SOLUTION OF EQUATIONS OF INTERNAL BALLISTICS FOR THE COMPOSITE CHARGE CONSISTING OF A MIXTURE OF GRAINS OF 'N' SIZES, SHAPES OR COMPOSITIONS

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This paper describes the four basic equations of internal ballistics which have been modified for composite charge consisting of N-charges. These equations have been solved numerically using Runge-Kutta method. A computer programme is developed which gives all the ballistic parameters from shot start to shot exit.

In order to keep the 'all burnt' point well within the muzzle even in case of the lowest charge in the multi-charge Gun/Howitzers and to keep the maximum pressure within the specified limits, composite charges which consist of a mixture of grains of two or more nominal sizes usually with the same composition, but quite often with different shapes are used in modern field guns. The general theory of composite charges has been discussed by Clemmow[1] and Corner[2]. Clemmow has discussed the two composite charges of the same composition, but of different shapes and sizes. Corner has considered the more general problem of two charges of different shapes, sizes and compositions by reducing the problem to that of a single equivalent charge with adjusted parameters. After Corner and Clemmow several authors viz Kapur[3,4,5], Venkatesan and Patni[6], Aggarwal[7], Gupta[8] and Tawakley[9] have discussed the problem of composite charges under different conditions. Most of the authors have attempted the analytical solution of the equations which are not integrable without several assumptions and approximations. This fails to provide accurate pressure-time and pressure-space curves which is the most important requirement for the design of new weapons especially which departs materially from the existing ones.

In the present method the four basic equations of internal ballistics namely, (*i*) rate of burning equation, (*ii*) 'Resals' energy equation, (*iii*) equation for the form funcions, and (*iv*) the equation of motion, have been modified for the composite charges consisting of a mixture of grains of 'n' sizes, shapes or compositions, and have been solved numerically using Runge-Kutta method. Pikes rate of burning constant β known to give good results with orthodox guns has been used with a linear law of burning. The assumptions about the linear rate of burning is not really necessary for the numerical solution and the author[10] has solved the internal ballistics equations for recoilless guns with a similar method taking the power law of burning in which case pressures being of a low order a linear law of burning is less accurate. A computer programme is developed which gives the ballistics design parameters f, p, x, v and 't' from the instant the shot starts upto the shot exit. The flow chart for the computer programme is shown in Appendix to this paper.

NOTATIONS

For the ith component charge, let F_i, C_i, D_i, β_i, γ_i, η_i, δ_i, z_i, f_i represent respectively the force constant, charge mass, propellant size, rate of burning constant, ratio of propellant gas specific heats, co-volume per unit mass, propellant density, fraction of charge burnt at time 't' and the fraction of web size remaining at time 't'.

Let A, K_0, x, W, P and V represent respectively the bore area, initial chamber capcity, shot travel at time 't'. the projectile mass (corrected for spin etc), mean gas pressure at time 't' and the shot velocity at any instant.

The subscripts B, 0 and 3 represent conditions at total charge burnt conditions, at shot start and at the muzzle.

BASIC EQUATIONS

Before 'all burnt'

The four basic equations for the internal ballistics of orthodox guns using a composite charge can be written as follows :—

(*i*) The law of burning away of the propellant grains or the equation for the form functions coefficients is :

$$z_i = (1 - f_i)\ (1 + \theta_i f_i) \tag{1}$$

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(*ii*) The equation for the rate of burning of the propellant grains is :

$$\frac{D_i}{\beta_i} \frac{df_i}{dt} = -P \tag{2}$$

(*iii*) The Resal's energy equation is :

$$\sum_{i=1}^{n} F_i C_i z_i = P \left[K_0 + Ax - \sum_{i=1}^{n} C_i z_i \eta_i - \sum_{i=1}^{n} \frac{C_i}{\delta_i} \left(1 - z_i \right) \right] +$$

$$+ \left(\frac{\bar{\gamma} - 1}{2} \right) \left[W + \tfrac{1}{3} \sum_{i=1}^{n} C_i \right] V^2 \tag{3}$$

$\bar{\gamma} = \gamma_1 = \gamma_2 = \gamma_3 \ldots \gamma_n$: since γ is practically same for all propellants.

