TWO DIMENSIONAL MOTION OF AN OSCILLATING AIRFOIL IN A FLUCTUATING ON COMING STREAM AT SUPERSONIC SPEED

J. P. SHARMA

Defence Science Laboratory, Delhi

(*Received* 2 *March* 1974)

The aerodynamic characteristics of a lifting body are studied for the case of a non-uniform oncoming stream at supersonic speed past a thin oscillating airfoil at a small angle of attack. The two motions are assumed independent of each other and the coupled motion is analysed using the theory of small disturbances. Analytic solution of the linearized problem is then obtained by applying Laplace transformation to it.

The majority of fluid flows encountered by the aerodynamicist are turbulent. As the statistical theory of turbulence applicable to subsonic regime is well developed, extensive work has been done on the response of airplane to atmospheric turbulence. This, however, is no longer so at supersonic speed; even though, supersonic aircraft cruising in the stratosphere occasionally encounter sudden and unexpected clear air turbulence (CAT) and the difficulty of control in such circumstances is, further, accentuated due to high speed of the aircraft[1]. The present study is, therefore, motivated by the desire to learn something about the effects of turbulence on the aerodynamic characteristics of a wing at such speed.

In the absence of any suitable statistical model for supersonic turbulence, the case of an unsteady oncoming stream past a thin oscillating airfoil at a small angle of attack is examined in the first instance. From the results of this case one may expect to get some indication of what an ambient atmosphere would do to the characteristics of a lifting body at supersonic speed.

NOTATIONS

$U(x, t)$	=	free stream velocity
U_∞	=	basic steady velocity
b	=	chord length
t	=	time
a_∞	=	speed of sound
Φ	=	complete velocity potential
ϕ	=	complete perturbation potential
V	=	volume occupied by the gas
t_1, t_2	=	time instants at which conditions are prescribed
ω	=	frequency of the oscillating airfoil
ω_0	=	frequency of the fluctuating flow
k	=	reduced frequency of airfoil
k_0	=	reduced frequency of flow
(x, y, z)	=	coordinate system (origin is fixed in the body)
h	=	vertical translation amplitude
$\bar{\epsilon}$	=	complex amplitude of the flow
ϵ_0	=	modulus
θ_0	=	phase angle
$W_a(x, t)$	=	prescribed down wash
$L(t)$	=	$L_\omega(t) + L_{\omega_0}(t)$ total unsteady lift
$L_\omega(t)$	=	lift due to the oscillating airfoil
$L_{\omega_0}(t)$	=	lift due to the fluctuating flow
$M(t)$	=	$M_\omega(t) + M_{\omega_0}(t)$ total pitching moment
$M_\omega(t)$	=	pitching moment due to the oscillating airfoil
$M_{\omega_0}(t)$	=	pitching moment due to the fluctuating flow
ρ_∞	=	density of the medium
γ	=	ratio of specific heat at constant pressure to at constant volume
M_∞	=	Mach number
J_0	=	Bessel function of order zero
$(r), (i)$	=	denote real and imaginary part respectively
$*$	=	at the head denotes non-dimensionalized form of the variable

GOVERNING EQUATIONS

We assume that there exists a disturbance in the atmosphere independently of the passage of airfoil through it. Therefore, the free stream is assumed to vary

$$U(x, t) = U_\infty \left[1 + \bar{\epsilon}\, e^{i \omega_0 \left(t - \frac{x}{U_\infty} \right)} \right], \quad \bar{\epsilon} = \frac{\epsilon_0\, e^{i \theta_0}}{U_\infty} . \tag{1}$$

The fluid flow considered is inviscid, compressible and irrotational; the airfoil is thin and of infinite span at zero incidence and executing small oscillations in vertical translation only (plunging motion). Since the gust motion is assumed irrotational, it is justifiable to assume the motion induced by the airfoil to be irrotational also. In view of this, the equation to be satisfied by the velocity potential Φ is written in the variational form[2]

$$\delta \int_{t_1}^{t_2} \iiint_V \left[\frac{a_\infty^2}{\gamma - 1} + \frac{1}{2} U^2(x, t) - \frac{\partial \Phi}{\partial t} - \frac{1}{2} \vec{q} \cdot \vec{q} \right]^{\frac{\gamma}{\gamma - 1}} dV\, dt = 0 \tag{2}$$

