STABILITY OF A NON-NEWTONIAN FLUID BETWEEN TWO CONCENTRIC ROTATING POROUS CYLINDERS

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The stability of non-Newtonian fluid confined between two concentric rotating porous cylinders has been examined. The critical Taylor number which determines the on set of instability has been determined as a function of 'a' (wave length) and S (cross viscous parameter). The variation of the critical Taylor number with S, suction parameter λ and radial velocity distribution have been shown. It has been found that the effect of suction at the outer cylinder is to stabilize the flow whereas the injection destabilizes the flow. The presence of suction or injection does not affect radial velocity curves or vortex cells.

Most analysis of the stability of flow between rotating cylinders and related flows with this geometry have been concerned with the stability, characteristics of rotationally symmetric disturbances. Taylor[1], Jeffreys[2], Meksyn[3], Chandrasekhar[4], Krueger & Diprima[5], and Kelly & Alison[6] have studied the viscous flow between the two cylinders. The experimental work of Coles[7] indicated that if the speed of the inner cylinder is increased sufficiently beyond the critical speed, a cellular motion with a wavy form in the azimuthal direction occurs. Rosanblat[8] has investigated the centrifugal instability of an inviscid flow for axisymmetric disturbances in the stability of unsteady periodic flows. Drazin[9] considered the stability of a vortex sheet in an oscillating magnetic field. Recently Pradhan[10] studied the "Stability of a spiral flow with cubic axial velocity profile" in the case of narrow gap approximation. Chan Man Fong[11], discussed the "Stability of flow of visco-elastic fluids between arbitrary spaced cylinders". Reddy[12] also discussed the stability problem for visco-elastic fluids in the presence of suction and injection. Bhaskara Rao[13] discussed the stability problem for non-Newtonian fluids. The present results are compared with the results of Bhaskara Rao, Bahl, and Reddy[12–14]. The results are coinciding with Bhaskara Rao's results in the absence of suction or injection and with Reddy's & Chan Man Fong's results in the absence of visco-elastic character and suction parameter. But the present results differ from Bahl's results. It was found that Bahl took $K_1 = \dfrac{a\lambda}{\alpha \dfrac{R_2}{d} + 2}$ instead of $K_1 = \dfrac{\alpha\lambda}{\alpha \dfrac{R_2}{d} + 2}$ in the computation. This may be the minor mistake which cannot be detected so easily; but it effects the complete stability of the fluid and gives exactly opposite results in the case of suction and injection.

In this paper the couette flow between two rotating cylinders when the suction and injection are applied at the outer cylinder respectively has been considered. It has been assumed that the spacing between the two cylinders is very small and the cylinders are rotating in the same, opposite directions and the outer cylinder is at rest. The critical Taylor number has been computed for different values of λ (suction parameter) and the behaviour of the radial velocity distribution is calculated. From the calculations of the critical Taylor number it is found that the presence of injection destabilises the flow. The flow is, however, stable when suction is applied. The present results were compared with the previous results in the absence of suction or injection. It has also been concluded that the radial velocity curves or vortex pattern are not affected due to the suction or injection (for $S = 0, 0\cdot001, 0\cdot005, 0\cdot01$).

EQUATIONS OF MOTION

The equations of motion and continuity in cylindrical polar co-ordinates are given by

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right] = \frac{\partial}{\partial r} \tau_{rr} + \frac{\partial}{\partial z} \tau_{rz} - \frac{\tau_{rr} - \tau_{\theta\theta}}{r} \quad (1)$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right] = \frac{\partial}{\partial r} \tau_{\theta r} + \frac{\partial}{\partial z} \tau_{\theta z} + 2 \frac{\tau_{\theta r}}{r} \quad (2)$$

$$\rho \left[\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = \frac{\partial}{\partial r} \tau_{\theta r} + \frac{\partial}{\partial z} \tau_{zz} + \frac{\tau_{zr}}{r} \tag{3}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 . \tag{4}$$