(*iv*) Equation for the motion of the projectile within the barrel is :

$$\left(W + \tfrac{1}{3} \sum_{i=1}^{n} C_i \right) \frac{dV}{dt} = A\,P \tag{4}$$

After 'all burnt'

(*i*) Equation of motion of the projectile remains the same which can be written as

$$W_1 \frac{VdV}{dx} = AP \text{ or,}$$

$$\frac{dV}{dx} = \frac{AP}{W_1 V} \tag{5}$$

where

$$W_1 = W + \tfrac{1}{3} \sum_{i=1}^{n} C_i$$

(*ii*) After 'all burnt', the expansion of the gases behind the shot is adiabatic

$$PK = P_B K_B$$

where K & K_B are the chamber volume behind the shot at any instant and at 'all burnt' respectively. therefore

$$P = P_B \left[\left(K_0 + Ax_B - \sum_{i=1}^{n} C_i \eta_i \right) \Big/ \left(K_0 + Ax - \sum_{i=1}^{n} C_i \eta_i \right) \right]^{\bar{\gamma}} \tag{6}$$

From (2) the following relationship between the web-fraction remaining to be burnt for various chrages *i.e.,* $f_1, f_2, \ldots \ldots f_{n-1}$ and f_n can be obtained:

$$\frac{D_1}{\beta_1} \frac{df_1}{dt} = \frac{D_2}{\beta_2} \frac{df_2}{dt} = \ldots \ldots \ldots \ldots \frac{D_{n-1}}{\beta_{n-1}} \frac{d(f_{n-1})}{dt} = \frac{D_n}{\beta_n} \frac{d(f_n)}{dt} = -P$$

therefore

$$f_r = 1 + \frac{\beta_r}{D_r} \frac{D_n}{\beta_n} \left(f_{n-1} \right); \text{ where } r = 1, 2 \ldots \ldots \ldots (n-1) \tag{7}$$

when the smallest size is burnt $f_1 = 0$

therefore

$$f_n = 1 - \left[(D_1/\beta_1) \Big/ (D_n/\beta_n) \right]$$

Similarly, when the r^{th} size charge is burnt

$$f_n = 1 - \left[(D_r/\beta_r) \Big/ (D_n/\beta_n) \right]$$

writing

$$\zeta_r = \left[(D_r/\beta_r) \Big/ (D_n/\beta_n) \right], \text{ where } r = 1, 2 \ldots \ldots (n-1)$$

The following important conclusion is made.

Before the charge of r^{th} size is burnt $f_n > 1 - \zeta_r$ and after that $f_n < 1 - \zeta_r$.

Using equations (7) in (1); z_i can be determined at any instant in terms of θ_r, ζ_r (the known constants) and the variable f_n only.

For $r = 1, 2, \ldots \ldots (n-1)$

$$z_r = \frac{(1-f_n)}{\zeta_r} \left[1 + \theta_r - \frac{\theta_r}{\zeta_r} \left(1 - f_n \right) \right]$$

Defining the functions :

$$\left.\begin{aligned} \phi(\theta_r, \zeta_r, f_n) &= \frac{1-f_n}{\zeta_r} \left[1 + \theta_r - \frac{\theta_r}{\zeta_r}\left(1 - f_n \right)\right] \\ z_1 &= \phi(\theta_1, \zeta_1, f_n) \\ z_2 &= \phi(\theta_2, \zeta_2, f_n) \\ z_{n-1} &= \phi(\theta_{n-1}, \zeta_{n-1}, f_n) \end{aligned}\right\} \quad (8)$$

the value of z_n (the largest size) is already known in terms of θ_n and f_n i.e.

$$z_n = (1-f_n)(1 + \theta_n f_n)$$

Combining equation (5) with equation (2) i.e.

$$\frac{D_n}{\beta_n} \frac{d(f_n)}{dt} = - P$$

we have

$$\frac{dV}{df_n} = - \frac{AD_n}{W_1 \beta_n}$$

or

$$V = \frac{AD_n}{W_1 \beta_n} (f_0 - f) \qquad (9)$$

(For convenience only f will be written henceforth for f_n which is taken to be the independent variable for numerical integration ; f_0 is the value of f_n at shot start)

writing $V = \frac{dx}{dt}$ and again combining this with (2)

$$\frac{dx}{df} = - \frac{D_n}{\beta_n} \cdot \frac{V}{P} \qquad (10)$$

Also for the determination of 't' equation (2) can be written as

$$\frac{dt}{df} = - \left(\frac{D_n}{\beta_n} \right) \Big/ P \qquad (11)$$

SUMMARY OF THE EQUATIONS TO BE SOLVED

Before 'all burnt'

$$\frac{dx}{df} = - \frac{D_n}{\beta_n} \frac{V}{P} = \psi_x (V, P)^* \qquad (12)$$