where

$$\Phi = \int U(x, t)\, dx + \phi \tag{3}$$

and

$$\vec{q} = \text{grad}\, \Phi . \tag{4}$$

The exact equation (2) is non-linear and is solved by applying an iteration in ϵ_0, which is a small parameter characterizing the size of the small disturbance, to it. Thus, giving the first order perturbation equation

$$a_\infty^2 \nabla^2 \phi = \frac{\partial^2 \phi}{\partial t^2} + 2 U_\infty \frac{\partial^2 \phi}{\partial x \partial t} + U_\infty^2 \frac{\partial^2 \phi}{\partial x^2} + i a_\infty^2 (\bar{\epsilon}\, \omega_0)\, e^{i \omega_0 \left(t - \frac{x}{U_\infty} \right)} \tag{5}$$

Equation (5) is hyperbolic in the present case and is being solved subject to the tangency condition

$$\left. \frac{\phi}{\partial z} \right|_{z = 0} = W_a(x, t), \; 0 \leqslant x \leqslant b \tag{6}$$

and the up stream condition

$$\phi = \phi_x = 0 \text{ at } x = 0 \left(\phi_x = \frac{\partial \phi}{\partial x} \right) \tag{7}$$

The nose of the airfoil is located at the origin in a moving frame of coordinate system, z-axis pointing vertically upwards.

Solution to Perturbation Equation

The non-homogeneous equation (5) is solved by letting

$$\phi = \psi + \chi \tag{8}$$

so that ψ satisfies

$$a_\infty^2 \nabla^2 \psi = \frac{\partial^2 \psi}{\partial t^2} + 2 U_\infty \frac{\partial^2 \psi}{\partial x \partial t} + U_\infty^2 \frac{\partial^2 \psi}{\partial x^2} \tag{9}$$

and χ

$$a_\infty^2 \nabla^2 \chi = \frac{\partial^2 \chi}{\partial t^2} + 2 U_\infty \frac{\partial^2 \chi}{\partial x \partial t} + U_\infty^2 \frac{\partial^2 \chi}{\partial x^2} + i a_\infty^2 (\bar{\epsilon}\, \omega_0)\, e^{i \omega_0 \cdot \left(t - \frac{x}{U_\infty} \right)} \tag{10}$$

The boundary conditions satisfied by ψ and χ are

$$\left. \frac{\partial \psi}{\partial z} \right|_{z = 0} = W_a(x, t), \; 0 \leqslant x \leqslant b \tag{11}$$

and

$$\left.\frac{\partial \chi}{\partial z}\right|_{z=0} = 0 \quad , \quad 0 \leqslant x \leqslant b \tag{12}$$

respectively.

Since the solution ψ will vanish up stream of the plane $x = 0$, the functions χ must also vanish there, i.e.

$$\chi = \chi_x = 0 \text{ at } x = 0 \left(\chi_x = \frac{\partial \chi}{\partial x} \right) \tag{13}$$

The solution to (9) subject to the boundary conditions (11) is well-known[3], while to that of (10) is, most readily, obtained by first separating the exponential time factor and then applying to it the Laplace transformation with respect to x. Therefore, after substituting

$$\chi = \chi_1 (x, z)\, e^{i \omega_0 \left(t - \frac{x}{U_\infty} \right)} \tag{14}$$

into (10), one gets

$$\frac{\partial^2 \chi_1}{\partial z^2} = (M_\infty^2 - 1) \frac{\partial^2 \chi_1}{\partial x^2} + 2 \left(\frac{i\, \omega_0}{U_\infty} \right) \frac{\partial \chi_1}{\partial x} + \left(\frac{\omega_0}{U_\infty} \right)^2 \chi_1 + i (\bar{\epsilon}\, \omega_0) \tag{15}$$

Denoting the Laplace Transform of χ_1 by $\bar{\chi}_1$, and applying it to the (15) one has

$$\frac{d^2 \bar{\chi}_1}{d z^2} = \left[(M_\infty^2 - 1)\, s^2 + 2 \left(\frac{i\, \omega_0}{U_\infty} \right) s + \left(\frac{\omega_0}{U_\infty} \right)^2 \right] \bar{\chi}_1 + i\, (\bar{\epsilon}\, \omega_0) \frac{1}{s} \tag{16}$$

where s is the transformed variable. At the same time, the boundary condition (12) becomes

$$\left.\frac{d \bar{\chi}_1}{d z}\right|_{z=0} = 0, \quad 0 \leqslant x \leqslant b \tag{17}$$

