# SOLUTION OF INTERNAL BALLISTIC EQUATIONS FOR RECOILLESS GUNS

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Internal ballistic equations for the three cases of recoilless guns, e.g., in which shot start is before, at, or after the nozzle start have been numerically solved using Runge-Kutta method. A computer program is developed that can be used to give ballistic design parameters, effect of loading conditions and Pressure/Time, Pressure/Velocity/Space curves for such type of guns. Pressure/Time/Velocity Spaces curves calculated for two guns with this method have also been shown.

After the first published work of Corner<sup>1</sup>, attempts have been made by several authors viz., Thiruvenkatachar & Venkatesan<sup>2</sup>, Aggarwal<sup>3</sup>, Asim Ray<sup>4</sup> and others to solve the internal ballistic problems of recoilless guns. Practically every one has assumed that the deviations of the ballistics from that of normal guns can be expressed as linear functions of  $\Psi$ . The same assumption has been recently made by Tawakley and Ahuja<sup>5</sup> who obtained expressions for calculating the basic internal ballistic quantities upto 'all burnt' for orthodox guns added with a correction term accounting for leakage. Tawakley and Ahuja<sup>5</sup> like Corner<sup>1</sup> and others have neglected the covolume term in the equation of state. To some extent this may suffice for routine ballistics work, but an accurate pressure-time, pressure-space curve and effect of loading conditions viz., variation in bursting strength of liner or disc for the opening of the nozzle, chamber capacity shot travel and nozzle throat area etc., are necessary for the design of such guns.

The first attempt to solve directly the basic internal equations of ballistics without any such assumptions was made by Dewis<sup>6</sup> who solved such equations by iteration. Though the details of the method or the computer program followed by him are not completely given it appears that in order to take P and dx/dt as the mean of the values at the beginning and end of a step Dewis has assumed a linear law of burning in which dx/dt is given by a simple relation :

$$\frac{dx}{dt} = \frac{AD(f_0 - f)}{\beta(W + \frac{1}{k}CN)}$$

It has been shown below that as many as five simultaneous differential equations connected with four other simple relations are to be solved simultaneously. It becomes a lengthy and complicated process to achieve the self consistency of these variables during iteration. In this paper the internal ballistic equations for the three cases of recoilless guns based on equations given by Corner have been numerically solved by Runge-Kutta method (Dewis<sup>6</sup> has discussed only the case where shot start occurs before nozzle opening). The following solution gives all the ballistic parameters of interest i.e.,  $P. f. t. V, \phi, N$  and xthroughout the motion of the projectile inside the barrel. The value of  $\int Pdt$  which is required for the calculations of recoil momentum of the gun is also obtained at each step from all burnt to complete gas exhaustion.

NOTATIONS

		<b>NOTATIONS</b>		
Symbol		Dimension	Meaning	
A		$L^2$	Gun bore area	
C		M	Charge weight	
U		$L^3$	Gun chamber capacity	
D	n an	$oldsymbol{L}$	Propellant web size	
f			Fraction of 'D' remaining at time 't'	
x		L	Shot travel at time 't'	
V		$LT^{-1}$	Shot velocity at time 't'	
φ		Aliante de la companya de la company Aliante de la companya	Fraction of charge burnt at time 't'	

