation neglecting the co-volume effect. However Clemmow solved the problem with single charge in an isothermal model. Recently Ray solved the problem considering isothermal condition and neglecting co-volume teım. He got an approximate solution of the above equation considering $\theta = 0$. Here we note that for solving these equations we are supposing C_2 and δ_2 (or C_2/δ_2) as known for Al involves these quantities.. However suppose the solutions of the above equations gives ξ_{B1}, η_{B1}, ζ_{B1} the values of ξ, η and ζ at the instant when the first component burns but as these quantities are required for the integration of the ballistic equations during the second stage of burning.

SECOND STAGE OF BURNING

Ballistic equations when the second component burns

$$F_1 C_1 + F_2 C_2 Z_2 = P\left\{K_0 + Ax - \frac{C_2}{\delta_2} - C_2 Z_2\left(b_2 - \frac{1}{\delta_2}\right) - C_1 b_1\right\} + \\ + \frac{1}{2}\omega_1 (\gamma - 1) v^2 + AP_R x (\gamma - 1) \tag{11}$$

Further we have the equation of motion

$$\omega_1 \frac{dv}{dt} = A (P - P_R) \tag{12}$$

The equation of burning

$$D_2 \frac{df_2}{dt} = -\beta_2 P \tag{13}$$

and the form function

$$Z_2 = (1 - f_2)(1 + \theta_2 f_2) \tag{14}$$

We are to obtain a solution of these equations with initial conditions $x = x_{B1}$, $P = P_{B1}$, $v = v_{B1}$ at $f_2 = 1$. Here x_{B1}, v_{B1} and P_{B1} are the values of x, v, P when the first component has just burnt out.

Let us assume that a solution of the above equation is possible with

$$P = P_{B1} \tag{15}$$

and seek conditions that his solution may give $x = x_{B1}$, $v = v_{B1}$ at $f_2 = 1$ and that the system of equations (11) to (14) may remain consistent for the solution $P = P_{B1}$.

Now from (12) and (13) with (15)

$$\frac{dv}{df_2} = -\frac{AD_2}{\beta_2 \omega_1}\ \frac{P_{B1} - P_R}{P_{B1}}$$

or

$$\frac{d}{df_2}\left[\frac{dx}{df_2} \cdot \frac{df_2}{dt}\right] = -\frac{AD_2}{\beta_2 \omega_1}\ \frac{P_{B1} - P_R}{P_{B1}} \tag{16}$$

By (13) and (15), the above equation reduces to

$$\frac{d^2x}{df_2^2} = \frac{AD_2^2}{\beta_2^2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}} \tag{17}$$

Integrating (16) with initial conditions that $v = v_{B1}$ at $f_2 = 1$ we have

$$v = v_{B1} + \frac{AD_2}{\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\left(1-f_2\right) \tag{18}$$

Taking (18) and (15) into consideration (11) may be written as

$$F_1 C_1 + F_2 C_2 Z_2 = P_{B1}\left\{K_0 + Ax - \frac{C_2}{\delta_2} - C_2 Z_2\left(b_2 - \frac{1}{\delta_2}\right) - C_1 b_1\right\} +$$

$$+\frac{1}{2}\,\omega_1(\gamma-1)\left\{v_{B1} + \frac{AD_2}{\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\,(1-f_2)\right\}^2 + AP_R\,x(\gamma-1) \tag{19}$$

The equations (19) and (14) will give x as a function of f_2 during the second stage of burning. Now we impose the conditions that (19) should give $x = x_{B1}$, $v = v_{B1}$ at $f_2 = 1$ and further that (19) be consistent with (17). Now $x = x_{B1}$, $f_2 = 1$ and $Z_2 = 0$ will satisfy (19) if

$$F_1 C_1 = P_{B1}\left\{K_0 + Ax - \frac{C_2}{\delta_2} - C_1 b_1\right\} + \frac{1}{2}\,\omega_1(\gamma-1)\,v_{B1}^2 + AP_R\,x_{B1}(\gamma-1)$$

which is true, for $x = x_{B1}$, $v = v_{B1}$, $P = P_{B1}$, $f_1 = 0$, $Z_1 = 1$ satisfy equation (1) and this equation for these values of these variables is identical with the above equation.