where ρ is the density u, v and w are the velocity components in the directions of r, θ and z respectively. The stress components for a non-Newtonian fluid are given by

$$\begin{bmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{bmatrix} = -p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \phi_1 \begin{bmatrix} e_{rr} & e_{r\theta} & e_{rz} \\ e_{\theta r} & e_{\theta\theta} & e_{\theta z} \\ e_{zr} & e_{z\theta} & e_{zz} \end{bmatrix} + \phi_2 \begin{bmatrix} e_{rr} & e_{r\theta} & e_{rz} \\ e_{\theta r} & e_{\theta\theta} & e_{\theta z} \\ e_{zr} & e_{z\theta} & e_{zz} \end{bmatrix} \begin{bmatrix} e_{rr} & e_{r\theta} & e_{rz} \\ e_{\theta r} & e_{\theta\theta} & e_{\theta z} \\ e_{zr} & e_{z\theta} & e_{zz} \end{bmatrix} \tag{5}$$

where

$$e_{rr} = 2 \frac{\partial u}{\partial r} \qquad e_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}$$

$$e_{\theta\theta} = 2 \frac{u}{r} \qquad e_{\theta z} = \frac{\partial v}{\partial z}$$

$$e_{zz} = 2 \frac{\partial w}{\partial r} \qquad e_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r}$$

In the presence of suction and injection the equations (1) to (5) admit the steady state solution

$$\left.\begin{aligned} u &= U(r) = c/r = R_1 u_1/r \\ v &= V(r) = A_1 r^{\lambda+1} + B_1/r \\ w &= 0 \end{aligned}\right\} \tag{6}$$

where $\lambda = R_1 u_1/\alpha$, $\alpha = \dfrac{\phi_1}{\rho}$, u_1 is the radial velocity of the fluid at $r = R_1$ (radius of the inner cylinder) and A_1 and B_1 are arbitrary constants. For the motion of an incompressible fluid between two concentric rotating cylinders of radii R_1 and R_2 ($R_2 > R_1$) the two constants A_1 and B_1 are related to the angular velocities of rotation Ω_1 and Ω_2 of the inner and outer cylinders as

$$\left.\begin{aligned} A_1 &= -\Omega_1 \eta^2 \frac{1 - \mu/\eta^2}{R_2^{\lambda} (1 - \eta^{\lambda+2})} \\ B_1 &= R_1^2 \Omega_1 \frac{1 - \mu\eta^{\lambda}}{1 - \eta^{\lambda+2}} \end{aligned}\right\} \tag{7}$$

where

$$\mu = \frac{\Omega_2}{\Omega_1} \quad \text{and} \quad \eta = \frac{R_1}{R_2}$$

PERTURBATION EQUATIONS

Assuming the disturbance to be symmetric about the axis of the cylinder, we can take the velocity components and the pressure as

$$\left.\begin{aligned} u &= e^{\sigma t}\, u(r) \cos pz \\ v &= e^{\sigma t}\, v(r) \cos pz \\ w &= e^{\sigma t}\, w(r) \sin pz \\ \hat{\omega} &= e^{\sigma t}\, \overset{\Delta}{\omega}(r) \cos pz \end{aligned}\right\} \tag{8}$$

where u, v and w are small compared to U and V. Substituting the perturbed quantities (8) in (1) to (4) and eliminating p after linearising the equations with respect to u, v and w the differential equations governing the marginal stability are given by