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Symbol	Dimension	Meaning
N		Fraction of Charge burnt at time 't' present in the gun
<b>T</b> <sub>0</sub> (° <b>K</b> )		Adiabatic flame temperature of propellant gases
$T(^{\circ}K)$		Gas temperature at time 't'
α	na da esta esta esta esta esta esta esta est	Pressure index for power law burning
$T^1$ (° $K$ ).		Ratio $T/T_{0}$
S	$L^2$	Ventury throat area
$rac{S_t}{W}$	M	Projectile mass
β	$L^2 TM^{-1}$	Propellant rate of burning constant
θ-		Propellant form factor
<b>n</b>	$\cdot L^3 M^{-1}$	Propellant gas co-volume
그는 그는 것이 아무지 않는 것을 많이	에는 것 같은 것은 것이 같은 것이 있는 것이 있다. 것이 있는 것이 있는 것이 있다. 같은 것은 것은 것이 같은 것이 같은 것이 있는 것이 같이 있는 것이 같이 있다.	Propellant gas specific heat ratio
<b>γ</b> 8	ML3	Propellant density
k R		Emperical Constant (Corner)
R	$L^2 T^{-2^{\circ}} K^{-1}$	Gas Constant
	$ML^{-1}T^{-2}$	Mean gas pressure at any time
Ρ χ ξ ζ		Factor for heat loss
· · · · · · · · · · · · · · · · · · ·		$NT^1 - \phi$
Ν. ζ. Α.		$\phi - N$
$\mathbf{\Psi}$		Nozzle discharge co-efficient
Uo	<b>L</b> <sup>8</sup>	Total internal capacity of the gun
Subscript	SS	Refers to condition at Shot start
-	NS	. Refers to condition at Nozzle start
		Refers to condition at Shot exit
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# INTERNAL BALLISTIC EQUATIONS

The ballistic equations to be considered are given below for the five possible conditions.

(a) Upto shot or nozzle start whichever is earlier the solution is that of a closed vessel and the equations are :

$$P\left[U - \frac{C}{\delta} - \left(\eta - \frac{1}{\delta}\right)C\phi\right] = C\phi RT_0$$

$$\phi = (1 - f) (1 + \theta f)$$
(1)
(2)

(b) If  $P_{NS} > P_{SS}$ , from shot start to nozzle start the system behaves as a conventional gun and equations are :

$$P\left[U + Ax - \frac{C}{\delta} - \left(\eta - \frac{1}{\delta}\right)C\phi\right] = C\phi RT_0 - \frac{1}{2}(\overline{\gamma} - 1)\left(W + \frac{C}{3}\right)V^2 \qquad (3)$$
$$\overline{\gamma} = 1 + (\gamma - 1)(1 + \gamma) \qquad (\text{Ref 1 Page 260})$$

where

$$\widehat{\gamma} = 1 + (\gamma - 1) (1 + \chi) \quad (\text{Ref 1 Page 260})$$

$$\left(W + \frac{C}{3}\right) \frac{dV}{dt} = AP \quad (4)$$

$$-D \frac{df}{dt} = \beta P^{\alpha}$$
 (5)

- $\phi$  is given as before by (2)
  - (c) If  $P_{NS} < P_{SS}$ , the equations from nozzle start to shot start are :

$$P\left[U - \frac{C(1-\phi)}{\delta} - NC\eta\right] = CNRT_0T^2\left[1 + \frac{kCN}{6W}\right]$$
(6)  
$$\frac{dN}{dt} = \frac{d\phi}{dt} - \frac{\psi S_t P(\overline{T}^1)^{-1/2}}{C(RT_0)^{1/2}}$$
(7)

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$$\frac{d}{dt} (NT^{1}) = \frac{d\phi}{dt} - \frac{\gamma' \psi S_{\epsilon} P (T^{1})^{1/2}}{C (RT_{0})^{1/2}}$$

$$\gamma' = \gamma + (\gamma - 1) \epsilon$$
(8)

where<sup>1</sup>

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$$= 1 + (\gamma - 1) \left[ 1 + \frac{\gamma P}{RT_0 T^1} \right]$$
 (9)

(d) If  $P_{NS} = P_{SS}$  or after shot as well as nozzle have started and propellant is still burning the equations are :

$$P\left[U + Ax - \frac{C(1-\phi)}{\delta} - NC\eta\right] = CNRT_0T^1\left(1 + \frac{kCN}{6W}\right)$$
(10)

$$\left(W+\frac{1}{2}kCN\right)\frac{dV}{dt} = AP$$
(11)

$$\frac{d}{dt} (NT^{1}) = -(\bar{\gamma} - 1) \left( \frac{AP}{CRT_{0}} \frac{dx}{dt} \right) + \frac{d\phi}{dt} - \frac{\gamma' \psi S_{t} P(T^{1})^{1/2}}{C (RT_{0})^{1/2}}$$
(12)

Equations (2), (5) & (7) hold good.