$$\frac{dt}{df} = -\frac{D_n}{\beta_n P} = \psi_t (P)^* \tag{13}$$

$$V = \frac{AD}{\beta_n W_1} (f_0 - f) = \psi_v (f)^* \tag{14}$$

From equation (3)

$$P = \left[\left(\sum_{i=1}^{n} F_i C_i z_i - \tfrac{1}{2} (\bar{\gamma} - 1) W_1 V^2 \right) \Big/ \left(K_0 + Ax - \sum_{i=1}^{n} C_i z_i \eta_i - \sum_{i=1}^{n} \frac{C_i}{\delta_i} (1 - z_i) \right) \right]$$

$$= \left[\left(\sum_{i=1}^{n} F_i C_i z_i - \tfrac{1}{2} (\bar{\gamma} - 1) W_1 V^2 \right) \Big/ \left(A (x + l) - \sum_{i=1}^{n} C_i z_i \left(\eta_i - \frac{1}{\delta_i} \right) \right) \right] = \psi_p (z_i, V, x)^*$$

where (15)

$$Al = K_0 - \sum_{i=1}^{n} \frac{C_i}{\delta_i}$$

$z_1, z_2 \ldots \ldots \ldots z_n$ are given by equations (8)

After 'all burnt'

$$\frac{dV}{dx} = \frac{AP}{W_1 V} = \phi_v (P, V)^* \tag{16}$$

$$P = P_B \left[\left(x_B + l' \right) \Big/ \left(x + l' \right) \right]^{\gamma} \tag{17}$$

where

$$Al' = K_0 - \sum_{i=1}^{n} C_i \eta_i$$

and the equation,

$$\frac{dt}{dx} = \frac{1}{V} = \phi_t (V) \tag{18}$$

SOLUTION OF THE EQUATIONS

From shot start upto 'all burnt'

Upto shot start the ballistic solution is that of a closed vessel. Putting x, V, t equal to zero at shot start f_0 is calculated by assuming the value of P at shot start as P_0 in (15). The values of z_1, z_2 .. z_n are put in terms of f from equation (8). If the value of f_0 so determined is $< 1 - \zeta_1$, it is understood that the shot does not start until the lowest size charge has burnt. As such z_1 is put equal to 1 and f_0 is determined again. This time the value of f_0 is again compared with $1 - \zeta_2$. If this value of f_0 is $<(1 - \zeta_2)$, z_2 is put equal to 1 and so on till the value of f_0 is less than $1 - \zeta_r$. After determining the f_0 the remaining web fraction to be burnt *i.e.* f_0 is divided into 'n' parts of equal step length 'h'. The differential equations (10) and (11) are than solved by Runge-Kutta method as follows:

Initial conditions :— $x = 0, V = 0, P = P_0, f = f_0, t = 0.$

$$k_1 = \psi_x (V, P) (-h) \qquad V = \psi_v (f)$$

$$l_1 = \psi_t (P) (-h) \qquad z_1 = \phi (\theta_1, \zeta_1, f)$$

* ψ and ϕ are the functions defined for ease of writing and also for computer programming.

$$x = x_0 + k_1/2 \qquad z_2 = \phi\,(\theta_2, \zeta_2, f)$$
$$f = f_0 - h/2 \qquad z_{n-1} = \phi\,(\theta_{n-1}, \zeta_{n-1}, f)$$
$$z_n = (1-f)\,(1+\theta_n f)$$

If $f < (1 - \zeta_1)$, $z_1 = 1$

This value of f is successively compared with $(1 - \zeta_r)$ $(r = 1, 2 \ldots\ldots\ldots r)$ till $f < 1 - \zeta_{r-1}$ but $> 1\,\zeta_r$. At this stage $z_1, z_2 \ldots\ldots\ldots z_{r-1}$ are all equal to 1 other values of $z_r, z_{r-1} \ldots\ldots z_n$ are given as before.

Taking these values of z_i $(i = 1, 2, \ldots\ldots\ldots n)$

$$P = \psi_p\,(z_1, z_2 \ldots\ldots\ldots z_n, V, x)$$

Now

$$k_2 = \psi_x\,(V, P)\,(-h) \quad ; \quad l_2 = \psi_t\,(P)\,(-h)$$

This process is repeated in the Runge-Kutta way to get the values of k_3, k_4, l_3 *and* l_4 to give values of

$$x = x_0 + (k_1 + 2k_2 + 2\,k_3 + k_4)/6 \quad ; \quad t = t_0 + (l_1 + 2l_2 + 2l_3 + l_4)/6$$

V and P are calculated at the end of this step i.e. $f = f - h$. The initial conditions are now changed to be the values of the value of x and t at the end of the previous step. The process is repeated till $f = 0$ i.e. upto 'all burnt'.

