ON PIEZOMETRIC EFFICIENCY IN AN ORTHODOX GUN WITH RESISTANCE

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It has been shown that the muzzle velocity and the piezometric efficiency have less values in case of a single charge of an orthodox gun with constant bore resistance than in case of the gun without resistance.

Recently Ray[1] in his paper obtained an approximate solution of the internal ballistics in an orthodox gun with constant bore resistance from shot-start to all burnt in an isothermal model. This solution of the internal ballistics is in fair agreement with the solution given by Hunt-Hinds[2] systems for values of M around unity (M is the central Ballistic Parameter). Kapur[3] in his paper discussed the ballistic effect by a concentrated bore resistance as well as by long stretches of bore resistance for the general non-isothermal model. The author tried to establish some results about the effect of constant bore resistance on the muzzle velocity and the piezometric efficiency. The muzzle velocity and the piezometric efficiency in an orthodox gun without resistance was discussed by the author[4] in his previous paper. An approximate expression for the muzzle velocity and piezometric efficiency in an orthodox gun with constant bore resistance are obtained and the results are compared with the results obtained in the case of orthodox gun having no resistance. It has been seen from the approximate solution that the muzzle velocity and piezometric efficiency both will decrease by the constant resistance. The whole discussion is confined only to tubular propellants but can be easily extended to cover quadratic form functions and co-volume.

BALLISTIC EQUATIONS FOR THE SINGLE CHARGE

The co-volume term is entirely neglected and the kinetic energy term in the energy equation is neglected during the burning of the charge. The shot start pressure is taken to be zero. Then the equations are

$$FCZ = p\,(K_0 + Ax - C/\delta) \tag{1}$$

$$\omega_1 v \frac{dv}{dx} = Ap \tag{2}$$

$$D \frac{df}{dt} = -\beta p \tag{3}$$

and

$$Z = (1 - f) \tag{4}$$

Making the following transformations

$$\xi = 1 + \frac{x}{l}\,,\ Al = K_0 - \frac{C}{\delta}$$

$$\eta = \frac{vAD}{FC\beta}\,,\ \zeta = \frac{p\,Al}{FC} \tag{5}$$

and

$$M = \frac{A^2 D^2}{FC\beta^2 \omega_1}$$

The above equations can be made dimensionless and the reduced equations are

$$Z = \zeta\, \xi \tag{6}$$

$$M\zeta = \eta \frac{d\eta}{d\xi} \tag{7}$$

$$\zeta = -\eta \frac{df}{d\xi} \tag{8}$$

$$Z = 1 - f \tag{9}$$

The above equations were solved by the method of crow given in H. M. S. O.[2]

The solution gives

$$f = 1 - \frac{\eta}{M} \tag{10}$$

$$\xi = e^{M(1-f)} \tag{11}$$

and ζ is obtained from (9), (10) and (11).

Now the burnt values are

$$\zeta_B = \frac{1}{e^M}\ ,\quad \eta_B = M,\quad v_B = \frac{FC\beta}{AD}\, M \tag{12}$$

and

$$x_B = l\,(e^M - 1)$$

Maximum pressure is given by

$$p_{max} = \frac{FC}{Al} \cdot \frac{1}{e^M} \tag{13}$$

now if $x_B/d > 1$, the shot may leave the muzzle before all burnt of the charge and if $x_B/d < 1$, the shot may leave the muzzle after all burnt of the charge. Where d is the length of the gun.

The piezometric efficiency is defined to be the ratio of the mean pressure to the maximum pressure.

The muzzle velocity in non-dimensional form is denoted by η_m and the expression for η_m is given by

$$\eta^2_m = M\left[M + \frac{2}{1-\gamma}\left\{ \left(\frac{1 + d/l}{e^M}\right)^{1-\gamma} - 1 \right\}\right] \tag{14}$$

for $\left(\frac{x_B}{d} < 1\right)$

The expression for the piezometric efficiency is given by

Piezometric efficiency =

$$\frac{l}{2d}\, e^M \left[M + \frac{2}{1-\gamma}\left\{ \left(\frac{1 + d/l}{e^M}\right)^{1-\gamma} - 1 \right\}\right] \tag{15}$$

for $\left(\frac{x_B}{d} < 1\right)$

The method of calculation of these two quantities for $(x_B/d > 1)$ was discussed by the author in his previous paper.

Ballistic equations for the single charge with constant resistance from shot-start to all burnt.