The solution $\bar{\chi}_1$ of (16) subject to the condition (17) reads

$$\bar{\chi}_1 = \frac{i\, (\bar{\epsilon}\, \omega_0)}{s\, (M_\infty^2 - 1) \left[s^2 + 2 \left(\frac{i\, \omega_0}{U_\infty} \right) \left(\frac{1}{M_\infty^2 - 1} \right) s + \left(\frac{\omega_0}{U_\infty} \right)^2 \left(\frac{1}{M_\infty^2 - 1} \right) \right]} \tag{18}$$

for $z \geqslant 0+$

Inversion of which yields[4]

$$\chi_1 = - \frac{i\, (\bar{\epsilon}\, \omega_0)}{(\omega_0 / U_\infty)^2} \left[1 - e^{- \frac{i\, \omega_0}{U_\infty (M_\infty^2 - 1)} x} \left\{ \cos \left(\frac{\omega_0\, M_\infty}{U_\infty (M_\infty^2 - 1)} x \right) + \right.\right.$$
$$\left.\left. + \frac{i}{M_\infty} \sin \left(\frac{\omega_0\, M_\infty}{U_\infty (M_\infty^2 - 1)} x \right) \right\} \right] \tag{19}$$

Defining now the reduced frequency of the fluctuating flow as

$$k_0 = \frac{b\, \omega_0}{U_\infty} \tag{20}$$

so that

$$\bar{\omega}_0 = \frac{k_0\, M_\infty^2}{(M_\infty^2 - 1)} \tag{21}$$

The complete solution of the first order problem is, then

$$\phi\, (x^*, 0+, t) = - \frac{b}{\sqrt{M_\infty^2 - 1}} \int_0^{x^*} W_a (\xi^*, t)\, e^{- i \bar{\omega} (x^* - \xi^*)} J_0 \left(\frac{\bar{\omega}}{M_\infty} (x^* - \xi^*) \right) \cdot d\, \xi^* -$$
$$- \frac{i\, (\bar{\epsilon}\, \omega_0)}{(\omega_0 / U_\infty)^2} \left[e^{- i \bar{\omega}_0 \left(\frac{M_\infty^2 - 1}{M_\infty^2} \right) x^*} - e^{- i \bar{\omega}_0 x^*} \left\{ \cos \left(\frac{\bar{\omega}_0}{M_\infty} x^* \right) + \right.\right.$$
$$\left.\left. + \frac{i}{M_\infty} \sin \left(\frac{\bar{\omega}_0}{M_\infty} x^* \right) \right\} \right] e^{i\, \omega_0 t} \tag{22}$$

where

$$W_a(\xi^*, t) = -i\,\omega\,h\,e^{i\,\omega}$$

and

$$x^* = x/b\,,\ \xi^* = \xi/b\,,\ k = \frac{b\,\omega}{U_\infty}\,,\ \bar{\omega} = \frac{k\,M_\infty^2}{M_\infty^2 - 1} \tag{23}$$

Lift Distribution over the Airfoil

Compatible with the second-degree terms in (5) and modified to include the disturbance in the flow field, the expression giving the approximate pressure relation becomes

$$p(x^*, t) = -\rho_\infty \left(\frac{\partial \phi}{\partial t} + \frac{U_\infty}{b} \frac{\partial \phi}{\partial x^*} \right) + \rho_\infty U_\infty^2\, e^{i\,\omega_0 \left(t - \frac{x}{U_\infty} \right)} \tag{24}$$

Total lift and pitching moment are now obtained by integrating

$$l(x^*, t) = -2\,p(x^*, t) \tag{25}$$

the amplitude of lift-distribution, between the limits 0 & 1. Thus,

$$L(t) = b \int_0^1 l(x^*, t)\, dx^* = L_\omega(t) + L_{\omega_0}(t)\,, \text{ say} \tag{26}$$

$$-M(t) = b \int_0^1 x^*\, l(x^*, t)\, dx^* = -M_\omega(t) - M_{\omega_0}(t)\,, \text{ say} \tag{27}$$