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(e) From the propellant 'all burnt' up to shot ejection, the equations are :

$$\left[ U + Ax - NC\eta \right] = CNRT_0 T^1 \left[ 1 + \frac{kCN}{6W} \right]$$
(13)

$$\frac{dN}{dt} = -\frac{\psi S_t P}{C (RT_0)^{1/2}} (T^1)^{-1/2}$$
(14)

$$\frac{d(NT^{1})}{dt} = (\bar{\gamma} - 1) \frac{AP}{CRT_{0}} \frac{dx}{dt} - \frac{\gamma' \psi S_{t} P(T^{1})^{1/2}}{C(RT_{0})^{1/2}}$$
(15)

and equation (11)

 $(W + \frac{1}{2} kCN) \frac{dV}{dt} = AP$ 

(f) After the shot ejection up to complete gas exhaustion the equations are :

$$P\left[U_{0} - NC\eta\right] = CNRT_{0}T^{1}$$
(16)

$$\frac{dN}{dt} = -\frac{\psi \left(S_t + A\right)P}{C \left(RT_0\right)^{1/2}} (T^1)^{-1/2}$$
(17)

$$\frac{d}{dt} (NT^{1}) = -\gamma' \frac{\psi (S_{t} + A) P (T^{1})^{1/2}}{C (RT_{0})^{1/2}}$$
(18)

# RECOIL MOMENTUM AND RECOIL ENERGY OF RCL GUNS

The thrust Co-efficient<sup>1,6</sup> of a nozzle is defined as :

$$\xi = \frac{F}{P_r S_r}$$

Where  $P_r$  is the reservoir pressure taken as the maximum pressure in the gun at that instant. The recoil momentum can be calculated as follows:

Case 
$$A : -$$

$$P_{NS} > P_{SS}$$

Forward momentum from shot start to nozzle start :

$$-A\int_{SS}^{NS} Pdt = -A\int_{SS}^{NS} -\frac{D}{\beta} \frac{df}{dt} = \frac{AD}{\beta} \left( f_{NS} - f_{SS} \right)$$

From nozzle start to 'all burnt'

$$= (\xi S_t - A) \int_{NS}^{Burnt} P dt = (\xi S_t - A) \int_{NS}^{Burnt} - \frac{D}{\beta} \frac{df}{dt}$$

from all hurnt to shot ejection

$$= (\xi S_t - A) \int_{Burnt}^{E} P dt$$

from shot ejection to complete gas exhaustion

$$= (\xi S_t - A) \int_E^\infty P dt$$

: Total forward momentum imparted to the gun

$$= \frac{D}{\beta} \Big[ \xi S_t f_{NS} - A f_{SS} \Big] + \Big( \xi S_t - A \Big) \int_{Burnt}^{\infty} P dt$$

Similarly for Case B :  $(P_{NS} < P_{SS})$  and Case C :  $(P_{NS} = P_{SS})$ .

It can be shown that the expression for recoil momentum is the same.  $\int Pdt$  from all burnt to gas exhaustion is determined by simple summation of  $P \times dt$ .

## METHOD OF SOLUTION

(a) From ignition to shot or nozzle start whichever is earlier :---

Taking:  $P = P_{SS}$  or  $P_{NS}$  (Lower of the two) the value of 'f' is calculated from equation (1) using the value of ' $\phi$ ' in terms of 'f' from (2). This value of 'f' gives  $f_{SS}$  or  $f_{NS}$ . In the case  $P_{NS} = P_{SS}$ ,  $f_{SS} = f_{NS}$ . At the end of this stage the ballistic parameters are V = 0, x = 0,  $P = P_{NS}$  or  $P_{SS}$ ,  $f = f_{NS}$  or  $f_{SS}$  and t = 0.