$$\begin{aligned} 2\left(Ar^{\lambda} + \frac{B}{r^2}\right) v = {} & \frac{\alpha}{p^2}(DD_* - p^2)^2 u - \frac{\lambda\alpha}{p^2 r} D^2 D_* u + \frac{\lambda\alpha}{p^2 r^2}(DD_* - p^2) u + \\ & + \frac{\lambda\alpha}{r} Du - \gamma \Bigg[\frac{16\lambda\alpha}{r^3} Du + \frac{2\lambda\alpha}{r^2}(DD_* + p^2) u + \frac{2}{p^2 r}\frac{3\lambda\alpha}{r^3}(DD_* + p^2) u - \\ & - \frac{2\lambda\alpha}{p^2 r^3} D(DD_* + p^2) u - \frac{4\lambda\alpha}{r^3}\left(\frac{1}{p^2} D^2 D_* + D\right) u - \frac{8\lambda\alpha}{r^2} D_* D u - \frac{8\lambda\alpha}{r^4} u + \\ & + \frac{2\lambda\alpha}{r^2 p^2} D^2 (DD_* + p^2) u + \left(A\lambda r^{\lambda} - \frac{2B}{r^2}\right)\left(D^2 - \frac{3}{2} D + \frac{4}{r^2} - p^2\right) v - \\ & - \left\{ A\lambda(\lambda+1)^2 r^{\lambda-2} - \frac{2B}{r^4} \right\} v \end{aligned} \tag{9}$$

$$\begin{aligned} A(\lambda+2) r^{\lambda} u + \frac{\lambda\alpha}{r} D_* u = {} & \alpha(DD_* - p^2) v + \gamma \Bigg[\left(A\lambda r^{\lambda} - \frac{2B}{r^2}\right)(DD_* - p^2) u + \\ & + 2\left(A\lambda^2 r^{\lambda-1} + \frac{4B}{r^3}\right) D_* u + \frac{4}{r}\left(A\lambda r^{\lambda} - \frac{2B}{r^2}\right) D_* u - \frac{2\lambda\alpha}{r^2} p^2 v \end{aligned} \tag{10}$$

where

$$\gamma = \frac{\phi_2}{\rho},\ D = \frac{d}{dr},\ D_* = \frac{d}{dr} + \frac{1}{r}$$

$$DD_* = D_* D - \frac{1}{r^2} = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}$$

with the boundary conditions

$$u = D_* u = v = 0 \quad \text{at } r = R_1 \text{ and } r = R_2 \tag{11}$$

NARROW GAP APPROXIMATION

When $d = R_2 - R_1 \ll (R_1 + R_2)/2 = R_0$ we need not distinguish between D and D_*[3]. Shifting the origin midway between the cylinders, the equations (9) and (10) with the transformations $u \rightarrow 2\,\Omega_1 \dfrac{a^2 d^2}{\alpha} u$ reduce to (upto the terms of the order d/R_0)

$$(D^2 - a^2)^2 u = [1 - (1-\mu)\zeta - (1-\mu) S (D^2 - a^2)]\, v \tag{12}$$

$$(D^2 - a^2) v = -Ta^2 u \left(1 - \frac{K_1}{2} + K_1 \zeta\right) \tag{13}$$

where the approximation used in this for $A_1\, r^\lambda$, B_1/r^2

and $A_1\, r^\lambda + \frac{B_1}{r^2}$ are

$$A_1\, r^\lambda = -\frac{\Omega_1}{\lambda + 2}\left[(1-\mu)\frac{R_0}{d} + \lambda\,(1-\mu)\,(\zeta - 1) - 2\right] \tag{14}$$

$$\frac{B_1}{r^2} = \frac{\Omega_1}{\lambda + 2}\left[(1-\mu)\frac{R_0}{d} - (1-\mu)\,2\zeta + \mu\lambda\right]$$

$$A_1\, r^\lambda + \frac{B_1}{r^2} = \Omega_1\,[\,1 - (1-\mu)\,\zeta\,]$$

where

$$\zeta = x + \frac{1}{2}$$

$$T = -2\frac{\Omega_1^{\,2}\, d^4}{\alpha^2}\left[(\mu - 1)\frac{R_0}{d} + 2\right]$$

$$K_1 = \frac{(\mu - 1)\,\lambda}{(\mu - 1)\frac{R_0}{d} + 2}\ ;\ K_2 = \frac{\mu - 1}{(\mu - 1)\frac{R_0}{d} + 2}\ ;\ S = -\frac{\gamma\, R_1}{2d^3}$$

The boundary conditions are

$$u = Du = v = 0 \quad \text{at} \quad x = \pm\frac{1}{2} \tag{15}$$

The equations (12), (13) and (15) determine the eigen value problem for T as a function of the parameters a, μ, λ and S.

