TANGENTIAL FLOW AND HEAT TRANSFER OF POWER-LAW FLUIDS IN AN ANNULUS BETWEEN TWO ROTATING CYLINDERS WITH SUCTION AND INJECTION

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(*Received* 9 *September* 1974 ; *revised* 23 *June* 1975)

The steady state tangential flow and heat transfer of a power-law fluid in an annulus between two rotating co-axial porous cylinders in presence of suction and injection has been considered in this paper. The effect of suction and injection on the flow fieid has been investigated. The heat transfer is considered in two cases: (*i*) the inner wall is thermally insulated and outer wall is maintained at a fixed temperature ; (*ii*) both the walls are maintained at fixed different temperatures, including the case when both the walls are at equal temperature. In the first case it has been found that the temperature increases with Brinkmans' number Br or flow behaviour index n and Br acts as a scale factor. In the second case the temperature increases with the increase of Br or n and decreases with increase of Peclet number (Pe'). In the second case the Nusselt numbers at the cylinders have been calculated.

The phenomenon arising out of the flow of a liquid in a rotating cylindrical annulus in presence of suction and injection has important engineering applications. The problem of heat transfer in an annular duct is also of great interest in chemical engineering and flow meters. Berman[1] considered the laminar flow in an annulus with porous walls. Bird and Fredrickson[2] have studied the flow behaviour of Non-Newtonian (Power Law) fluid in an annulus. Tangential flow in a rotating annulus with viscous heat generation has been discussed by Bird and Stewart[3]. The present paper deals with the study of the steady state tangential flow and heat transfer of a power-law fluid in an annulus between two rotating co-axial cylinders with suction on one wall and injection on the other. The main flow is maintained by the rotation of the cylinders. The effect of suction and injection on the flow depends on a dimensionless number S. Heat transfer is considered in two cases (*i*) the inner cylinder is insulated and the outer cylinder is maintained at fixed temperature and its equilibrium temperature and the consequent temperature distribution within the annulus is determined, (*ii*) both the walls are maintained at fixed temperatures. In the latter case the heat transfer rate at the cylinder is also obtained. In the first case the equilibrium temperature decreases as Brinkman number Br decreases and the material properties such as the thermal conductivity and specific heat do not alter the nature of the temperature distribution. In the second case the temperature increases as Br increases or Peclet number (Pe') decreases and the effect of rotation of the cylinders is to increase the temperature while that of suction is to reduce it. Also the temperature increases as the flow behaviour index increases. It is also found that for larger values of Brinkman number there exists a maximum temperature in the fluid.

The fluid is assumed to be incompressible and the fluid parameters are assumed to be constant so as to enable the use of velocity distribution independent of heat transfer phenomenon. The equation of motion is solved by perturbation technique, considering the suction/injection parameter S as perturbation parameter. The resulting velocity distribution is inserted into the energy equation which on solving gives temperature distribution. This problem is of interest in connection with heat effects in viscometry and friction bearings and has other engineering applications.

FORMULATION OF THE PROBLEM AND REDUCTION OF EQUATIONS

The basic equations governing the flow of Power-Law fluids are the constitutive equation between stress components τ_{ij} and strain rate components e_{ij} given by

$$\tau_{ij} = m \left| \sum_{q=1}^{3} \sum_{p=1}^{3} e_{pq}\, e_{qp} \right|^{\frac{n-1}{2}} e_{ij} \, , \tag{1}$$

the momentum equation

$$\rho \left(\frac{\partial v_i}{\partial t} + v^j v_{i,j} \right) = -p, i + \tau^i_{ij} \tag{2}$$

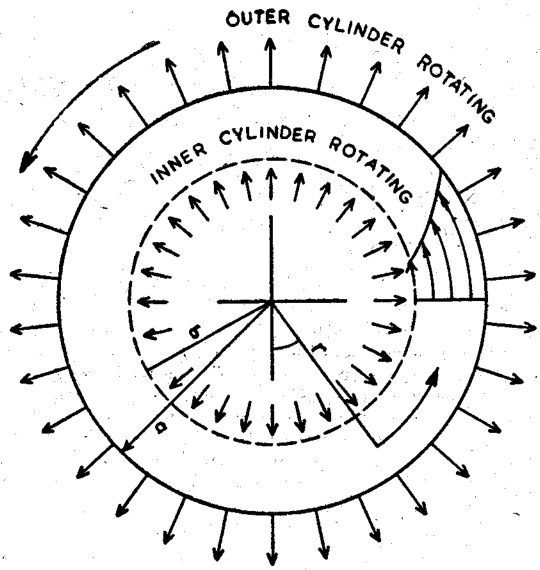

Fig. 1—Tangential flow of power-law fluids in an annulus between two rotating co-axial cylinders.

and the continuity equation

$$v^i,_i = 0 \tag{3}$$

where m, n, ρ, are respectively the consistency, flow behaviour index, density of the fluid and v^i the velocity vector and comma denotes co-variant differentiation. The fluid behaviour is pseudoplastic, Newtonian or dilatant according as $n <, =$ or > 1.

The energy equation describing the transport of thermal energy in terms of transport properties of the fluid is

$$\rho C_p \frac{DT}{Dt} = k \nabla^2 T + \Phi \tag{4}$$

where C_p, k and ρ are the specific heat, thermal conductivity and density (all considered constant) of the fluid, T is the temperature and Φ the dissipation function given by

$$\Phi = \tfrac{1}{2} S^i_j d^j_i \tag{5}$$

Let a Power-Law fluid with rheological equation of state (1) flow tangentially in the annulus between two infinite co-axial porous cylinders rotating about the common axis with uniform angular velocities ω_1 and ω_2 in presence of suction on one wall and injection on the other. Cylindrical polar coordinates (r, θ, z) are used and the cylinders are identified with the surfaces $r = a_1$ and $r = b_1$ (Fig. 1). Due to axial symmetry all derivatives with respect to θ vanish and the velocity field is assumed as

$$v^i_i = [u, v, 0] = [u(r), v(r), 0] \tag{6}$$

which depends upon r only and is independent of θ and z.

The physical components of stresses are

$$\left.\begin{aligned}
\tau_{rr} &= m \left| \sum_{q=1}^{3} \sum_{p=1}^{3} e_{pq}\, e_{qp} \right|^{\frac{n-1}{2}} 2 \frac{\partial u}{\partial r}, \\
\tau_{\theta\theta} &= m \left| \sum_{q=1}^{3} \sum_{p=1}^{3} e_{pq}\, e_{qp} \right|^{\frac{n-1}{2}} 2 \frac{u}{r}, \\
\tau_{r\theta} &= m \left| \sum_{q=1}^{3} \sum_{p=1}^{3} e_{pq}\, e_{qp} \right|^{\frac{n-1}{2}} r \frac{\partial}{\partial r}\left(\frac{v}{r}\right), \\
\tau_{zz} &= \tau_{\theta z} = \tau_{rz} = 0,
\end{aligned}\right\} \tag{7}$$

where

$$\left| \sum_{q=1}^{3} \sum_{p=1}^{3} e_{pq}\, e_{qp} \right|^{\frac{n-1}{2}} = \left| 2 \left(\frac{\partial u}{\partial r}\right)^2 + 2 \left(\frac{u}{r}\right)^2 + \left\{ r \frac{\partial}{\partial r}\left(\frac{v}{r}\right) \right\}^2 \right|^{\frac{n-1}{2}} \tag{8}$$

The simplified equations of motion governing the flow are

$$\rho \left(u \frac{\partial u}{\partial r} - \frac{v^2}{r} \right) = - \frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} , \tag{9}$$

$$\rho \left(u \frac{\partial v}{\partial r} + \frac{uv}{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \tau_{r\theta} \right), \tag{10}$$

$$0 = - \frac{\partial p}{\partial z} \tag{11}$$

Equation (11) readily gives $p = p(r)$, which shows that pressure is a function of r only.