(b) Case A:  $P_{NS} > P_{SS}$ . From shot start to nozzle start, the equations (3), (4), & (5) can be put as follows:

$$\begin{aligned} \frac{dx}{df} &= -\frac{DV}{\beta P^{\alpha}} &= \Psi_{x}(V, P) \\ \phi &= (1 - f) (1 + \theta f) \\ \frac{dV}{df} &= -\frac{ADP^{1-\alpha}}{\beta (W + C/3)} &= \Psi_{x}(P) \\ \frac{dt}{df} &= -\frac{D}{\beta P^{\alpha}} - \cdots = \Psi_{t}(P) \\ P &= \frac{C\phi RT_{0} - \frac{1}{2}(\gamma - 1) (W + C/3)V^{2}}{u + Ax - C/\delta - (n - 1/\delta) \cdot C\phi} &= \Psi_{P}(\phi, V, x) \end{aligned}$$

for convenience the functions have been defined as above. The Runge-Kutta steps for the solution of above equations are :

(i) Initial conditions : 
$$x = x_0 = 0$$
,  $V = V_0 = 0$ ,  $t = t_0 = 0$ ,  $P = P_{SS}$ ,  $f = f_0 = f_{SS}$   
 $k_1 = \Psi_x (V_0, P) h$   $V = V_0 + m_1/2$   
 $l_1 = \Psi_t (P) \cdot h$   $f = f_0 - h/2$ 

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$$\begin{array}{ll} m_1 = \Psi_v\left(P\right) \cdot h & \phi = (1-f)\left(1+\theta f\right) \\ x = x_0 + k_1/2 & P = \Psi_P\left(\phi, V, f\right) \\ [h = -df \text{ is the step length}]. \end{array}$$

Using new values of V and P,  $k_2$ ,  $l_2$  and  $m_2$  are again determined. Similarly  $k_3$ ,  $l_3$ ,  $m_3$  and  $k_4$ ,  $l_4$  and  $m_4$  are calculated in the usual way of Runge-Kutta steps<sup>7</sup> giving:

$$\begin{aligned} x &= x_0 + \left(k_1 + 2 k_2 + 2 k_3 + k_4\right) / 6 \\ V &= V_0 + \left(m_1 + 2 m_2 + 2 m_3 + m_4\right) / 6 \\ t &= t_0 + \left(l_1 + 2 l_2 + 2 l_3 + l_4\right) / 6 \end{aligned}$$

The value of ballistic parameters after the first step, i.e.,  $f = f_0 - h$  and V, P, x and t as calculated above are taken as the initial conditions for the second step. The process is repeated till  $P > P_{NS}$ . The parameters at  $P = P_{NS}$  can be found by inter-polation.

(c) Case B :--  $P_{NS} < P_{SS}$ . From nozzle start, to shot start the equations (6), (7), (8) & (9) can be put as follows :

$$P=rac{CNT_{0}T^{1}\left(1+rac{kCN}{6W}
ight)}{U-C\left(1-\phi
ight)/\delta-NC\eta}=arP_{P}\left(\phi,\,N,\,T^{1}
ight)$$
 say

$$\frac{d(\zeta)}{df} = \frac{\Psi S_t P^{1-\alpha} D (T^1)^{-1/2}}{\beta C (RT_0)^{1/2}} = \Psi_{\zeta} (PT^1) \text{ say}$$
$$\frac{d(\xi)}{df} = \frac{\gamma \Psi S_t P (T^1)^{1/2} D}{\beta C (RT_0)^{1/2}} = \Psi_{\xi} (P, T^1) \text{ say}$$

 $\gamma'$  has been kept equal to  $\gamma$  in view of Corner's comment<sup>1</sup>

$$\phi = (1 - f) (1 + \theta f)$$

Here the initial conditions are  $P = P_{NS}$ ,  $f = f_{NS}$ ,  $N = \phi_{NS}$ ,  $\xi = 0$ ,  $\zeta = 0$  and  $T^1 = 1$ , the Runge-Kutta method is used to give the value of  $\xi$  and  $\zeta$ , N and  $T^1$  as follows:

$$k_{1} = \Psi_{\zeta} (P, T^{1}) \cdot h$$

$$l_{1} = \Psi_{\xi} (P, T^{1}) \cdot h$$

$$f = f_{0} - h/2$$

$$\phi = (1 - f) (1 + \theta f)$$

$$\zeta = \zeta_{0} + k_{1}/2$$

$$\xi = \xi_{0} + l_{1}/2$$

$$N = \phi - \zeta$$

$$T^{1} = (\xi + \phi) / N$$

$$P = \Psi_{P} (P, T^{1})$$

The Runge-Kutta process is repeated to get  $k_2$ ,  $l_2$ ,  $k_3$ ,  $l_3$  and  $k_4$ ,  $l_4$  to give the final value of all these variables up to  $P = P_{SS}$ .