METHOD OF SOLUTIONS

Diprima[10] and Kurzweg[11] have employed the Galerkin technique to solve the simultaneous equations associated with the hydrodynamic and hydromagnetic stability of flows between the concentric rotating cylinders. Here the Galerkin technique has been used to study the eigen value problem (12) to (15). We take the trial functions satisfying the boundary conditions (15) and suitable for the present analysis, upto the first and second order approximation as

$$u = a_1\,(1 - 4\,\zeta^2)^2 + a_2\,\zeta\,(1 - 4\,\zeta^2)^2$$

$$v = b_1\,(1 - 4\,\zeta^2) + b_2\,\zeta\,(1 - 4\,\zeta^2) \tag{16}$$

where a_1, a_2, b_1 and b_2 are constants. Applying the arthogonolity condition[11]

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} [u,\ v]\ L\,(u,\ v)\, d\zeta = 0 \tag{17}$$

where $L\,(u, v)$ is the error matrix obtained on substituting (16) in (12) and (13) we get the secular equation as

$$\begin{bmatrix} G11 & 0 & G13 & G14 \\ 0 & G22 & G23 & G24 \\ G31\,T & G32\,T & G33 & 0 \\ G41\,T & G42\,T & 0 & G44 \end{bmatrix} = 0 \tag{18}$$

where

$$G11 = 16\,(a^4 + 24\,a^2 + 504)$$

$$G13 = 6\ [3\,(1+\mu) + 2\,(1-\mu)\,S\,(3a^2 + 28)]$$

$$G14 = (\mu - 1)$$

$$G22 = 8\,(a^4 + 88\,a^2 + 3960)$$

$$G23 = 22\,(\mu - 1)$$

$$G24 = 11\,[(1+\mu) + 2\,(1-\mu)\,S\,(a^2 + 36)]$$

$$G31 = 3\left[1 - \frac{K_1}{2} + K_2\right]a^2$$

$$G32 = K_1\,a^2/12$$

$$G33 = 7\,(a^2 + 10)$$

$$G41 = a^2\,K_1$$

$$G42 = \left(1 - \frac{K_1}{2} + K_2\right)a^2$$

$$G44 = 3\,(a^2 + 42)$$

The maximum value of T with respect to 'a' (the wave number) for fixed μ, λ and S is the critical Taylor number T_c which governs the on set of instability. The corresponding wave number is denoted by a_c. For different values of λ and S, (18) is solved for T_c for a range of values of μ given in Table 1. It is also seen from Table 1 that the critical Taylor values are computed for $R_0/d = 1000, 100$ where T_c and a_c depend on λ the injection (or suction) parameter.

From Table 1 it can be seen that the critical Taylor number T_c decreases in the case of injection and increases in the case of suction for fixed μ and S. It shows that the viscous or cross viscous fluids destabilise with injection and are stable in the case of suction. The effect of suction and injection on the Newtonian and non-Newtonian (cross viscous) fluids (of stability) are same for $\mu = 1$ (if the cylinders are co-rotating). Table 1 is shown for R_0/d=1000 and R_0/d=100. The Taylor number in case of R_0/d=100 is less when compared to $R_0/d = 1000$. Hence the fluid is more stable in the case of $R_0/d = 1000$ than in the case of $R_0/d = 100$. It is also found from Table 1 that the presence of suction or injection effects the non-Newtonian fluid more than compared to the Newtonian fluid. The present results compared with Chandrasekhar's results in the case of Newtonian fluids and Bhaskar Rao's results in the case of Newtonian and non-Newtonian fluids in the absence of suction or injection. The present authors have used the same method as used by Bhaskar Rao. But Bhaskar Rao deleted one term $K_2 = (\mu - 1)\Big/\left(\frac{R_0}{d}(\mu - 1) + 2\right)$ which appears in the expansion of the terms Ar^λ and $\frac{B}{r^2}$. Due to the effect of the above term the difference in the Taylor number is clearly shown although it does not effect the stability much.