The equation of continuity compatible to velocity field is

$$\frac{\partial u}{\partial r} + \frac{u}{r} = 0 \tag{12}$$

In our case the energy equation (4) reduces to

$$\rho C_p u \frac{\partial T}{\partial r} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \Phi \tag{13}$$

where

$$\Phi = m \left| 2 \left(\frac{\partial u}{\partial r} \right)^2 + 2 \left(\frac{u}{r} \right)^2 + \left\{ r \frac{\partial}{\partial r} \left(\frac{v}{r} \right) \right\}^2 \right|^{\frac{n-1}{2}} \times \left[2 \left(\frac{\partial u}{\partial r} \right)^2 + 2 \left(\frac{u}{r} \right)^2 + \left\{ r \frac{\partial}{\partial r} \left(\frac{v}{r} \right) \right\}^2 \right] \tag{14}$$

The boundary conditions on the velocity field are

$$\left. \begin{aligned} u &= u_{a_1}, \; v = a_1 \omega_1 \text{ at } r = a_1 \\ u &= u_{b_1}, \; v = b_1 \omega_2 \text{ at } r = b_1 \end{aligned} \right\} \tag{15}$$

where u_{a_1} and u_{b_1} are some constants.

Introducing the transformations

$$\left. \begin{aligned} & U = u/U_0 , \; V = v/U_0 , \; P = p/\rho U_0^2 \\ & a = a_1/L , \; b = b_1/L , \; R = r/L, \\ & \Omega_1 = \omega_1 L/U_0 , \; \Omega_2 = \omega_2 L/U_0 , \; M = U_0^{2-n} L^n/\nu \end{aligned} \right\} \tag{16}$$

where U, V, P, a, b, R, Ω_1, Ω_2 are the dimensionless quantities, M is the Reynolds' number, U_0, L_0 are the reference velocity and reference length, and ν is the kinematic viscosity. Using (16) in (12) and integrating, we get

$$U = S/R \tag{17}$$

where S is a dimensionless parameter, positive for suction at the outer wall and injection at the inner wall and negative for their opposite orders.

Using (7), (8), (16) and (17), the equations (9) and (10) transforms to

$$\frac{S^2}{R^2} + \frac{V^2}{R} = \frac{\partial P}{\partial R} + \frac{2S}{R^2} \frac{1}{M} \frac{\partial}{\partial R} \left| \frac{4 S^2}{R^4} + \left\{ R \frac{\partial}{\partial R} \left(\frac{V}{R} \right) \right\}^2 \right|^{\frac{n-1}{2}} , \tag{18}$$

$$S \frac{\partial}{\partial R} \left(VR \right) = \frac{1}{M} \frac{\partial}{\partial R} \left[R^2 \left| \frac{4S^2}{R^4} + \left\{ R \frac{\partial}{\partial R} \left(\frac{V}{R} \right) \right\}^2 \right|^{\frac{n-1}{2}} R \frac{\partial}{\partial r} \left(\frac{V}{R} \right) \right] \tag{19}$$

The boundary conditions can be rewritten as

$$\left. \begin{aligned} U_a &= u_{a_1}/U_0 , \; V = a\Omega_1 \text{ at } R = a, \\ U_b &= u_{b_1}/U_0 , \; V = b\Omega_2 \text{ at } R = b. \end{aligned} \right\} \tag{20}$$

Integrating (19), we get

$$R^3 \left| \frac{4S^2}{R^4} + \left\{ R \frac{\partial}{\partial R} \left(\frac{V}{R} \right) \right\}^2 \right|^{\frac{n-1}{2}} \frac{\partial}{\partial R} \left(\frac{V}{R} \right) = MSVR + MD \tag{21}$$

where D is a integration parameter depending on S.

S is assumed to be very small such as $S \ll 1$, and so series solution for obtaining tangential velocity can be used.

Let

$$V = \sum_{q=0}^{\infty} S^q V_q, \quad D = \sum_{q=0}^{\infty} S^q D_q \tag{22}$$

and the new boundary conditions are

$$\left. \begin{aligned} V_0 &= a\Omega_1, \; V_q = 0 \; (q = 1, 2, \ldots, \infty) \text{ at } R = a \\ V_0 &= b\Omega_2, \; V_q = 0 \; (q = 1, 2, \ldots, \infty) \text{ at } R = b \end{aligned} \right\} \tag{23}$$

Substituting (22) in (21), expanding the left hand side in powers of S and equating coefficients of S and term independent of S, we get

$$R^{n+2} \left[\frac{\partial}{\partial R} \left(\frac{V_0}{R} \right) \right]^{n-1} \frac{\partial}{\partial R} \left(\frac{V_0}{R} \right) = MD_0 \tag{24}$$

$$nR^{n+2} \left[\frac{\partial}{\partial R} \left(\frac{V_0}{R} \right) \right]^{n-1} \frac{\partial}{\partial R} \left(\frac{V_1}{R} \right) = MV_0 R + D_1 M \tag{25}$$

$$R^{n+2} \left[\frac{\partial}{\partial R} \left(\frac{V_0}{R} \right) \right]^{n-1} \left[n \frac{\partial}{\partial R} \left(\frac{V_2}{R} \right) + \frac{n-1}{2} \, \frac{4R^{-6} + n \left\{ \frac{\partial}{\partial R} \left(\frac{V_1}{R} \right) \right\}^2}{\frac{\partial}{\partial R} \left(\frac{V_0}{R} \right)} \right] = MV_1 R + MD_2 \tag{26}$$

etc.

Integrating these differential equations, we get

$$V_0 = C_1 R - \frac{n}{2} (D_0 M)^{\frac{1}{n}} R^{1 - \frac{2}{n}}, \tag{27}$$

$$V_1 = \frac{M (D_0 M)^{\frac{1}{n} - 1}}{2} \left[\frac{C_1}{n-1} R^{3 - \frac{2}{n}} + \frac{nR^{3 - \frac{4}{n}}}{2(2-n)} (D_0 M) - D_1 R^{1 - \frac{3}{n}} \right] + C_2 R \tag{28}$$

$$\begin{aligned} V_2 = {} & \frac{n-1}{2n-1} \, \frac{R^{-3+n}}{(D_0 M)} - \frac{(n-1)(n-2)+2}{16(n-2)(2n-3)} n (D_0 M)^{\frac{3}{n} - 2} M^2 R^{5 - \frac{6}{n}} - \\ & - \frac{n-1}{4n(2n-1)} C_1^2 M^2 (D_0 M)^{\frac{1}{n} - 2} R^{5 - \frac{2}{n}} + \frac{(n-1)^2 + 1}{8(n-1)^2} \cdot \\ & \cdot C_1 M^2 (D_0 M)^{\frac{2}{n} - 2} R^{5 - \frac{4}{n}} + \frac{(n-1) D_2 - D_1}{4(n-2)} M^2 (D_0 M)^{\frac{2}{n} - 2} R^{3 - \frac{4}{n}} - \\ & - \frac{C_2 - (n-1) C_1}{2(n-1)} M (D_0 M)^{\frac{1}{n} - 1} R^{3 - \frac{2}{n}} - \frac{1}{2} D_2 M (D_0 M)^{\frac{1}{n} - 1} R^{1 - \frac{2}{n}} + C_3 R, \end{aligned} \tag{29}$$