(d) This section deals with the solution up to the all burnt of the propellant. The equations (10), (11) & (12) neglecting heat losses through barrel can be put as follows:

$$P = \frac{CNRT_0 T^1 \left(1 + \frac{kCN}{6W}\right)}{\left[U + Ax - \frac{C(1-\phi)}{\delta} - NC \eta\right]} = \Psi_P(x, \phi, N, T^1)$$
  
$$\frac{dV}{df} = -\frac{ADP^{1-\alpha}}{\beta (W + \frac{1}{2} kCN)} = \Psi_v(P, N)$$
  
$$\phi = (1-f) (1+\theta f)$$
  
$$\frac{dt}{df} = -\frac{\beta}{D} P^{-\alpha} = \Psi_t(P)$$

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$$\begin{aligned} \frac{d\zeta}{df} &= -\frac{\psi \, S_t \, P^{1-\alpha} \, (T^1)^{-1/2} \, D}{\beta \, C \, (RT_0)^{1/2}} \, = \Psi_{\zeta} \, (P, \, T^1) \\ \frac{d\xi}{df} &= \frac{(\gamma - 1) \, A P^{1-\alpha} \, D V}{\beta \, C \, RT_0} + \frac{\gamma \, \psi S_t \, P^{1-\alpha} \, D \, (T^1)^{1/2}}{\beta \, (RT_0)^{1/2}} = \Psi_{\xi} \, (P, \, V, \, T^1) \\ N &= \phi - \zeta \\ T^1 &= (\xi + \phi)/N \\ \frac{dx}{df} &= -\frac{V D}{\beta \, P^\alpha} = \Psi \, (V, P) \end{aligned}$$

The last equation has been obtained by writing dx/dt = V and then dividing by equation (5). It may be noted that in the above set of equations there are five differential equations all with 'f' as independent variable connected with four simple equations giving  $P, \phi, N$  and  $T^1$ . The initial conditions are :

- (i) Case A :-  $P = P_{NS}, f = f_{NS}, V = V_{NS}, t = t_{NS}$  (given at the end of section 5(b),  $T^1 = 1$ ,  $\phi = \phi_{NS}, N = \phi_{NS}$  and  $\xi = \zeta = 0$ .
- (ii) Case B :-  $P = P_{SS}$ ,  $f = f_{SS}$ , V = 0,  $\phi = \phi_{SS}$ , t = 0, x = 0. N,  $\xi$ ,  $\zeta$  and  $T^1$  are given at the end of section  $5(\alpha)$ .
- (iii) Case C :- when  $P_{NS} = P_{SS} :- P = P_{NS} = P_{SS}$ ,  $f = f_{NS}$ , (Calculated) in section 5(a),  $\phi = \phi_{NS}$ ,  $t, x, V, \xi, \zeta = 0$ ;  $T^1 = 1$ . The suffix '0' is used for initial conditions.

The steps of Runge-Kutta method followed are as below:

$$\begin{array}{ll} k_1 &= \Psi_{\xi} & (P_0, V_0, T_0^{-1}) (-h) & \xi &= \xi_0 + k_1/2 \\ l_1 &= \Psi_{\xi} & (P_0, T_0^{-1}) (-h) & \zeta &= \zeta_0 + l_1/2 \\ m_1 &= \Psi_x & (V_0, P_0) (-h) & N &= \phi - \zeta \\ n_1 &= \Psi_v & (P_0, N_0) (-h) & T^1 &= (\xi + \phi)/N \\ O_1 &= \Psi_t & (p_0) (-h) & x &= x_0 + m_1/2 \\ f &= f_0 - h/2 & V &= V_0 + n_1/2 \\ \phi &= (1 - f) (1 + \theta f) & P &= \Psi_p & (x, \phi, N, T^1) \end{array}$$