Now differentiating (19) and (14) with respect to f_2 we have

$$F_2 C_2\,\frac{dZ_2}{df_2} = P_{B1}\left\{A\,\frac{dx}{df_2} - C_2\left(b_2 - \frac{1}{\delta_2}\right)\frac{dZ_2}{df_2}\right\} -$$

$$-\left(\gamma-1\right)\frac{AD_2}{\beta_2}\,\frac{P_{B1}-P_R}{P_{B1}}\left\{v_{B1} + \frac{AD_2}{\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\left(1-f_2\right)\right\} +$$

$$+ A\left(\gamma-1\right)P_R\,\frac{dx}{df_2}$$

and

$$\frac{dZ_2}{df_2} = \left(1-f_2\right)\theta_2 - \left(1+\theta_2 f_2\right)$$

and these last two equations with (13) and (15) give

$$\left\{F_2 C_2 + P_{B1} C_2\left(b_2 - \frac{1}{\delta_2}\right)\right\}\frac{dZ_2}{df_2} = -\frac{AD_2}{\beta_2}\left\{1+(\gamma-1)\,\frac{P_R}{P_{B1}}\right\}\frac{dx}{dt} -$$

$$-(\gamma-1)\,\frac{AD_2}{\beta_2}\,\frac{P_{B1}-P_R}{P_{B1}}\left[v_{B1} + \frac{AD_2}{\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\,(1-f_2)\right]$$

Now

$$v = \frac{dx}{dt} = v_{B1} \text{ at } f_2 = 1$$

we have

$$(1+\theta_2)\left[F_2 C_2 + P_{B1} C_2\left(b_2 - \frac{1}{\delta_2}\right)\right] = \frac{AD_2}{\beta_2}\,v_{B1}\left\{1+(\gamma-1)\,\frac{P_R}{P_{B1}}\right\} +$$

$$+(\gamma-1)\,\frac{AD_2}{\beta_2}\,v_{B1}\,\frac{P_{B1}-P_R}{P_{B1}}$$

Hence, we get

$$1+\theta_2=\frac{\gamma A\;\dfrac{D_2}{\beta_2}\;v_{B1}}{F_2\,C_2+P_{B1}\,C_2\left(b_2-\dfrac{1}{\delta_2}\right)} \tag{20}$$

Now in order to satisfy the condition that (17) and (19) should be consistent we differentiate (19) twice with respect to f_2.

Now we have

$$\frac{d^2 Z_2}{df_2^2}\left[F_2\,C_2+P_{B1}\,C_2\left(b_2-\frac{1}{\delta_2}\right)\right]=\frac{A^2\,D_2^2}{\beta_2^2\,\omega_1}\left(1-\frac{P_R}{P_{B1}}\right)+(\gamma-1)\frac{A^2\,D_2^2}{\beta_2^2\,\omega_1}\left(1-\frac{P_R}{P_{B1}}\right)^2+$$
$$+\frac{A^2\,D_2^2}{\beta_2^2\,\omega_1\,P_{B1}}(\gamma-1)\,P_R\left(1-\frac{P_R}{P_{B1}}\right)$$

Since
$$\frac{d^2 Z_2}{df_2^2}=-2\,\theta_2 .$$

We have

$$2\,\theta_2=\frac{-\dfrac{A^2\,D_2^2}{\beta_2^2\,\omega_1}\left[1-\dfrac{P_R}{P_{B1}}+\left(\gamma-1\right)\left(1-\dfrac{P_R}{P_{B1}}\right)^2+(\gamma-1)\dfrac{P_R}{P_{B1}}\left(1-\dfrac{P_R}{P_{B1}}\right)\right]}{F_2\,C_2+P_{B1}\,C_2\left(b_2-\dfrac{1}{\delta_2}\right)} \tag{21}$$

The simultaneous satisfaction of the Equations (20) and (21) gives the condition that $P=P_{B1}$ may be the solution of the equations (11) to (14). From (21) it is evident that θ_2 must be negative also $\dfrac{P_R}{P_{B1}}$ is considered to be less than unity in order to have the forward motion of the shot. Hence the second propellant component of moderated charge should have progressively increasing burning surface.

Let us introduce the following dimensionless constants

$$\frac{F_2\,C_2}{F_1\,C_1}=\beta_0,\quad \frac{D_2/\beta_2}{D_1/\beta_1}=\alpha_0,\quad B_2=\left(b_2-\frac{1}{\delta_2}\right)\frac{C_2}{Al}$$

and

$$1-\frac{P_R}{P_{B1}}=P_R'$$

We have the required conditions

$$1+\theta_2=\frac{\gamma\,\alpha_0\,\eta_{B1}}{\beta_0+\zeta_{B1}\,B_2} \tag{22}$$

and

$$2\,\theta_2=-\frac{\gamma\,M_1\,\alpha_0^2\,P_R'}{\beta_0+\zeta_{B1}\,B_2} \tag{23}$$

Equations (22) and (23) actually give two relations connecting four parameters α_0, β_0, θ_2 and B_2 defining second propellant component, the properties and mass of the first component being assumed to be known. B_2 involves C_2, b_2 and δ_2 of which C_2 and δ_2 or $\dfrac{C_2}{\delta_2}$ were as noted earlier, supposed to be known in the integrations of the equations for the first stage of burning. When b_2 is given B_2 may be taken to be known for a given gun. Again β_0 involves F_2 and C_2 and with suitable choice of the propellant one may suppose that β_0 is also known. Hence we may look upon (22) and (23) as two equations for θ_2 and α_0. To complete our solution we should show that, as given by (22) and (23) α_0 is positive and θ_2 satisfies $-1 < \theta_2 \leqslant 1$ for practical values of constants involves in (22) and (23).