An approximation for the velocity field perturbations is obtained by solving the four simultaneous equations whose determinant has been given in (18), from which $\frac{a_2}{a_1}$ is obtained. Using the values of a_c and T_c to the corresponding values of μ for $R_0/d = 1000$, $-a_2/a_1$ is calculated and are given in Table 2.

The radial velocity perturbation (16) has been normalised to unity and the values are tabulated. The presence of suction or injection will not disturb the radial velocity either in the Newtonian fluids or in the non-Newtonian fluids. In case of non-Newtonian fluids, the variation at the fourth decimal place is found to be negligible. The calculations for $R_0/d = 1000$ and $R_0/d = 100$ are clearly shown in the Table 3.

Table 1

Variation of critical Taylor number T_C

μ \ λ	S=0					S=·001				S=·01				
	CH-SEKR			BH-RAO		BH-RAO				BH-RAO				
	—1·5	0	0	0	1·5	—1·5	0	0	1·5	—1·5	0	0	1·5	R_0/d
1·00	3·12 1750	3·12 1708	3·11 1750	3·12 1750	3·12 1750	3·12 1750	3·11 1750	3·12 1750	3·12 1750	3·12 1750	3·11 1750	3·12 1750	3·12 1750	1000
1·00	3·12 1750	3·12 1708	3·11 1750	3·12 1750	3·12 1750	3·12 1750	3·11 1750	3·12 1750	3·12 1750	3·12 1750	3·11 1750	3·12 1750	3·12 1750	100
0·50	3·12 2327	3·12 2275	3·12 2331	3·12 2329	3·12 2330	3·13 2298	3·13 2302	3·13 2299	3·13 2301	3·25 2060	3·20 1821	3·25 2061	3·21 2063	1000
0·50	3·12 2289	3·12 2275	3·12 2331	3·12 2307	3·12 2325	3·13 2260	3·13 2302	3·13 2278	3·13 2296	3·25 2026	3·20 1821	3·25 2042	3·25 2058	100
0·25	3·12 2787	3·12 2725	3·13 2788	3·12 2789	3·12 2792	3·14 2725	3·15 2730	3·14 2727	3·14 2729	3·35 2255	3·40 2259	3·35 2256	3·35 2258	1000
0·25	3·12 2742	3·12 2725	3·13 2788	3·12 2764	3·12 2786	3·14 2681	3·15 2730	3·14 2702	3·14 2724	3·35 2218	3·40 2259	3·35 2236	3·35 2253	100
0·00	3·13 3468	3·12 3390	3·13 3475	3·13 3471	3·13 3474	3·17 3351	3·17 3348	3·17 3344	3·17 3347	3·49 2479	3·50 2484	3·49 2481	3·49 2483	1000
0·00	3·13 3412	3·12 3390	3·13 3475	3·13 3439	3·13 3467	3·17 3287	3·17 3348	3·17 3313	3·17 3340	3·49 2439	3·50 2484	3·49 2458	3·49 2478	100
—0·25	3·14 4567	3·13 4462	3·13 4573	3·14 4571	3·14 4547	3·21 4299	3·19 4377	3·21 4302	3·21 4306	3·68 2723	3·70 2738	3·68 2735	3·68 2737	1000
—0·25	3·14 4492	3·13 4462	3·13 4573	3·14 4529	3·14 4567	3·21 4228	3·19 4377	3·21 4263	3·21 4299	3·68 2689	3·70 2738	3·68 2710	3·68 2732	100
—0·50	3·19 6581	3·20 6417	3·20 6589	3·19 6586	3·19 6589	3·30 5942	3·20 6237	3·30 5947	3·30 5952	3·95 3006	3·90 3012	3·95 3009	3·95 3011	1000
—0·50	3·12 6497	3·20 6417	3·20 6589	3·14 6501	3·14 6517	3·30 5843	3·20 6237	3·30 5894	3·30 5948	3·95 2957	3·90 3012	3·95 2982	3·98 3006	100
—1·00	3·98 19603	4·00 18680	3·98 19640	3·98 19623	3·98 19643	4·22 14728	4·02 14700	4·22 14743	4·22 14757	4·74 3506	4·40 3539	4·74 3509	4·74 3512	1000
—1·00	3·96 19253	4·00 18680	3·98 19460	3·98 19446	3·98 19643	4·22 14466	4·02 14700	4·22 14610	4·22 14757	3·74 3448	4·40 3539	4·74 3478	4·74 3508	100