In the same way  $k_2$ ,  $k_3$ ,  $k_4$ ,  $l_2$ ,  $l_3$ ,  $l_4$ ,  $m_2$ ,  $m_3$ ,  $m_4$ ,  $n_2$ ,  $n_3$ ,  $n_4$  and  $O_2$ ,  $O_3$ ,  $O_4$  are determined giving :

$$\begin{split} \xi &= \xi_0 + (k_1 + 2 k_2 + 2 k_3 + k_4)/6 \\ \xi &= \zeta_0 + (l_1 + 2 l_2 + 2 l_3 + l_4)/6 \\ x &= x_0 + (m_1 + 2 m_2 + 2 m_3 + m_4)/6 \end{split} \qquad \qquad V = V_0 + (n_1 + 2 n_2 + 2 n_3 + n_4)/6 \\ t &= t_0 + (O_1 + 2 O_2 + 2 O_3 + O_4)/6 \end{split}$$

The values of P, N,  $T^1$ , and  $\phi$  are also calculated from the above equations the values of all the nine variables at the end of 1st step, i.e., at f = f - h and taken as the initial values for the second step and the process is repeated till f = 0. If at any stage (f - h) < 0, by interpolation the variables at f = 0 are calculated.

(e) From 'all burnt' to shot ejection, the set of equation (13), (14) & (15) can be written as follows :

$$P = \frac{CN \ RT_0 \ T^1 \left(1 + \frac{kCN}{6W}\right)}{(U + Ax - NC \eta)}$$
$$\frac{dt}{dN} = -\frac{C \ (RT_0)^{1/2} \ (T^1)^{1/2}}{\Psi S_t \ P}$$
$$\frac{d\xi}{dN} = \frac{(\gamma - 1) \ A \ (T^1)^{1/2} \ V}{\Psi S_t \ (RT_0)^{1/2}} + \gamma T^1$$
$$\frac{dV}{dN} = \frac{AC \ (RT_0)^{1/2} \ (T^1)^{1/2}}{(W + \frac{1}{2} \ kCN) \ \Psi S_t}$$
$$\frac{dx}{dN} = \frac{C \ (RT_0)^{1/2} \ (T^1)^{1/2}V}{\Psi \ S_t \ P}$$

Taking the values of P, t,  $\xi$ , V, x, N,  $T^{1}(\phi = 1)$  from the end of last section (d) the Runge-Kutta method is applied as before. N being the independent variable in this case, the step length is taken as  $dN = -h^{1}$ .

This process is continued till  $x = x_3$ , i.e., the total shot travel. The values of parameters p, t, V,  $x, N, T^1$  are calculated at  $x = x_3$ .

(f) From shot ejection upto complete gas exhaustion :

Writing 
$$\triangle = \frac{O}{U_0}$$

Equations (16), (17) & (18) can be written as

$$P = \frac{RT_0 \triangle N T^1}{1 - \triangle N n}$$
(19)

$$\frac{dN}{dt} = -\frac{\psi}{C} \frac{(S_t + A) P}{(BT)^{1/2} (T^{1})^{1/2}}$$
(20)

$$\frac{d}{dN} (NT^1) = \gamma^1 T^1$$
(21)

or

$$N \ \frac{dT^1}{dN} = (\gamma^1 - 1)T^1$$

giving

$$\frac{dT^{1}}{T^{1}} = (\gamma^{1} - 1) \quad \frac{dN}{N}$$
$$= (\gamma - 1) \left[ \frac{1}{N} + \frac{\Delta \eta}{1 - N \Delta \eta} \right] dN$$

integrating from shot exit.

$$\begin{bmatrix} \log T^{1} \end{bmatrix} = (\gamma - 1) \left[ \left( \log N \right)_{N_{E}}^{N} - \log \left( \sum_{N_{E}}^{N} - N \bigtriangleup \eta \right) \right]$$

or

$$\frac{T^{1}}{T_{E}} = \left[\frac{N}{N_{E}} \cdot \frac{1 - N_{E} \bigtriangleup \eta}{1 - N \bigtriangleup \eta}\right]^{\gamma - 1}$$
(22)