The co-volume term is entirely neglected and kinetic energy term in the energy equation is neglected during the burning of the charge.

Then the equations are

$$FCZ = Ap\,(x + l) \tag{16}$$

$$D\,\frac{df}{dt} = -\,\beta p \tag{17}$$

$$\omega\, v\,\frac{dv}{dx} = Ap_s - AA_0 \tag{18}$$

Where p_s is the pressure at the shot base and AA_0 is the constant resistance,

and

$$Z = 1 - f$$

The initial conditions are

$$x = v = 0,\quad p = p_0,\quad Z = Z_0 \quad \text{at } f = f_0$$

Considering the instant of shot-start from (18)

$$A\,(p_s)_0 = AA_0$$

so by Lagrange's correction namely

$$p_s = \frac{p}{1 + \dfrac{C}{3w}}$$

We have

$$AA_0 = A\,(p_s)_0 = \frac{Ap_0}{1 + C/3w}$$

Therefore the equation (18) can be written as

$$\omega_1\, v\,\frac{dv}{dx} = A\,(p - p_0) \tag{19}$$

where

$$\omega_1 = 1{\cdot}05\,\omega + \frac{1}{3}\,C$$

Now with the usual substitutions

$$\xi = 1 + \frac{x}{l}\,,\qquad \zeta_0 = \frac{p_0\,Al}{FC}$$

$$\eta = \frac{vAD}{FC\beta}\,,\qquad \zeta = \frac{pAl}{FC}$$

and

$$M = \frac{A^2 D^2}{FC\beta^2\,\omega_1}$$

The equations (16) to (19) reduce to

$$Z = \zeta \, \xi \tag{20}$$

$$\eta \, \frac{df}{d\xi} = - \zeta \tag{21}$$

$$\eta \, \frac{d\eta}{d\xi} = M (\zeta - \zeta_0) \tag{22}$$

$$Z = (1 - f) \tag{23}$$

and the initial conditions are now

$$\xi = 1, \quad \eta = 0, \quad \zeta = \zeta_0 \text{ at } Z = Z_0 \text{ or at } f = f_0$$

Recently Roy discussed a method of solving the above equations and obtained an approximate solution of the equations. But an approximate solution has been found which compares favourably with exact numerical solution for values of M around unity.

From (20) considering the initial values, we have

$$Z_0 = \zeta_0 \tag{24}$$

From (20) to (23), we have

$$\frac{d\eta}{dZ} = M \left(1 - \frac{\zeta_0}{\zeta} \right) \tag{25}$$

and

$$\frac{d}{dZ} \left(\frac{Z}{\zeta} \frac{d\zeta}{dZ} \right) = - M \left(1 - \frac{\zeta_0}{\zeta} \right) \tag{26}$$

This is a differential equation for the pressure ζ and as the equation is non-linear the approximate solution is obtained by successive approximations.

Since $\zeta = \zeta_0$ initially, as a first approximation we put on the right hand side of (26) and get

$$\frac{d}{dZ} \left(\frac{Z}{\zeta} \frac{d\zeta}{dZ} \right) = 0$$

Integrating with the initial conditions, we have

$$\zeta = Z \tag{27}$$

For the second approximation we put Z for ζ on the right hand side of (26)

Then
$$\frac{d}{dZ} \left(\frac{Z}{\zeta} \frac{d\zeta}{dZ} \right) = - M \left(1 - \frac{\zeta_0}{Z} \right)$$

Integrating subject to the initial condition, we have

$$\log \frac{\zeta}{\zeta_0} = \log \frac{Z}{Z_0} - M (Z - Z_0) + M Z_0 \log \frac{Z}{Z_0} + M \frac{\zeta_0}{2} \left(\log \frac{Z}{Z_0} \right)^2 \tag{28}$$

and

$$\xi = \frac{Z}{\zeta} \tag{29}$$

$$\eta = M\,(Z - Z_0) - M\zeta_0 \log \frac{Z}{Z_0}$$

At all burnt the values of the ballistic variables are

$$\eta_B = M\,(1 - Z_0) + M\zeta_0 \log Z_0$$

$$\xi_B = \zeta_0 \exp \left\{ \frac{M}{2}\, \zeta_0 (\log Z_0)^2 - M\, Z_0 \log Z_0 - M\,(1 - Z_0) - \log Z_0 \right\} \tag{30}$$

and

$$\xi_B = \frac{1}{\zeta_B}$$

Then, we get

$$v_B = \frac{FC\beta}{AD}\, M \left[(1 - Z_0) + \zeta_0 \log Z_0 \right] \tag{31}$$

$$x_B = l \left(\frac{1}{\zeta_B} - 1 \right) \tag{32}$$

and

$$p_B = \frac{FC}{Al}\, \zeta_0 \exp \left[\zeta_0 (\log Z_0)^2 - M\, Z_0 \log Z_0 - M\,(1 - Z_0) - \log Z_0 \right] \tag{33}$$