Table 2

μ \ λ	S=0			S=·001			S=·01		
	−1·5	0	1·5	−1·5	0	1·5	−1·5	0	1·5
—1·0	5·0638	5·064	5·0642	4·1169	4·1172	4·1175	1·1175	1·1172	1·1169
0	0·4391	0·439	0·4389	0·4280	0·4279	0·4278	0·3439	0·3437	0·3435
0·25	0·2642	0·2641	0·2640	0·2599	0·2598	0·2597	0·2274	0·2273	0·2272
0·5	0·1471	0·147	0·1469	0·1457	0·1456	0·1455	0·1351	0·1350	0·1349

Table 3

Radial velocity distribution for $\nu = -1 \cdot 0$

$\zeta \backslash \lambda$	$S=0$			$S=\cdot 001$			$S=\cdot 005$			$S=\cdot 01$		
	—1·5000	0·0000	1·5000	—1·5000	0·0000	1·5000	—1·5000	0·0000	1·5000	—1·5000	0·0000	1·5000
—0·4500	0·0811	0·0812	0·0812	0·0769	0·0769	0·0769	0·0620	0·0620	0·0620	0·0524	0·0523	0·0525
—0·4300	0·1477	0·1477	0·1477	0·1402	0·1402	0·1402	0·1140	0·1140	0·1140	0·0969	0·0968	0·0968
—0·4200	0·1858	0·1858	0·1858	0·1765	0·1766	0·1766	0·1441	0·1441	0·1441	0·1229	0·1228	0·1228
—0·4000	0·2688	0·2688	0·2689	0·2560	0·2560	0·2561	0·2108	0·2108	0·2108	0·1810	0·1809	0·1808
—0·3500	0·4944	0·4944	0·4944	0·4738	0·4739	0·4740	0·9997	0·3996	0·3996	0·3492	0·3490	0·3488
—0·3000	0·7074	0·7075	0·7075	0·6833	0·6834	0·6834	0·5924	0·5924	0·5923	0·5278	0·5275	0·5273
—0·2500	0·8739	0·8739	0·8739	0·8520	0·8530	0·8521	0·7627	0·7627	0·7626	0·6944	0·6941	0·6939
—0·2000	0·9737	0·9737	0·9737	0·9603	0·9604	0·9604	0·8931	0·8930	0·8930	0·8330	0·8327	0·8325
—0·1500	0·9990	0·9990	0·9990	0·9998	0·9998	0·9998	0·9733	0·9733	0·9733	0·9328	0·9327	0·9325
—0·1000	0·9519	0·9518	0·9518	0·9712	0·9711	0·9711	1·0000	1·0000	1·0000	0·9884	0·9883	0·9882
—0·0500	0·8422	0·8421	0·8420	0·8823	0·8822	0·8820	0·9750	0·9750	0·9750	0·9982	0·9982	0·9983
0·0000	0·6857	0·6856	0·6855	0·7466	0·7464	0·7462	0·9045	0·9045	0·9046	0·9645	0·9646	0·9648
0·0500	0·5020	0·5018	0·5017	0·5812	0·5810	0·5807	0·7979	0·7980	0·7981	0·8923	0·8926	0·8929
0·1000	0·3121	0·3119	0·3117	0·4050	0·4047	0·4044	0·6671	0·6672	0·6673	0·7893	0·7897	0·7901
0·1500	0·1367	0·1365	0·1363	0·2367	0·2364	0·2361	0·5246	0·5248	0·5249	0·6645	0·6649	0·6654
0·2000	—0·0060	—0·0062	—0·0064	0·0933	0·0930	0·0927	0·3833	0·3834	0·3836	0·5281	0·5286	0·5291
0·2500	—0·1024	—0·1026	—0·1028	—0·0120	—0·0123	—0·0126	0·2548	0·2549	0·2550	0·3906	0·3911	0·3915
0·2700	—0·1262	—0·1264	—0·1265	—0·0416	—0·0418	—0·0421	0·2092	0·2093	0·2094	0·3376	0·3381	0·3385
0·3000	—0·1457	—0·1458	—0·1460	—0·0717	—0·0719	—0·0721	0·1485	0·1486	0·1487	0·2623	0·2627	0·2631
0·3500	—0·1376	—0·1377	—0·1379	—0·0855	—0·0856	—0·0858	0·0708	0·0709	0·0710	0·1525	0·1528	0·1531
0·3800	—0·1130	—0·1131	—0·1132	—0·0751	—0·0752	—0·0753	0·0389	0·0390	0·0390	0·0988	0·0990	0·0992
0·4000	—0·0911	—0·0911	—0·0912	—0·0625	—0·0626	—0·0627	0·0236	0·0236	0·0237	0·0690	0·0691	0·0693