Using boundary conditions (23), we get

$$[D_0 M]^{\frac{1}{n}} = \frac{2}{n} \, \frac{\Omega_1 - \Omega_2}{a^{2/n} - b^{2/n}} a^{2/n} b^{2/n} \tag{30}$$

$$C_2 = \frac{\Omega_1 a^{2/n} - \Omega_2 b^{2/n}}{a^{2/n} - b^{2/n}} \tag{31}$$

$$D_1 = \frac{n}{2(n-2)} (D_0 M)^{1/n} \left[\frac{a^{2-\frac{4}{n}} - b^{2-\frac{4}{n}}}{a^{2/n} - b^{2/n}} a^{2/n} b^{2/n} - \right.$$

$$\left. - \frac{n-2}{n-1} \frac{\Omega_1 a^{2/n} - \Omega_2 b^{2/n}}{\Omega_1 - \Omega_2} \frac{a^{2-\frac{2}{n}} - b^{2-\frac{2}{n}}}{a^{2/n} - b^{2/n}} \right], \tag{32}$$

$$C_2 = \frac{M}{2} \left[\frac{n}{4(n-2)} (D_0 M)^{\frac{2-n}{n}} \frac{a^{2-\frac{2}{n}} - b^{2-\frac{2}{n}}}{a^{2/n} - b^{2/n}} - \right.$$

$$\left. - \frac{C_1}{n-1} (D_0 M)^{\frac{1}{n}-1} \frac{a^2 - b^2}{a^{2/n} - b^{2/n}} \right], \tag{33}$$

$$\left[a^{-\frac{2}{n}} - b^{-\frac{2}{n}} \right] D_2 = \frac{2(n-1)}{2n-1} \left(\frac{D_0 M}{M}\right)^{1-\frac{2}{n}} \left[a^{\frac{2}{n}-4} - b^{\frac{2}{n}-4} \right] -$$

$$- \frac{(n-1)(n-2)+2}{8(n-2)(2n-3)} \left[a^{4-\frac{6}{n}} - b^{4-\frac{6}{n}} \right] nM (D_0 M)^{\frac{2}{n}-1} -$$

$$- \frac{(n-1) C_1^2}{2n(2n-1) D_0} \left[a^{4-\frac{2}{n}} - b^{4-\frac{2}{n}} \right] +$$

$$+ \frac{(n-1)^2+1}{4(n-1)^2} \frac{C_1}{M} \left[a^{4-\frac{4}{n}} - b^{4-\frac{4}{n}} \right] (D_0 M)^{\frac{1}{n}-1} +$$

$$+ \frac{(n-1) D_0 - D_1}{2(n-2) M} \left[a^{2-\frac{4}{n}} - b^{2-\frac{4}{n}} \right] (D_0 M)^{\frac{1}{n}-1} +$$

$$+ \frac{C_2 - (n-1) C_1}{n-1} \left[a^{2-\frac{2}{n}} - b^{2-\frac{2}{n}} \right] \tag{34}$$

$$\left[a^{\frac{2}{n}} - b^{\frac{2}{n}} \right] C_3 = \frac{n-1}{2n-1} \left[b^{-4+\frac{4}{n}} - a^{-4+\frac{4}{n}} \right] (D_0 M)^{-\frac{1}{n}} +$$

$$+ \frac{(n-1)(n-2)+2}{16(n-2)(2n-3)} n \left[a^{4-\frac{4}{n}} - b^{4-\frac{4}{n}} \right] (D_0 M)^{\frac{3}{n}-2} +$$

$$+ \frac{(n-1) C_1^2}{4n(2n-1) M_1^2} \left[a^4 - b^4 \right] (D_0 M)^{\frac{1}{n}-2} -$$

$$- \frac{1+(n-1)^2 C_1}{8(n-1)^2 M^2} \left[a^{4-\frac{2}{n}} - b^{4-\frac{2}{n}} \right] (D_0 M)^{\frac{2}{n}-2} -$$

$$- \frac{(n-1) D_0 - D_1}{4(n-2) M^2} \left[a^{2-\frac{2}{n}} - b^{2-\frac{2}{n}} \right] (D_0 M)^{\frac{2}{n}-2} -$$

$$- \frac{C_2 - (n-1) C_1}{2(n-1) M} \left[a^2 - b^2 \right] (D_0 M)^{\frac{1}{n}-1}, \tag{35}$$

Hence the tangential velocity in the annulus is given by

$$V = C_1 R - \frac{n}{2} (D_0 M)^{\frac{1}{n}} R^{1 - \frac{2}{n}} +$$

$$+ S \left[C_2 R - \frac{M (D_0 M)^{\frac{1}{n} - 1}}{2} \left\{ \frac{n}{2(n-2)} (D_0 M)^{n} R^{3 - \frac{4}{n}} + \right.\right.$$

$$\left.\left. + \frac{C_1}{n-1} R^{3 - \frac{2}{n}} - D_1 R^{1 - \frac{2}{n}} \right\} \right] +$$

$$+ S^2 \left[C_3 R + \frac{n-1}{2n-1} \frac{R^{\frac{3}{n} - 3}}{(D_0 M)^{1/n}} - \frac{(n-1)(n-2)+2}{16(n-2)(2n-3)} n (D_0 M)^{\frac{3}{n} - 2} R^{5 - \frac{4}{n}} - \right.$$

$$- \frac{n-1}{4n(2n-1)} C_1^2 M^2 (D_0 M)^{\frac{1}{n} - 2} R^{5 - \frac{4}{n}} +$$

$$+ \frac{(n-1)^2 + 1}{8(n-1)^2} C_1 M^2 (D_0 M)^{\frac{2}{n} - 2} R^{5 - \frac{4}{n}} +$$

$$+ \frac{(n-1) D_0 - D_1}{4(n-2)} M^2 (D_0 M)^{\frac{2}{n} - 2} R^{3 - \frac{4}{n}} + \frac{C_2 - (n-1) C_1}{2(n-1)} M (D_0 M)^{\frac{1}{n} - 1} R^{3 - \frac{2}{n}} -$$

$$\left. - \frac{M D_2}{2} (D_0 M)^{\frac{1}{n} - 1} R^{1 - \frac{2}{n}} \right] + O(S^3). \qquad (36)$$

The shear distribution $\tau_{r\theta}(r)$ is given by

$$\frac{\tau_{r\theta}}{\rho U_0^2} = \frac{D}{R^2} + \frac{SV}{R}$$

$$= \frac{D_0}{R} + S \left[D_1 R^{-2} - \frac{n}{2} (D_0 M)^{\frac{1}{n}} R^{-\frac{2}{n}} + C_1 \right] +$$

$$+ S^2 \left[\frac{D_2}{R^2} - \frac{nM}{4(n-2)} (D_0 M)^{\frac{2}{n} - 1} R^{2 - \frac{4}{n}} + \right.$$

$$\left. + \frac{C_1 M}{2(n-1)} (D_0 M)^{\frac{1}{n} - 1} R^{2 - \frac{2}{n}} - \frac{D_1 M}{2} (D_0 M)^{\frac{1}{n}} R^{-\frac{2}{n}} \right] + O(S^3). \qquad (37)$$

Shear at the outer and inner walls are given respectively by

$$\frac{\tau_{r\theta}}{\rho U_0^2} = - \tau_{R\theta} \Big|_{R = a}$$

$$= - \left[\frac{D_0}{\,} - S \left\{ C_1 + \frac{D_1}{a^2} - \frac{n}{2} \frac{(D_0 M)^{\frac{1}{n}}}{a^{2/n}} \right\} + \right.$$

$$+ S^2\left[C_2 + \frac{D_2}{a^2} - \frac{nM}{4(n-2)} (D_0M)^{\frac{1}{n}-1} a^{2-\frac{4}{n}} + \right.$$

$$\left. + \frac{MC_1}{2(n-1)} (D_0M)^{\frac{1}{n}-1} a^{2-\frac{2}{n}} - \frac{MD_1}{2} (D_0M)^{\frac{1}{n}-1} a^{-\frac{2}{n}} \right] + O(S)^3. \tag{38}$$

$$\frac{\tau_{r\theta}}{\rho U_0^2} = + \tau_{R\theta} \Big|_{R=b}$$

$$= \frac{D_0}{b^2} + S\left\{ C_1 + \frac{D_1}{a^2} - \frac{n}{2} \frac{(D_0M)^{\frac{1}{n}}}{b^{2/n}} \right\} +$$

$$+ S^2 \left[C_2 + \frac{D_2}{b^2} - \frac{nM}{4(n-2)} (D_0M)^{\frac{1}{n}-1} b^{2-\frac{4}{n}} + \frac{MC_1}{2(n-1)} (D_0M)^{\frac{1}{n}-1} b^{2-\frac{2}{n}} - \right.$$

$$\left. - \frac{MD_1}{2} (D_0M)^{\frac{1}{n}-1} b^{-\frac{2}{n}} \right] + O(S^3). \tag{39}$$