The expressions for $L_\omega(t)$ and $-M_\omega(t)$ which give, respectively, contribution to lift and pitching moment due to the oscillating airfoil in a uniform stream are the same as given in reference[3]; while $L_{\omega_0}(t)$ and $-M_{\omega_0}(t)$ which give respectively, additional contribution to lift and pitching moment due to the fluctuating stream have their real and imaginary parts, as

$$\frac{L_{\omega_0}(r)}{\rho_\infty\, b\, U_\infty^2} = \left(\frac{2\,\bar{\epsilon}}{M_\infty\, k_0} \right) \left[\cos \bar{\omega}_0 \sin\left(\frac{\bar{\omega}_0}{M_\infty} \right) - \frac{1}{M_\infty} \sin \bar{\omega}_0 \sin\left(\frac{\bar{\omega}_0}{M_\infty} \right) - \right.$$
$$\left. - M_\infty \sin k_0 \right] \tag{28}$$

$$\frac{L_{\omega_0}(i)}{\rho_\infty\, b\, U_\infty^2} = -\left(\frac{2\,\bar{\epsilon}}{M_\infty\, k_0} \right) \left[\sin \bar{\omega}_0 \sin\left(\frac{\bar{\omega}_0}{M_\infty} \right) + \frac{1}{M_\infty} \cos \bar{\omega}_0 \cos\left(\frac{\bar{\omega}_0}{M_\infty} \right) - \right.$$
$$\left. - \frac{1}{M_\infty} - M_\infty (1 - \cos k_0) \right] \tag{29}$$

$$\frac{-M_{\omega_0}(r)}{\rho_\infty\, b^2\, U_\infty^2} = \left(\frac{2\,\bar{\epsilon}}{M_\infty\, k_0} \right) \left[\cos \bar{\omega}_0 \sin\left(\frac{\bar{\omega}_0}{M_\infty} \right) - \frac{1}{M_\infty} \sin \bar{\omega}_0 \cos\left(\frac{\bar{\omega}_0}{M_\infty} \right) + \right.$$
$$+ \frac{1}{k_0} \left\{ -\sin \bar{\omega}_0 \sin\left(\frac{\omega_0}{M_\infty} \right) + \frac{2}{M_\infty} \left(1 - \cos \bar{\omega}_0 \cos\left(\frac{\bar{\omega}_0}{M_\infty} \right) \right) - \right.$$
$$\left. \left. - \frac{1}{M_\infty^2} \sin \bar{\omega}_0 \sin\left(\frac{\bar{\omega}_0}{M_\infty} \right) \right\} + \frac{1}{k_0} \left\{ k_0 \sin k_0 - (1 - \cos k_0) \right\} \right] \tag{30}$$

$$\frac{-M_{\omega_0}(i)}{\rho_\infty\, b^2\, U_\infty^2} = -\left(\frac{2\,\bar{\epsilon}}{M_\infty k_0} \right) \left[\sin \bar{\omega}_0 \sin\left(\frac{\omega_0}{M_\infty} \right) + \frac{1}{M_\infty} \cos \omega_0 \cos\left(\frac{\bar{\omega}_0}{M_\infty} \right) + \right.$$

$$+ \frac{1}{k_0} \left\{ \cos \bar{\omega}_0 \sin \left(\frac{\bar{\omega}_0}{M_\infty} \right) - \frac{2}{M_\infty} \sin \bar{\omega}_0 \cos \left(\frac{\bar{\omega}_0}{M_\infty} \right) + \right.$$

$$\left. + \frac{1}{M_\infty^2} \cos \omega_0 \sin \left(\frac{\bar{\omega}_0}{M_\infty} \right) \right\} - \frac{1}{k_0} (k_0 \cos k_0 - \sin k_0) \Big] \tag{31}$$

Case (1) : If $\omega_0 = 0$, i.e., there is a sudden change from U_∞ to $U_\infty (1 + \bar{\epsilon})$ in the free stream velocity, a quasi-steady state, the values of the total lift and pitching moment are obtained by taking limit as $\omega_0 \rightarrow 0$

Hence,

$$\frac{L\omega_0 = 0}{\rho_\infty b U_\infty^2} = - 2\epsilon_0 e^{i\theta_0} \left[1 - \frac{1}{(M_\infty^2 - 1)} \right] \tag{32}$$

and

$$\frac{M\omega_0 = 0}{\rho_\infty b^2 U_\infty^2} = \epsilon_0 e^{i\theta_0} \qquad (33$$

Expression (32), obviously, gives zero lift for $M_\infty^2 = 2$, no matter what is the value of the velocity pitching moment, on the other hand, remains unaffected by the change in Mach number.