For constant values of S, μ and ψ it is found that the disturbance in the vortex cells is practically nil when suction or injection is applied, in the case of narrow gap approximation. It is calculated for different values of μ (1 to —4). Hence it can be concluded that the presence of suction or injection does not have any effect on vortex cells at the onset of instability. Since the variation is negligible it is not shown here either by graphs or by tables.

CONCLUSIONS

1. The fluid is more unstable for narrow gap approximation in the presence of suction or injection.

2. The Taylor number is constant when the cylinders are co-rotating equally ($\mu = 1{\cdot}0$) for any fluid even in the presence of suction or injection.

3. The radial velocity is not having any countable change in the presence of suction or injection. It is having little effect for non-Newtonian fluids in the presence of suction or injection.

4. It is concluded with so many calculations (for $\mu = 1$ to -4) the vortex cells are having very little effect which cannot be shown on the figure in the presence of suction or injection.

REFERENCES

1. TAYLOR, G.I., *Phil. Trans. A.*, **223** (1923), 289.
2. JEFFRYS, H., *Phil. Mag.*, **7** (1926), 833.
3. MEKSYN, D., *Proc. Roy. Soc.*, **A. 187** (1946), 115.
4. CHANDRASEKHAR, S., *Mathematica*, **1** (1954), 5.
5. KRUGER, E.R. & DIPRIMA, R.C., *J. Fluid Mech.*, **19**, Part 4 (1964), 528–538.
6. KELLY, R.E. & ALISON, M. CHEERS, *J. Mech. & Appl. Math.*, **33** (1970), 127.
7. COLES, D., *J. Fluid Mech.*, **21** (1965), 385.
8. ROSANBLAT, S., *J. Fluid Mech.*, **33** (1968), 321.
9. DRAZIN, P. G., *J. Mech. & Appl. Math.*, **20** (1967), 201.
10. GOP KRISHAN PRADHAN, *Indian J. Pure & Appl. Math.*, **4** (1973), 379.
11. CHAN MAN FONG, C.F., *J. Appl. Math. & Physics*, **21** (1970), 977.
12. REDDY, Y. B., *Tensor, N.S.*, **27** (1973), 138.
13. BHASKARA RAO, *Appl. Sci. Res. Section A*, **14** (1965), 199.
14. BAHL, S. K., *Def. Sci. J.*, **20** (1970), 89.