After 'all burnt'

The remaining shot travel after 'all burnt' i.e. $(x_3 - x_B)$ is divided into equal number of step length each equal to h'.

The solution is progressed as follows :—

Initial conditions : $x = x_B$, $V = V_B$, $P = P_B$, $t = t_B$ and $f = 0$

$$k_1 = \phi_v\,(P_B, V_B)\,(h') \qquad V = V_B + k_1/2$$
$$l_1 = 1/V_B \qquad k_2 = \phi_v\,(P, V)\,(h')$$
$$P = P_B \left[(x_B + l') \Big/ \overline{(x + h'/2 + l')} \right]^{\gamma} \qquad l_2 = 1/V$$

In this way k_3, k_4, l_3 and l_4 are calculated which gives

$$V = V_B + (k_1 + 2\,k_2 + 2\,k_3 + k_4)\,/\,6 \quad ; \quad t = t_B + (l_1 + 2\,l_2 + 2\,l_3 + l_4)\,/\,6$$

the initial conditions are changed and the process is repeated till $x = x_3$. Having determined the ballistic parameter as above it is now simple to draw pressure/time, pressure/velocity/space curves.

From the above it may be noticed that the above solution is very easy, provided a computer is used. The equations that have been solved above, have been derived from the fundamental theory of internal ballistics as given by Corner[2] for single charge without any other approximation and assumptions. As such the above method gives results which matches closely with that of the experimental values. The author tested the above method with composite charges consisting of three sizes and shapes for which experimental values were available. The results being classified information are not reproduced here.

FLOW CHART FOR THE COMPUTER PROGRAMME

A flow chart for the computer programme to calculate the ballistic parameters t, V, P, x and f from start to shot exit for composite charges consisting of three sizes, shapes or composition is given in Appendix. The programme can be extended to cover more number of sizes etc., if required.

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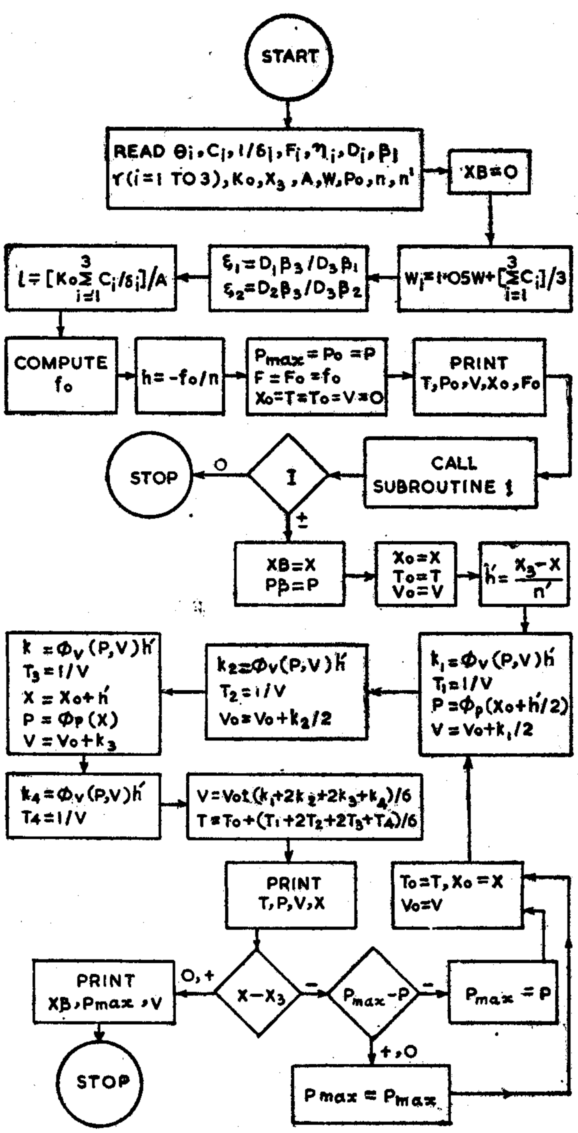
APPENDIX

FLOW CHART FOR THE COMPUTER PROGRAMME

MAIN PROGRAMME



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