Again after all burnt the gas expands adiabatically and the corresponding equations are

$$\omega_1 v \frac{dv}{dx} = A\,(p - p_0)$$

$$(p - p_0)\, V^{\gamma} = \text{constant}$$

Where V represents the volume

or

$$p - p_0 = (p_B - p_0) \left(\frac{x_B + l}{x + l} \right)^{\gamma}$$

Hence

$$v \frac{dv}{dx} = \frac{A}{\omega_1}\, (p_B - p_0)\, (x_B + l)^{\gamma} \cdot \frac{1}{(x + l)^{\gamma}}$$

Integrating one gets

$$v^2 = {v_B}^2 + \frac{2A}{\omega_1}\, \frac{p_B - p_0}{1 - \gamma}\, (x_B + l)^{\gamma} \left[(x + l)^{1-\gamma} - (x_B + l)^{1-\gamma} \right]$$

since $x = x_B$ when $v = v_B$ which from (31) and (32) reduces to

$$v^2 = \frac{F^2 C^2 \beta^2 M^2}{A^2 D^2} \left\{ (1 - Z_0) + \zeta_0 \log Z_0 \right\}^2 +$$

$$+ \frac{2\,A}{\omega_1}\, \frac{FC}{Al}\, \frac{\zeta_B - \zeta_0}{1 - \gamma}\, \frac{l^{\gamma}}{\zeta_B{}^{\gamma}} \left[(x + l)^{1-\gamma} - \left(\frac{l}{\zeta_B} \right)^{1-\gamma} \right]$$

Where ζ_B is given by (30)

If V_m be the muzzle velocity of the shot, we have

$$V_m^2 = \frac{FC}{\omega_1}\left[M\left\{(1-Z_0)+\zeta_0 \log Z_0\right\}^2 + \right.$$

$$\left. + \frac{2}{\gamma-1}(\zeta_B-\zeta_0)\left\{1-\left(1+\frac{d}{l}\right)^{1-\gamma}\zeta_B^{\,1-\gamma}\right\}\right]$$

The muzzle velocity in non-dimensional form can be written as

$$\eta_m^2 = M\left[M\left\{(1-Z_0)+\zeta_0 \log Z_0\right\}^2 + \right.$$

$$\left. + \frac{2}{\gamma-1}(\zeta_B-\zeta_0)\left\{(1-\left(1+\frac{d}{l}\right)^{1-\gamma}\zeta_B^{\,1-\gamma}\right\}\right] \quad (34)$$

$$\left(\text{for } \frac{x_B}{d} < 1\right).$$

If $\frac{x_B}{d} > 1$ i.e. the shot leaves the muzzle before all burnt of the charge, then from (29) for $x = d$ the value of Z is to be calculated and then

$$\eta_m = M(Z - Z_0) - M\,\zeta_0 \log \frac{Z}{Z_0}$$

corresponding muzzle velocity.

If $\bar{p}$ be the mean pressure which applied to the shot base for the total shot travel would give the muzzle velocity.

The corresponding equation is given by

$$\tfrac{1}{2}\,\omega_1 V_m^2 = A\,d\,\bar{p}$$

or

$$\bar{p} = \frac{\omega_1 V_m^2}{2\,A\,d}$$

Then putting the value of V_m^2, we have

$$\bar{p} = \frac{FC}{2\,A\,d}\left[M\left\{(1-Z_0)+\zeta_0 \log Z_0\right\}^2 + \right.$$

$$\left. + \frac{2}{\gamma-1}(\zeta_B-\zeta_0)\left\{1-\left(1+\frac{d}{l}\right)^{1-\gamma}\zeta_B^{\,1-\gamma}\right\}\right] \quad (35)$$