Putting values of P and  $T^1$  from equations (20) and (22), (21) can be written as:

$$\frac{dt}{dN} = \frac{-C \left\{\frac{1}{N_{E}} - \bigtriangleup \eta\right\}^{\frac{1-\gamma}{2}} \left[\frac{1}{N} - \bigtriangleup \eta\right]^{\frac{\gamma+1}{2}}}{\psi \left(S_{t} + A\right) \left(RT_{0}\right)^{1/2} \left(T^{1}_{E}\right)^{1/2} \bigtriangleup}$$

giving<sup>2</sup>

$$t = t_E - \frac{C\left\{\frac{1}{N_E} - \bigtriangleup\eta\right\}^{\frac{1-\gamma}{2}}}{\psi(S_t + A)(RT_0)^{1/2}(T_E^1)^{1/2}\bigtriangleup} \int_{N_E}^{N} \left(\frac{1}{N} - \bigtriangleup\eta\right)^{\frac{\gamma+1}{2}} dN$$

After the shot ejection the parameters of interest are P, t (for calculating  $\int Pdt$  and time for complete gas exhaustion). Initial conditions are  $P_E$ ,  $N_E$ ,  $T_E^1$  and  $t_E$  (these values at the end of previous section).

The following functions are defined

$$\begin{split} \Psi_{P}\left(N,\,T^{1}\right) &= \frac{RT_{0} \bigtriangleup N \,T^{1}}{1-\bigtriangleup N \,\eta} \\ \Psi_{T^{1}}\left(N\right) &= T^{1}_{E}\left[\frac{N}{N_{E}} \cdot \frac{1-N_{E}\bigtriangleup \eta}{1-N\bigtriangleup \eta}\right]^{\gamma-1} \\ \Psi_{T^{1}}\left(N\right) &= \frac{C\left\{\frac{1}{N_{E}}-\bigtriangleup \eta\right\}^{\frac{1-\gamma}{2}}}{\psi\left(S+A\right)\left(RT_{0}\right)^{1/2}\left(T^{1}_{E}\right)^{1/2}\bigtriangleup \left(\frac{1}{N_{E}}-\bigtriangleup \eta\right]^{\frac{\gamma-1}{2}}} \end{split}$$

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$$N = N_{E} - h'$$

$$P = \Psi_{P} (N, T^{1})$$

$$T^{1} = \Psi_{T^{1}} (N)$$

$$dt = \left[ \Psi_{i} (N_{E}) + 4 \Psi_{i} \left( N_{E} - \frac{h'}{2} \right) + \Psi_{i} (N_{E} - h') \right] \cdot \left( - \frac{h'}{6} \right)$$

$$N_{E} - h'$$

$$\int_{P} P dt = P_{*} dt \quad \text{(from above)}$$

This process is repeated till  $N \simeq 0$  the value of  $\int Pdt$  is obtained by summation.

FLOW CHART FOR THE COMPUTER PROGRAM

For calculating the complete internal ballistics up to gas exhaustion, the flow chart for the Computer Program are shown in Appendix "A". Appendix "B" gives the Pressure/Velocity/Space curves calculated by this method for two guns.

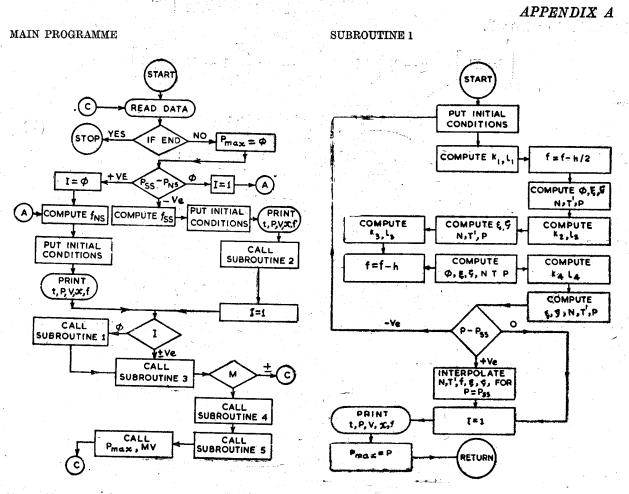


Fig. 1-Flow chart for the computer programme.