HEAT TRANSFER

Introducing the transformation

$$T = T_w + T_c\, \theta(R) \text{ and } R = r/L \tag{40}$$

where T_w and T_c denote the wall and characteristic temperatures to be appropriately chosen for the different situations. The dimensionless form of the energy equation (13) is then given by

$$Pe' \frac{S}{R} \frac{\partial\theta}{\partial R} = \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial\theta}{\partial R} \right) + B_r \left[\frac{4S^2}{R^4} + \left\{ R \frac{\partial}{\partial R}\left(\frac{V}{R} \right) \right\}^2 \right]^{\frac{n+1}{2}} \tag{41}$$

where

$$Pe' \left(= \frac{PC_p U_0 L}{k} \right) \text{ and } B_r \left(= \frac{m\, U_0^{n+1}}{kT_c\, L^{n-1}} \right)$$

are respectively the Peclet number and Brinkman number for power-law fluids.

Boundary Conditions

Case I : When the inner cylinder is insulated and outer is maintained at temperature T_a, we put $T_w = T_a$ and choose T_c as some characteristic temperature in (40). The boundary conditions are then

$$\frac{d\theta}{dR} = 0, \text{ at } R = b;\ \theta = 0, \text{ at } R = a. \tag{42}$$

Case II (a) : When the inner and outer cylinders are at fixed temperatures T_b and T_a ($T_a > T_b$) put $T_w = T_a$, $T_c = T_a - T_b$. The boundary conditions then become

$$\theta = 0,\ R = b;\ \theta = 1,\ R = a. \tag{43}$$

Case II (b) : When both the cylinders are at the same temperature, say T_a, we set $T_w = T_a$ and T_c is once again some reference temperature as in case I. Thus the boundary conditions are

$$\theta = 0 \text{ at } R = b \text{ and } R = a. \tag{44}$$

Now using (22), (24), (25) and (26) in equation (41) and on simplification, we get

$$\frac{d^2\theta}{dR^2} + \frac{1 - SPe'}{R}\frac{d\theta}{dR} = - B_r (D_0 M)^{\frac{1}{n}} R^{-2-\frac{2}{n}} \Big[MD_0 + + \frac{n+1}{n} S\Big\{ MV_0 R + MD_1\Big\} + + \frac{n+1}{n-1} S^2 \Big\{ MV_1 R + MD_2 - (D_0 M)^{1-\frac{1}{n}} R^{1+\frac{2}{n}} \frac{\partial}{\partial R}\left(\frac{V_2}{R}\right)\Big\}\Big] + O(S^3). \quad (45)$$

Substituting the values of V_0, V_1 and V_2 from (27)—(29) in (45) and integrating we get

$$\theta = \frac{T - T_b}{T_a - T_b} = M(D_0 M)^{\frac{1}{n}} Br \Bigg[- \frac{D_0}{\frac{2}{n}\left(\frac{2}{n} + SPe'\right)} R^{-\frac{2}{n}} -$$

$$- \frac{n+1}{2} S \Bigg\{ \frac{C_1}{\left(2 - \frac{2}{n}\right)\left(2 - \frac{2}{n} - SPe'\right)} R^{2-\frac{2}{n}} - \frac{n(D_0 M)^{\frac{1}{n}}}{2\left(2 - \frac{4}{n}\right)\left(2 - \frac{4}{n} - SPe'\right)} R^{2-\frac{4}{n}} +$$

$$+ \frac{D_1}{\frac{2}{n}\left(\frac{2}{n} + SPe'\right)} R^{-\frac{2}{n}} \Bigg\} -$$

$$- \frac{n+1}{n-1} S^2 \Bigg\{ \frac{(n-1)(n-4)M}{16(n-2)\left(2 - \frac{2}{n}\right)\left(4 - \frac{6}{n} - SPe'\right)} (D_0 M)^{\frac{2}{n}-1} R^{4-\frac{6}{n}} -$$

$$- \frac{n^3 - 2n^2 + n + 2}{8(n-1)^2(n+1)\left(4 - \frac{4}{n} - SPe'\right)} MC_1 (D_0 M)^{\frac{1}{n}-1} R^{4-\frac{4}{n}} +$$

$$+ \frac{n-1}{4n(2n-1)\left(4 - \frac{2}{n} - SPe'\right)} MC_1^2 (D_0 M)^{\frac{1}{n}-1} R^{4-\frac{2}{n}} +$$

$$+ \frac{(n-1)(C_1 + C_2)}{2(n-1)\left(2 - \frac{2}{n} - SPe'\right)} R^{2-\frac{2}{n}} -$$

$$+ \frac{D_1 - (n-1)D_0}{4(n-2)\left(2 - \frac{4}{n} - SPe'\right)} (D_0 M)^{\frac{1}{n}-1} MR^{2-\frac{4}{n}} -$$

$$- \frac{D_1 M}{2\left(1 - \frac{2}{n}\right)\left(1 - \frac{2}{n} - SPe'\right)} (D_0 M)^{\frac{1}{n}-1} R^{1-\frac{2}{n}} +$$

$$+ \frac{2(n-1)}{(2n-1)\left(4 - \frac{2}{n} + SPe'\right)} \frac{(D_0 M)^{1-\frac{2}{n}}}{M} R^{\frac{2}{n}-4} +$$

$$+ \frac{(n-1) D_2 R^{-\frac{2}{n}}}{2\left(\frac{2}{n} + SPe'\right)} \Bigg\} \Bigg] + \frac{A_1 R^{SPe'}}{SPe'} + A_2 \quad (46)$$

$$= Br\,\phi_R(R, n, S, Pe') + \frac{A_1\, R^{SPe'}}{SPe'} + A_2 \;, \quad \text{(say)}. \tag{47}$$

Case I : Solution for the case when the inner cylinder is insulated and the outer is thermally conducting :

Using the boundary conditions (42) in (46) the temperature distribution in the annulus is given by

$$\theta = B_r \left[\phi_R(R, n, S, Pe') + \frac{F(b, n, S, Pe') - \phi_a(a, n, S, Pe')}{a^{SPe'} - b^{SPe'}} - \frac{F(b, n, S, Pe')\, a^{SPe'} - \phi_a(a, n, S, Pe')\, b^{SPe'}}{a^{SPe'} - b^{SPe'}} \right. \tag{48}$$

where

$$\left.\begin{aligned} &\frac{d}{dR}\left\{ \phi_R(R, n, S, Pe') \right\} = F(R, n, S, Pe'), \\ &\left.\frac{d\phi_R}{dR}\right|_{R=a} = F(a, n, S, Pe')\,; \quad \left.\frac{d\phi_R}{dR}\right|_{R=b} = F(b, n, S, Pe') \end{aligned}\right\} \tag{49}$$

Case II (a) : Solution for the case when both the cylinders are at different temperatures.