Case (2) : If $\omega_0 = \omega$, the values of the expressions (28) to (31) are obtained by replacing $\bar{\omega}_0$ by $\bar{\omega}$.

NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

It is for the Case (2) that the values of the mid-chord derivatives are calculated for Mach numbers 1·2, 1·6 and 2·0; and for $\epsilon_0 = 0·0125$, $\theta_0 = 0$, $h = 0·5\, b$. The results are given in the form of graphs, separately, for real and imaginary components of the lift and pitching, to facilitate their comparison with those of Temple and Jahn.

As is usual mid-chord derivatives are calculated after transforming lift and pitching moment to an axis distance $\frac{1}{2}\, b$ aft of the leading edge. We have, therefore, plotted : $L^{(r)}$, $L^{(i)}$, $- M^{(r)}$ and $- M^{(i)}$ defined as

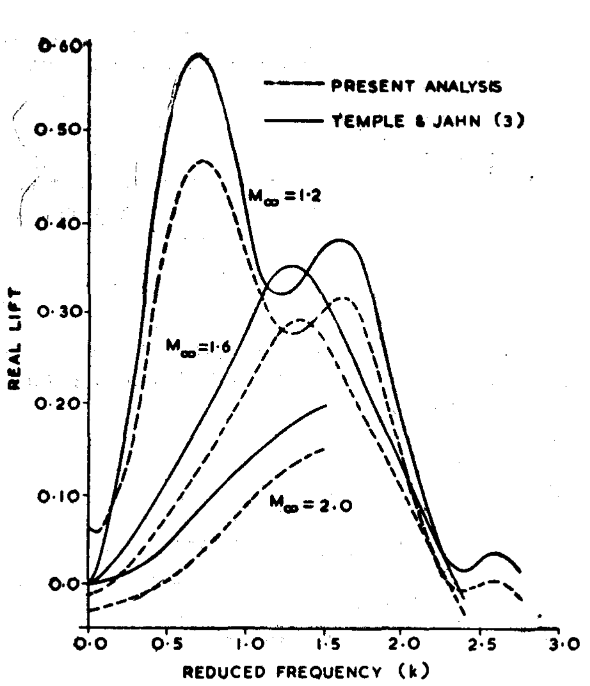

Fig. 1—Values of the real part of the lift against k for $M_\infty = 1.2, 1.6,$ and 2.0.

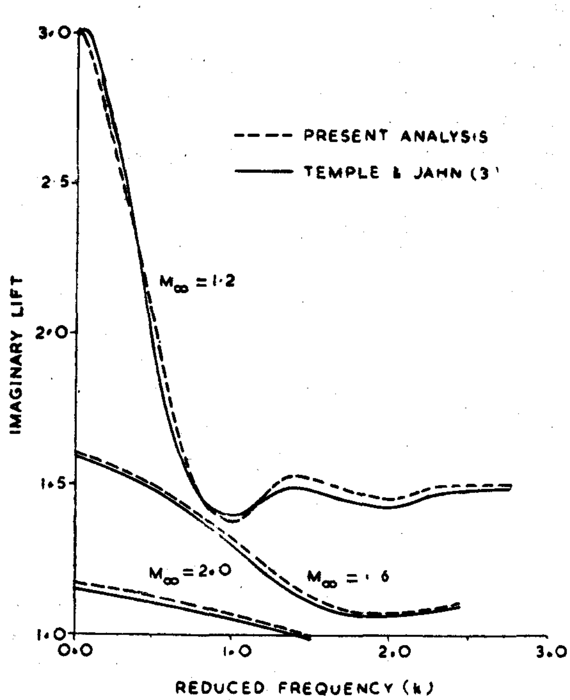

Fig. 2—Value of the imaginary part of the lift against k for $M_\infty = 1.2, 1.6$ and 2.0.