Eliminating θ_2 from (22) and (23) one gets

$$\gamma M_1 \alpha_0^2 P_R' + 2 \gamma \alpha_0 \eta_{B1} - 2 (\beta_0 + \zeta_{B1} B_2) = 0$$

Taking the root with positive sign before the radical one we have

$$\alpha_0 = -\frac{\eta_{B1}}{M_1 P_R'} + \left[\left(\frac{\eta_{B1}}{M_1 P_R'} \right)^2 + \frac{2 (\beta_0 + \zeta_{B1} B_2)}{\gamma M_1 P_R'} \right]^{\frac{1}{2}} \tag{24}$$

and

$$\theta_2 = \frac{\gamma \alpha_0 \eta_{B1}}{\beta_0 + \zeta_{B1} B_2} - 1 \tag{25}$$

Recently Ray in his paper obtained some numerical results about the ballistic variables for the Orthodox gun with constant bore resistance in an isothermal model for the tubular propellant only.

As the numerical results of the ballistic variables for the orthodox gun with constant bore resistance for the general non-isothermal model are not known, to get some idea about the results we consider the numerical values from Ray's Table. Also we take $P_R' \leqslant 1$ as explained before.

Let $M_1 = 1, \quad \gamma = 1{\cdot}25, \quad \theta_1 = 0, \quad B_1 = 0$ and $\zeta_0 = 0{\cdot}1$

Then we take from tables in Ray's paper

TABLE 1

$\zeta_{B1} = 0{\cdot}665, \eta_{B1} = 0{\cdot}655$, taking $\beta_0 = 1$ and $B_2 = 0{\cdot}25$

P_R' (Resistance)	α_0 (Size)	θ_2 (Shape)
0·9	0·8840	—0·3793
0·7	0·9453	—0·3365
0·5	1·044	—0·2670
0·3	1·134	—0·0453
0·1	1·295	—0·0183

TABLE 2

$\zeta_0 = 0{\cdot}3$

P_R'	α_0	θ_2
0·9	1·1546	—0·6125
0·7	1·2675	—0·5746
0·5	1·427	—0·5212
0·3	1·685	—0·4347
0·1	2·225	—0·2534

Let $M_1 = 2, \quad \gamma = 1{\cdot}25, \quad \theta_1 = 0, \quad B_1 = 0$ and $\zeta_0 = 0{\cdot}1$

We take from tables in Ray's paper

TABLE 3

$\zeta_{B1} = 0{\cdot}468, \eta_{B1} = 1{\cdot}237$, taking $\beta_0 = 1, B_2 = 0{\cdot}25$

P_R'	α_0	θ_2
0·9	0·5227	—0·2762
0·7	0·5503	—0·2381
0·5	0·5730	—0·2066
0·3	0·6270	—0·1318
0·1	0·6850	—0·0516

Here we notice that in all the cases α_0 is positive and θ_2 lies between -1 and $+1$. It will be of interest to know how far the shot travels when the second component burns. From (18), (13) and (15) we have

$$-\frac{dx}{df_2}\,\frac{\beta_2 P_{B1}}{D_2} = v_{B1} + \frac{AD_2}{\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\,(1-f_2)$$

Integrating this taking $x = x_{B1}$ at $f_2 = 1$

$$\frac{\beta_2 P_{B1}}{D_2}(x - x_{B1}) = v_{B1}(1-f_2) + \frac{AD_2}{2\,\beta_2\,\omega_1}\,\frac{P_{B1}-P_R}{P_{B1}}\,(1-f_2)^2$$

Let $x = x_{B2}$ when the second component burns out i.e. $f_2 = 0$. Therefore

$$x_{B2} - x_{B1} = \frac{D_2}{\beta_2 P_{B1}}\left[v_{B1} + \frac{AD_2}{2\,\beta_2\,\omega_1}(1-P_R')\right]$$

or

$$\frac{x_{B2}-x_{B1}}{x_{B1}} = \frac{D_2}{\beta_2 P_{B1}\, x_{B1}}\left[v_{B1} + \frac{AD_2}{2\,\beta_2\,\omega_1}(1-P_R')\right]$$

$$= \frac{1}{\zeta_{B1}(\xi_{B1}-1)}\left[\alpha_0\,\eta_{B1} + \frac{M_1\,\alpha_0^2}{2}(1-P_R')\right]$$

Therefore,

$$\text{travel ratio} = \frac{\text{travel during the second stage of burning}}{\text{travel during the first stage of burning}}$$

$$= \frac{1}{\zeta_{B1}(\xi_{B1}-1)}\left[\alpha_0\,\eta_{B1} + \frac{M_1\,\alpha_0^2}{2}(1-P_R')\right] \qquad (26)$$

Also from (18) we can show that the velocity ratio

$$\frac{v_{B2}}{v_{B1}} = 1 + \alpha_0\,\frac{M_1}{\eta_{B1}}(1-P_R') \qquad (27)$$

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