Using (43) in equation (46) we get

$$\theta = B_r\, \phi_R(R, n, S, Pe') + \frac{1 - B_r\,[\phi_a(a, n, S, Pe') - \phi_b(b, n, S, Pe')]}{a^{SPe'} - b^{SPe'}}\, R^{SPe'} + \frac{B_r\,[\phi_a(a, n, S, Pe')\, b^{SPe'} - \phi_b(b, n, S, Pe')\, a^{SPe'}] - b^{SPe'}}{a^{SPe'} - b^{SPe'}} \tag{50}$$

Rate of heat transfer per unit area to the inner cylinder is given by

$$q_h = h(T_a - T_b) = k \left.\frac{\partial T}{\partial r}\right|_{r=b_1} = K\, \frac{T_a - T_b}{L} \left[\frac{\partial \theta}{\partial R}\right]_{R=b} \tag{51}$$

where $h(r)$ is the coefficient of heat transfer, hence the non-dimensional heat transfer rate is given by

$$\left(\frac{d\theta}{dR}\right)_{R=b} = \frac{L\, h(b)}{k} = (Nu)_{R=b} \tag{52}$$

and from (44)

$$\left(\frac{d\theta}{dR}\right)_{R=b} = Br\, F(b, n, S, Pe') + \frac{1 - B_r\,[\phi_a(a, n, S, Pe') - \phi_b(b, n, S, Pe')]\, b^{SPe'-1}\, SPe'}{a^{SPe'} - b^{SPe'}} \tag{53}$$

Rate of heat transfer per unit area at the outer wall is given by

$$q_{a_1} = -h(a)\,[T_a - T_b] = -k \left.\frac{\partial T}{\partial r}\right|_{r=a_1} = -k\, \frac{T_a - T_b}{L} \left.\frac{\partial \theta}{\partial R}\right|_{R=a} \tag{54}$$

Hence non-dimensional heat transfer rate is

$$\left.\frac{d\theta}{dR}\right|_{R=a} = Br\, F\,(a, n, S, Pe') + \\ + \frac{1 - Br\,[\phi_a\,(a, n, S, Pe') - \phi_b\,(b, n, S, Pe')}{a^{SPe'} - b^{SPe'}}\, a^{SPe'-1}\, SPe' \tag{55}$$

where $F\,(a, n, S, Pe')$ and $F\,(b, n, S, Pe')$ are defined in (49).

Case II(b) : Solution for the case when both cylinders are at equal temperature.

Equation (47) under the boundary conditions (44) becomes

$$\theta = Br\,\phi_R\,(R, n, S, Pe') - Br\,\frac{\phi_a\,(a, n, S, Pe') - \phi_b\,(b, n, S, Pe')}{a^{SPe'} - b^{SPe'}}\, R^{SPe'} + \\ + Br\,\frac{\phi_a\,(a, n, S, Pe')\, b^{SPe'} - \phi_b\,(b, n, S, Pe')\, a^{SPe'}}{a^{SPe'} - b^{SPe'}} \tag{56}$$

The temperature profiles given in (46) and (56) differ by the terms

$$\frac{R^{SPe'} - b^{SPe'}}{a^{SPe'} - b^{SPe'}}$$

and also these start with zero and end with the zero at the inner and outer walls respectively, so the profiles are the same D_0, we need not plot them again for equal temperature case.

The Nusselt number (Nu) at the inner and outer cylinders are the same as (53) and (55) respectively with terms

$$\frac{SPe'\, b^{SPe'-1}}{a^{SPe'} - b^{SPe'}} \quad \text{and} \quad \frac{SPe'\, a^{SPe'-1}}{a^{SPe'} - b^{SPe'}}$$

respectively omitted and Nu tends to zero as Br tends to zero.

DISCUSSION OF RESULTS

Convergence of the solutions : Basically it is assumed that he perturbation parameter S (Suction/injection parameter) is very small such as $S << 1$. Therefore series solution given in the expression (22) for obtaining velocity is justified where any succeeding term is far less than its preceeding term : As such throughout the treatment of the problem the inequality $V_0 > SV_1 > S^2V_2 > \ldots > S^nV_nD$ is used. Again the velocity profile is continuous throughout the range ($b \leqslant R \leqslant a$) of the annulus and its value

TABLE 1

VARIATION OF VELOCITY WITH FLOW BEHAVIOUR INDEX (n) AND SUCTION/INJECTION PARAMETER (S)

	n=1/3				n=1				n=4/3			
R	S=0 V_o	S=0·01 V	S=0·1 V	S=0·2 V	S=0 V_o	S=0·01 V	S=0·1 V	S=0·2 V	S=0 V_o	S=0·01 V	S=0·1 V	S=0·2 V
1	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000
1·1	0·48650	0·48170	0·42634	0·33918	0·25454	0·25372	0·24596	0·23607	0·22666	0·22607	0·22105	0·21602
1·2	0·81086	0·80514	0·73658	0·62424	0·48888	0·48616	0·47393	0·45832	0·44410	0·44308	0·43434	0·42554
1·3	1·04680	1·04151	0·97547	0·86339	0·70769	0·70548	0·68948	0·66897	0·65425	0·65294	0·64168	0·63034
1·4	1·23320	1·22873	1·17089	1·06859	0·91429	0·91300	0·89247	0·87125	0·85827	0·85680	0·84415	0·83143
1·5	1·39001	1·38644	1·33862	1·25234	1·11110	1·10940	1·09176	1·06448	1·05726	1·04213	1·04277	1·02970
1·6	1·52840	1·52570	1·48809	1·41834	1·30000	1·29791	1·28150	1·26109	1·25209	1·23780	1·23840	1·22607
1·7	1·65530	1·65340	1·62584	1·57332	1·48235	1·47971	1·46804	1·44915	1·44320	1·43096	1·43146	1·42080
1·8	1·77480	1·77360	1·75550	1·72040	1·65930	1·65794	1·64902	1·63466	1·63130	1·62213	1·62251	1·61450
1·9	1·88910	1·88500	1·87969	1·86197	1·83160	1·82502	1·82810	1·81856	1·81685	1·81176	1·81197	1·81600
2	2	2	2	2	2	2	2	2	2	2	2	2

TABLE 2

VARIATION OF TANGENTIAL VELOCITY WITH (—S)

R	n=1/3		n=4/3	
	S=—0·01	S=—0·1	S=—0·01	S=—0·1
1	0·00000	0·00000	0·00000	0·00000
1·1	0·49130	0·51966	0·22725	0·23282
1·2	0·81657	0·84708	0·44513	0·45483
1·3	1·05209	1·07738	0·65558	0·66806
1·4	1·23767	1·25651	0·85976	0·87378
1·5	1·39272	1·40650	1·05879	1·07316
1·6	1·53110	1·53925	1·25354	1·26710
1·7	1·65721	1·66171	1·44451	1·45621
1·8	1·77600	1·77801	1·63232	1·64111
1·9	1·88967	1·89020	1·81738	1·82223
2	2·00000	2·00000	2·00000	2·00000

lies between $a\,\Omega_1$ and $b\,\Omega_2$. Consequently the expression (36) for the velocity profile is rapidly convergent, which is further illustrated by a numerical examples (Tables 1 and 2). By the same reasoning the expressions (37-39), (45) and (46) are also convergent.