Fig. 2—Ballistic parameters from nozzle start to start.

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### REFERENCES

1. CORNER, J., "Theory of the Interior Ballistics of Guns" (John Wiley & Sons, New York), 1950, p. 243.

2. THIBUVENKATACHAR, V. R. & VENKATESAN, N. S., Proc. Nat. Inst. Sci., India, 19 (1953), 829-837.

3. AGGARWAL, S. P., Proc. Nat. Inst, Sci., India, 21 (1956), 91-97.

4. RAY, ASIM., Def. Sci. J., 17 (1967), 13-20.

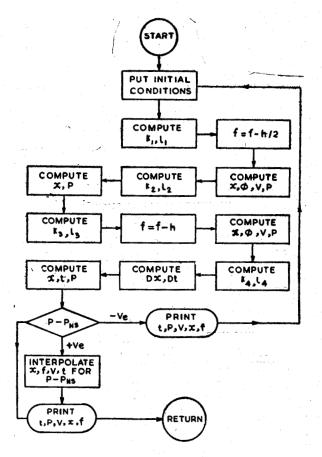
5. TAWAKLEY, V. B. & AHUJA, J. S., Sciences et Techniques de Armament 4 fasc 1972.

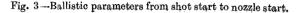
6. DEWIS, A. J., RARDE (UK) Memorandum 7/71.

7. SCARBOROUGH, JAMES, B., "Numerical Mathematical Analysis" (The John Hopkins Press, Baltimore), 1955, p. 300"

SUBROUTINE 2

### SUBROUTINE 3





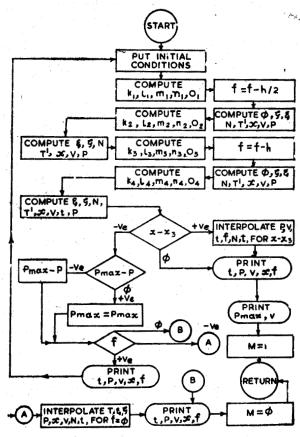
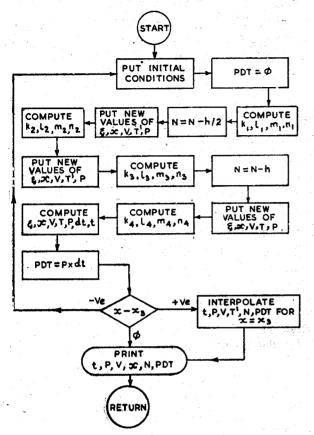


Fig. 4—Ballistic parameters from shot and nozzle start upto all burnt or shot exit whichever is earlier.

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#### SUBROUTINE 4





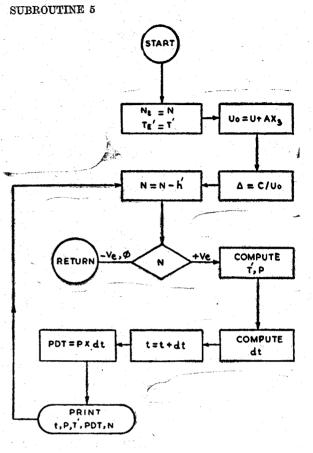
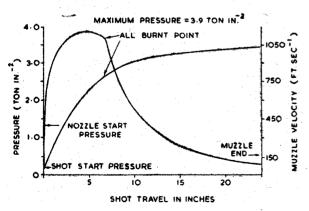
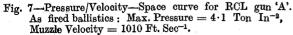


Fig. 6-Ballistic parameter from shot exit to complete gas exhaustion.





APPENDIX B

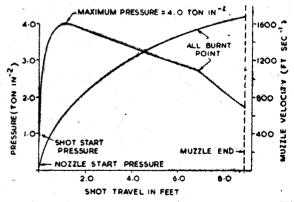


Fig. 8—Pressure/Velocity—Space curve for RCL gun 'B'. As fired ballistics : Max. Pressure = 4.1 Ton In<sup>-3</sup> Muzzle Velocity = 1673 Ft. Sec<sup>-1</sup>,