For maximum pressure, we put $\frac{d\zeta}{dZ} = 0$

which gives

$$1 - M(Z - Z_0) + M\zeta_0 \log \frac{Z}{Z_0} = 0 \quad (36)$$

If Z_1 the root of the equation (36) be less than unity, then as $Z = Z_1$ the pressure will be maximum and since $Z = Z_1$ satisfied (36), we must have

$$1 - M(Z_1 - Z_0) + M\zeta_0 \log \frac{Z_1}{Z_0} = 0 \quad (37)$$

Also it can easily be seen that $\frac{d^2\zeta}{dZ^2} < 0$ at $Z = Z_1$

Hence the maximum pressure ζ_{max} is given by

$$\zeta_{max} = \exp\left\{-1 + \log Z_1 + \frac{M\zeta_0}{2}\left(\log \frac{Z_1}{Z_0}\right)^2\right\} \qquad (33)$$

If however Z_1 be greater than unity, the maximum pressure will occur at all burnt and then

$$\zeta_{max} = \zeta_B = \zeta_0 \exp\left\{\frac{M}{2}\zeta_0 (\log Z_0)^2 - M Z_0 \log Z_0 - M(1 - Z_0) - \log Z_0\right\} \qquad (39)$$

Then

$$p_{max} = \frac{FC}{Al} \exp\left\{-1 + \log Z_1 + \frac{M\zeta_0}{2}\left(\log \frac{Z_1}{Z_0}\right)^2\right\} \qquad (40)$$

for $Z_1 < 1$

and

$$p_{max} = \frac{FC}{Al}\zeta_B \qquad \text{for } Z_1 > 1 \qquad (41)$$

where Z_1 is given by (37).

The piezometric efficiency is defined to be the ratio of the mean pressure to the maximum pressure and the corresponding expressions are given by

When $Z_1 < 1$

Piezometric efficiency =

$$= \frac{l}{2d}\,\frac{M\left\{(1 - Z_0) + \zeta_0 \log Z_0\right\}^2 + \dfrac{2}{\gamma - 1}(\zeta_B - \zeta_0)\left\{1 - \left(1 + \dfrac{d}{l}\right)^{1-\gamma}\zeta_B{}^{1-\gamma}\right\}}{e^{-1 + \log Z_1} + \dfrac{M\zeta_0}{2}\left(\log \dfrac{Z_1}{Z_0}\right)^2} \qquad (42)$$

$\left(\text{for } \frac{x_B}{d} < 1\right)$

When $Z_1 > 1$

Piezometric efficiency =

$$= \frac{l}{2d}\,\frac{M\left\{(1 - Z_0) + \zeta_0 \log Z_0\right\}^2 + \dfrac{2}{\gamma - 1}(\zeta_B - \zeta_0)\left\{1 - \left(1 + \dfrac{d}{l}\right)^{1-\gamma}\zeta_B{}^{1-\zeta}\right\}}{\zeta_B} \qquad (43)$$

$\left(\text{for } \frac{x_B}{d} < 1\right)$

where ζ_B is determined by (30) and Z_1 is determined by (37).

TABLE 1

ORTHODOX GUN WITHOUT RESISTANCE

$\frac{l}{d}$	M.V.	P.E.
0·4	1·230	0·8238
0·3	1·400	0·8008
0·2	1·562	0·6646
0·1	1·870	0·4768

Now the muzzle velocity and piezometric efficiency in an orthodox gun for single charge with and without resistance have been calculated numerically for tubular propellants for different values of $\frac{l}{d}$ at $M = 1$. For constant resistance we consider only two cases $\zeta_0 = 0{\cdot}3$ and $\zeta_0 = 0{\cdot}1$. Muzzle velocity is represented by M. V. and piezometric efficiency is represented by P.E.

TABLE 2

ORTHODOX GUN WITH RESISTANCE FOR $\zeta_0 = 0{\cdot}3$

$\frac{l}{d}$	M.V.	P.E.
0·4	1·199	0·3246
0·3	1·277	0·2761
0·2	1·378	0·2142
0·1	1·526	0·1315

TABLE 3

ORTHODOX GUN WITH RESISTANCE FOR $\zeta_0 = 0{\cdot}1$

$\frac{l}{d}$	M.V.	P.E.
0·4	1·227	0·4516
0·3	1·303	0·3816
0·2	1·403	0·2952
0·1	1·566	0·1840

From these tables we may conclude that both the muzzle velocity and piezometric efficiency in an orthodox gun will decrease with constant bore resistance in comparison to these quantities to the gun without resistance.

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