Velocity field : The radial component of velocity is given by (27) and varies inversely as the radial distance. In the absence of the cross flow ($S = 0$) the tangential velocity is given by

$$V_o = CR - \frac{n}{2}\left(D_o M\right)^{\frac{1}{n}} R^{1-\frac{2}{n}} \tag{57}$$

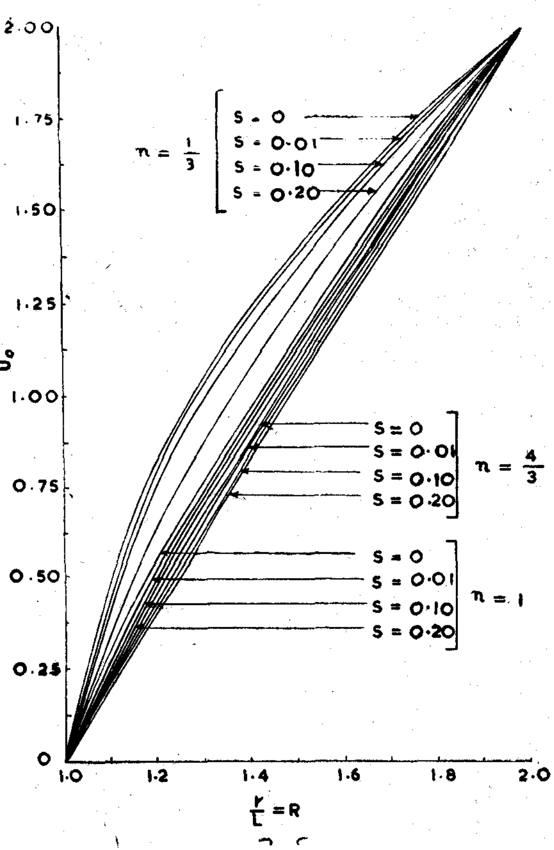

Fig. 2—Variation of velocity of power-law fluids in a rotating annulus with suction and injection.

which shows that whether the walls are solid or porous, even the first order solution exhibits the effect of Power-Law index n on the flow pattern. Putting $n=1$ in (57) we obtain the results obtained by Bird *et al*[3].

In order to get an understanding of the qualitative response of the tangential velocity to an increase in flow behaviour index n, and the suction and injection parameter S we consider a particular case in which the outer cylinder is rotating with uniform angular velocity Ω_1, while the inner cylinder is stationary, $\Omega_2 = 0$. For numerical computation we choose $M = 1$, $a = 2$; $b = 1$, $\Omega_2 = 0$, $\Omega_1 = 1$. $S = \pm\, 0{\cdot}01, \pm\, 0{\cdot}1, 0{\cdot}2$; V is calculated for $n=1/3$, 1, 4/3 and its values are tabulated in Tables 1 and 2 The plot of V versus R for $n = 1/3$, 1, 4/3, $S = +$ 0·01, 0·1, + 0·2 is plotted in Fig. 2. From the tables and figures it is found that velocity decreases as the flow behaviour index, n, increases. Also when S is positive, an increase in S decreases velocity as seen from Table 1 and Fig. 2, while for opposit order of S the response of velocity is reversed (Table 2).

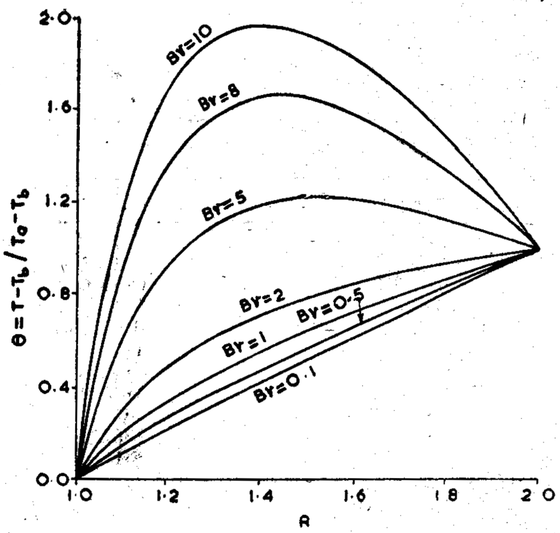

Fig. 3—Response of temperature in pseudoplastic fluid ($n = 1/3$) to an increase in Brinkman number Br at fixed Peclet number $Pe' = 10$ in a rotating annulus.

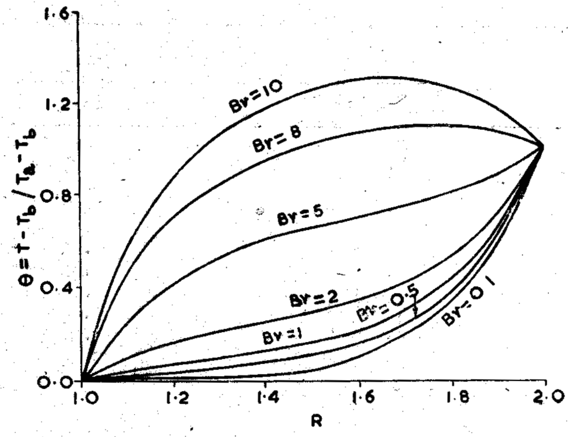

Fig. 4—Response of temperature in pseudoplastic fluid ($n = 1/3$) to increase in Brinkman number (Br) at fixed $Pe' = 100$ in a rotating annulus.

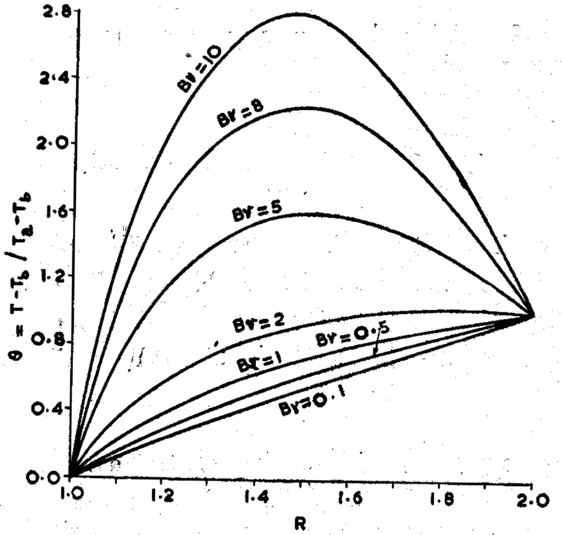

Fig. 5—Response of temperature in Newtonian fluids ($n=1$) in a rotating annulus to an increase in Brinkman number (Br) at fixed $Pe' = 10$.

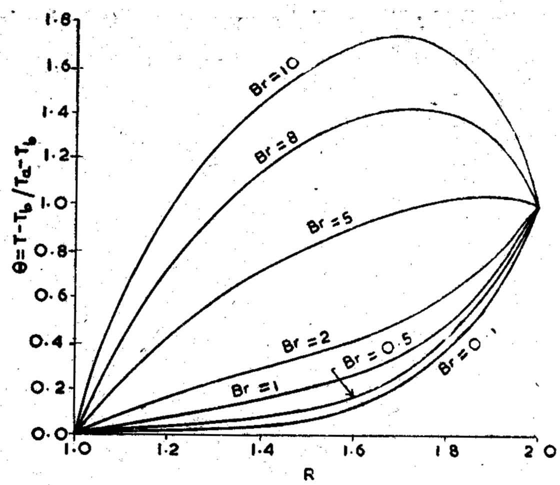

Fig. 6—Response of temperature in Newtonion fluid ($n = 1$) to increase in Brinkman number (Br) at $Pe' = 100$ in rotating annulus.

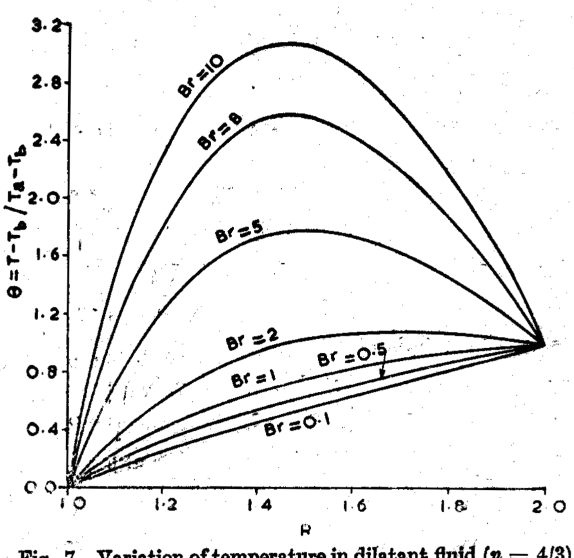

Fig. 7—Variation of temperature in dilatant fluid ($n = 4/3$) with Brinkman number (Br) at fixed $Pe' = 10$.