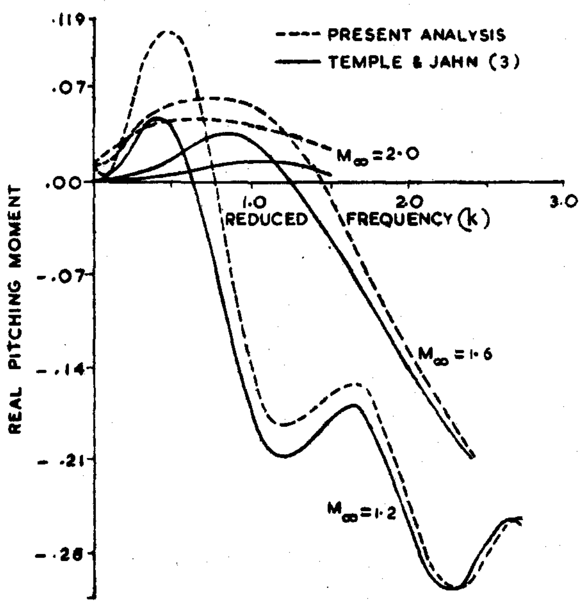

Fig. 3—Values of the real part of the pitching moment against k for $M_\infty = 1.2,\ 1.6$ and 2.0.

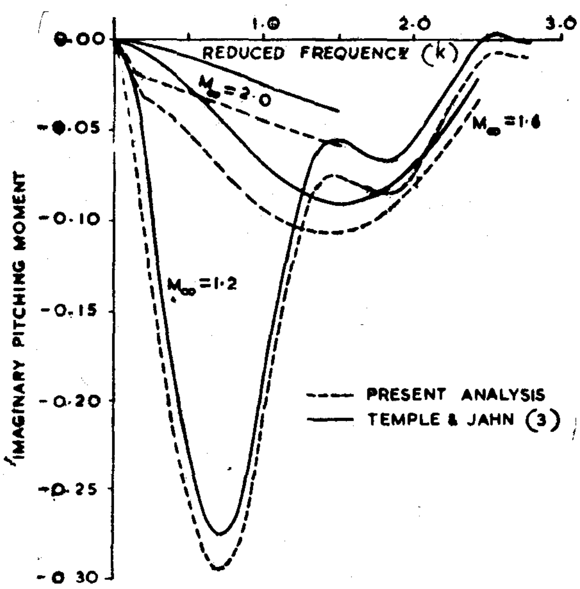

Fig. 4—Value of the imaginary part of the pitching moment against k for $M_\infty = 1.2,\ 1.6$ and 2.0.

$$L^{(r)} = l_z + \frac{b}{h} L_{\omega_0}(r) \quad L^{(i)} = l_{\dot z} + \frac{b}{hk} L_{\omega_0}(i) \tag{34}$$

$$-M^{(r)} = -m_z - \frac{b}{h} M_{\omega_0}(r), \; -M^{(i)} = -m_{\dot z} - \frac{b}{hk} M_{\omega_0}(i) \tag{35}$$

against the values of the reduced frequency $k\,(= k_0)$; where l_z , $l_{\dot z}$, $-m_z$ and $-m_{\dot z}$ have the same meaning as in reference[3].

The numerical values obtained, here, show that the additional term due to the disturbances in the atmosphere contributes significantly to the values of the lift and pitching moment. It is observed that the effect of the additional term is to decrease the value of lift acting on the airfoil. In the case of pitching moment the tendency of the additional term is to increase its magnitude for lower values of the reduced frequency $(0 \leqslant k \leqslant 1.5)$, and to decrease for higher frequency range $(1.5 < k \leqslant 3.0)$. Moreover, these effects increase with the increase in Mach number. This clearly shows that outside disturbances must be taken into account for any correct evaluation of the forces and moments on a wing in a moderately supersonic flight.

ACKNOWLEDGEMENTS

The author is grateful to Dr. R.R. Aggarwal for suggesting the problem and to Dr. C.D. Ghildyal for his continuous guidance. The author is also grateful to the Director, Defence Science Labortatory, Delhi for permission to publish this paper.

REFERENCES

1. JONES, R.F., *Science Journal*, Feb. (1967), 34-39.
2. BISPLINGHOFF, R.L. & ASHLEY, H., "Principles of Aeroelasticity" (John-Wiley and Sons, New York), 1962, p. 70.
3. TEMPLE, G. & JAHN, H.A., "Flutter at Supersonic Speeds : Derivative Coefficient for a Thin Aerofoil at Zero Incidence." British A.R.C. (Report & Memorandum No. 2140), 1945.
4. BATEMAN, H., "Tables of Integral Transforms", (Mc Graw Hill Book Company, New York), 1954, p. 230.