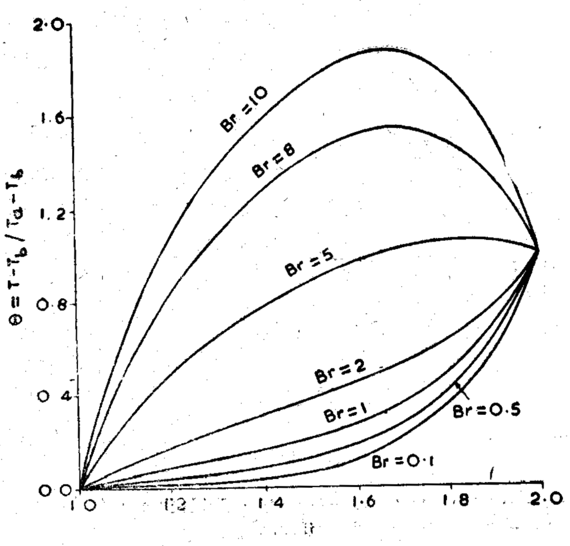

Fig. 8—Variation of temperature in dilatant fluid ($n = 4/3$) with Brinkman number (Br) at fixed $Pe' = 100$.

Temperature Field : If dissipation function is ignored, the energy equation (13) becomes

$$\frac{d^2\theta}{dR^2} + \frac{1 - SPe'}{R} \cdot \frac{d\theta}{dR} = 0$$

which, on integration along with (17) gives

$$\theta = \frac{R^{SPe'} - b^{SPe'}}{a^{SPe'} - b^{SPe'}}$$

which shows that in the absence of dissipation function heat transfer is independent of flow parameter n, and it has the same value whether we consider the Newtonian or Non-Newtonian power-law fluid. In the absence of cross-flow i.e. $S = 0$, heat transfer is

$$\theta = M\, Br\, (D_o\, M)^{\frac{1}{n}} \left[- \frac{n^2}{4} D_o R^{\left(-\frac{2}{n}\right)} \right] + A_1 \log R + A_2$$

which, when $n = 1$, gives the results obtained by Bird and Stewart[3].

In order to assess the qualitative response of heat transfer due to the effects of n, Br, Pe', we give numerical values to these parameters. We take $n = 1/3, 1, 4/3$; $Pe' = 10, 100$; $Br = 0{\cdot}1, 0{\cdot}5, 1, 5, 8, 10$; $S = 0{\cdot}1$, $M = 1$, $a = 2$, $b = 1$, $\Omega_2 = 0$, $\Omega_1 = 1$. The variation of θ with n, Br; Pe' is given in Tables 3 — 5. The plots of θ versus R for different values of n, Br, Pe' for case I are given in Figs. 9 (a, b, c) and for case II in Figs. 3, 4, 5, 6, 7 and 8. The graphs depict themselves the behaviour of the temperature variation in pseudoplastic, Newtonian and dilatant fluids at different Brinkman number and Peclet number at fixed $S = 0{\cdot}1$, $M = 1$, in the case when outer cylinder is rotating and inner cylinder is stationary. On the basis of the comparative study of the data given in the Tables 3, 4 and 5 and that of the temperature profiles given in the Figs. 3-8 the following facts are found.

TABLE 3

VARIATION OF θ WITH Br AND Pe' FOR $n=1/3$ $S=0{\cdot}1$

R/Br	0·1	0·5	1	2	5	8	10	Pe'
1	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	10
	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	100
1·1	0·11034	0·15171	0·20342	0·30685	0·61712	0·92738	1·13423	10
	0·00705	0·02904	0·05652	0·10148	0·27636	0·44124	0·55116	100
1·2	0·21481	0·27405	0·34811	0·49622	0·94066	1·38490	1·68113	10
	0·01368	0·04810	0·09113	0·17719	0·43537	0·69355	0·86567	100
1·3	0·31618	0·38091	0·46182	0·62364	1·10910	1·59457	1·91827	10
	0·02488	0·06467	0·11688	0·22129	0·53452	0·84775	1·05657	100
1·4	0·41567	0·47835	0·55670	0·71341	1·18353	1·65366	1·96707	10
	0·03875	0·08469	0·14210	0·25693	0·60142	0·94559	1·17557	100
1·5	0·51434	0·57173	0·64345	0·78690	1·21725	1·64760	1·93450	10
	0·06733	0·11518	0·17499	0·29461	0·65347	1·01233	1·25157	100
1·6	0·61219	0·66095	0·72191	0·84383	1·20958	1·57532	1·81916	10
	0·11825	0·16566	0·22491	0·34342	0·69895	1·05448	1·29150	100
1·7	0·70956	0·74779	0·79559	0·89119	1·17798	1·46477	1·65597	10
	0·20685	0·25087	0·30589	0·41594	0·74609	1·07624	1·29634	100
1·8	0·80659	0·83299	0·86599	0·93199	1·12999	1·32799	1·45998	10
	0·35731	0·39409	0·44007	0·53203	0·80791	1·08379	1·26770	100
1·9	0·90340	0·91700	0·93401	0·96802	1·07005	1·17208	1·24010	10
	0·60316	0·62711	0·65579	0·71316	0·88527	1·05738	1·17212	100
2	1	1	1	1	1	1	1	10
	1	1	1	1	1	1	1	100

TABLE 4

VARIATION OF θ WITH Br AND Pe' FOR $n=1$, $S=0\cdot1$

R/Br	0·1	0·5	1	2	5	8	10	Pe'
1	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	10
	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	100
1·1	0·11789	0·15894	0·21789	0·33578	0·68945	1·04312	1·27890	10
	0·00743	0·03104	0·06239	0·10429	0·28941	0·46025	0·60293	100
1·2	0·21830	0·29154	0·38308	0·56616	1·11540	1·66464	2·03080	10
	0·01468	0·04952	0·09307	0·18018	0·44147	0·70281	0·87697	100
1·3	0·32207	0·41037	0·52074	0·74148	1·40370	2·06590	2·50740	10
	0·02510	0·07194	0·13049	0·24759	0·59889	0·95019	1·18439	100
1·4	0·42227	0·51134	0·62269	0·84538	1·51345	2·18152	2·62269	10
	0·04204	0·07309	0·16636	0·30449	0·71888	1·13327	1·40953	100
1·5	0·52213	0·61063	0·72126	0·94252	1·60630	2·27008	2·71260	10
	0·07183	0·13346	0·21051	0·36460	0·82687	1·28813	1·59732	100
1·6	0·61934	0·69671	0·79343	0·98686	1·56715	2·14744	2·53436	10
	0·12355	0·18732	0·26703	0·42645	0·90471	1·38297	1·70181	100
1·7	0·71594	0·77969	0·85938	1·01876	1·49690	1·97504	2·29380	10
	0·21191	0·27469	0·35117	0·50678	0·97010	1·43450	1·74374	100
1·8	0·81132	0·85659	0·91319	0·02638	1·36590	1·70552	1·93190	10
	0·36331	0·41631	0·48258	0·61510	1·01266	1·41022	1·67259	100
1·9	0·90627	0·93134	0·96268	1·02536	1·21340	1·40144	1·53190	10
	0·60982	0·64404	0·68694	0·77274	1·03014	1·28754	1·45914	100
2	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	10
	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	100

TABLE 5

VARIATION OF θ WITH Br AND Pe' FOR $n=4/3$, $S=0\cdot1$

R/Br	0·1	0·5	1	2	5	8	10	Pe'
1	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	10
	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	100
1·1	0·11315	0·16574	0·23148	0·36296	0·75740	1·15184	1·41480	10
	0·00785	0·03302	0·06447	0·10737	0·31611	0·50484	0·63066	100
1·2	0·21905	0·29527	0·39054	0·58109	1·15272	1·72435	2·10544	10
	0·01577	0·05855	0·11202	0·21896	0·53978	0·86060	1·07448	100
1·3	0·32555	0·42775	0·55551	0·81102	1·57755	2·34408	2·85510	10
	0·02511	0·07561	0·13873	0·26496	0·64368	1·02241	1·27489	100
1·4	0·42678	0·53580	0·66760	0·93521	1·75802	2·54083	3·07604	10
	0·04232	0·10242	0·17754	0·32778	0·77850	1·22922	1·52969	100
1·5	0·52576	0·62880	0·75760	1·01520	1·78800	2·56080	3·07604	10
	0·07205	0·13851	0·22160	0·38777	0·88628	1·38479	1·71713	100
1·6	0·62294	0·71469	0·82933	1·05876	1·74690	2·43504	2·89380	10
	0·12376	0·19279	0·27907	0·45163	0·96931	1·48699	1·83111	100
1·7	0·71873	0·79366	0·88733	1·07466	1·63665	2·19864	2·57330	10
	0·21268	0·27923	0·36242	0·52880	1·02794	1·52708	1·85984	100
1·8	0·81336	0·86683	0·93365	1·06730	1·46825	1·86920	2·13650	10
	0·36265	0·41989	0·49033	0·63219	1·40778	1·48334	1·76706	100
1·9	0·90708	0·93539	0·97078	1·04156	1·25390	1·46624	1·60780	10
	0·60811	0·64440	0·68976	0·78049	1·05271	1·32488	1·50634	100
2	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	10
	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	100

The temperature at a point in the annulus increases with the increase in n or with the increase in Br and it decreases with the increase in Pe'. It can be easily seen from Figs. 3, 5 and 7 that for all n and for all values of Br the profiles are concave downwards for small value of Pe' (in our case $Pe' = 10$) and tends to a straight line when $Br = 0$ and $SPe' = 1$. Figs 4, 6, and 8 show that for large values of Pe' (in our case $P'e = 100$) the profiles are first concave upwards for lower values of Br ($Br < 2$ for $n = 1, 4/3$ and $Br < 5$ for $n = 1/3$) and afterwards they become concave downwards for higher values of Br ($Br > 2$ for $n = 1, 4/3$ and $Br > 5$ for $n = 1/3$). If Br is large enough there exists a unique temperature maximum, and its position shifts away and away in the fluid with the increase in Br from the outer rotating wall (at higher temperature. This phenomenon is more pronounced as Pe' decreases. Also the temperature maximum increases as Pe' decreases.

It can be easily seen from (50) that the outer wall is heated or cooled according as

$$Br > \text{ or } < SPe' \Big/ \Big[SPe' \left\{ \phi_a\left(a, n, S, Pe'\right) - \phi_b\left(b, n, S, Pe'\right) \right\} - \left\{1 - \left(\frac{b}{a}\right)^{SPe'}\right\} a F \left(a, n, SPe' \right) \Big].$$

The temperature distribution given by (48) is plotted in Figs. 9(a, b, c) for $n = 1/3, 1, 4/3$ respectively at $Pe' = 10$, $S = 0{\cdot}1$ and $Br = 0{\cdot}1, 0{\cdot}5, 1, 2, 5, 8, 10$. It is found that the equilibrium temperature of the inner insulated cylinder increases with the increase in Br or with the increase in n and the temperature distribution in the annulus between the cylinders becomes more and more uniform for small values of Br. In this case Br acts as scale factor showing that the thermal conductivity and the specific heat of the fluid do not alter the nature of the temperature distribution.

Hence, it is concluded that the temperature in the rotating annulus increases with the increase of Br and decreases with the increase of Pe'; consequently the effect of rotation of the cylinder is to increase the

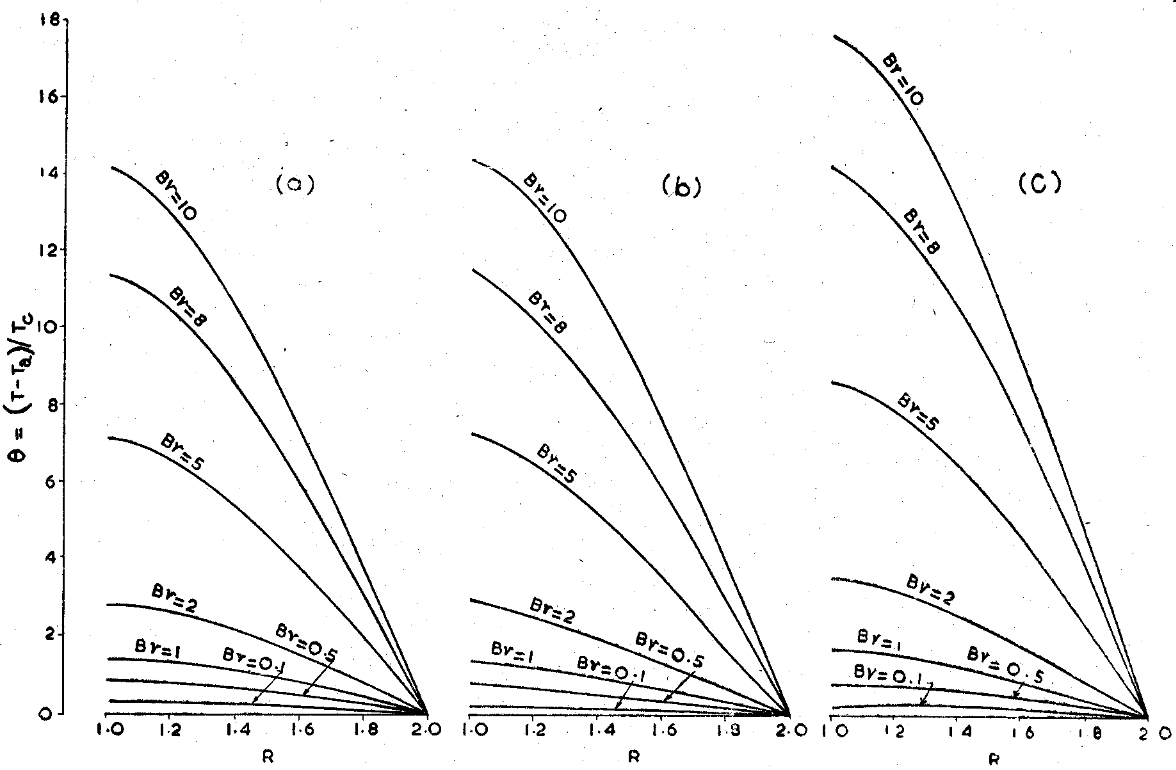

Fig. 9 (a, b, c)—Temperature distribution in power-law fluids in a rotating cylinderical annulus (inner wall insulated) for various Brinkman numbers (Br.) Where for : (a) $n=1/3$, $Pe'=10$, $S=0{\cdot}1$. (b) $n=1$, $Pe'=10$, $S=0{\cdot}1$. (c) $n=4/3$. $Pe'=10$, $S=0{\cdot}1$.

temperature in the annulus and that of suction is to reduce it. Also temperature increases with n and an unique temperature maximum always exists for large values of Br.

ACKNOWLEDGEMENT

The author is grateful to Shri Prakash Narain Singh for his help in numerical calculations and to the referee for suggesting the convergence of the solutions.

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