# SOLUTION OF INTERNAL BALLISTIC EQUATIONS FOR RECOILLESS GUNS 

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#### Abstract

Internal ballistic equations for the three cases of recoilless guns, e.g., in which shot start is before, at, or after the nozzle start have been numerically solved using Runge-Kutta method. A computer program is developed that can be used to give ballistic design parameters, effect of loading conditions and Pressure/Time, Pressure/Velocity/Space curves for such type of guns. Pressure/Time/Velocity Spaces curves calculated for two guns with this method have also been shown.


After the first published work of Corner ${ }^{1}$, attempts have been made by several authors viz., Thiruvenkatachar \& Venkatesan ${ }^{2}$, Aggarwal ${ }^{3}$, Asim Ray ${ }^{4}$ and others to solve the internal ballistic problems of recoilless guns. Practically cvery one has assumed that the deviations of the ballistics from that of normal guns can be expressed as linear functions of $\Psi$. The same assumption has been recently made by Tawakley and Ahuja. ${ }^{5}$ who obtained expressions for calculating the basic internal ballistic quantities upto 'all burnt' for orthodox guns added with a correction term accounting for leakage. Tawakley and Ahuja ${ }^{5}$ like Corner ${ }^{1}$ and others have neglected the covolume term in the equation of state. To some extent this may suffice for routine ballistics work, but an accarate pressure-time, pressure-space curve and effect of loading conditions viz., variation in bursting strength of liner or dise for the opening of the nozzle, chamber capacity shot travel and nozzle throat area etc., are necessary for the design of such guns.

The first attempt to solve directly the basic internal equations of ballistics without any such assumptions was made by Dewis ${ }^{6}$ who celved such equations by iteration. Though the details of the method or the computer program followed by him are not completely given it appears that in order to take $\boldsymbol{P}$ and $d x / d t$ as the mean of the values at the beginning and end of a step Dewis has assumed a linear law of burning in which $d x / d t$ is given by a simple relation :

$$
\frac{d x}{d t}=\frac{A D\left(f_{0}-f\right)}{\beta\left(W+\frac{1}{2} k C N\right)}
$$

It has been shown below that as many as five simultaneous differential equations connected with four other simple relations are to be solved simultaneously. It becomes a lengthy and complicated process to achieve the self consistency of these variables during iteration. In this paper the internal ballistic equations for the three cases of recoilless guns based on equations given by Corner have been numerically solved by Runge-Kutta method (Dewis ${ }^{6}$ has discussed only the case where shot start occurs before nozzle opening). The following solution gives all the ballistic parameters of interest i.e., P.f. $t . V, \phi, N$ and $x$ throughout the motion of the projectile inside the barrel. The value of $\int P d t$ which is required for the calculations of recoil momentum of the gun is also obtained at each step from all burnt to complete gas exhaustion.

NOTATIONS

| Symbol | Dimension |
| :---: | :---: |
| $A$ | $L^{2}$ |
| $C$ | $M$ |
| $U$ | $L^{3}$ |
| $D$ | $L$ |
| $f$ | - |
| $x$ | $L$ |
| $V$ | $L T^{-1}$ |
| $\phi$ | - |

## Meaning

Gun bore area
Charge weight
Gun chamber capacity
Propellant web size
Fraction of ' $D$ ' remaining at time ' $t$ '
Shot travel at time ' $t$ '
Shot velocity at time ' t '
Fraction of charge burnt at time ' $t$ '

Symbol

| N | - | Fraction of Charge burnt at time ' $t$ ' present in the gun |
| :---: | :---: | :---: |
| $T_{0}\left({ }^{\circ} \mathrm{K}\right)$ | - | Adiabatic flame temperature of propellant gases |
| $\left.T{ }^{\circ} \mathrm{K}\right)$ | - | Gas temperature at time ' $t$ ' |
| $\alpha$ | - | Pressure index for power law burning |
| $T^{1}\left({ }^{\circ} \mathrm{K}\right)$. | - | Ratio $T / T_{0}$ |
| $S_{t}$ | $L^{2}$ | Ventury throat area |
| W | $M$ | Projectile mass |
| $\beta$ | $L^{2} T M^{-1}$ | Propellant rate of burning constant |
| $\theta$ | - | Propellant form factor |
| $\eta$ | $L^{3} M^{-1}$ | Propellant gas co-volume |
| $\gamma$ | - | Propellant gas specific heat ratio |
| $\delta$ | $M L^{-3}$ | Propellant density |
| 1 | - | Emperical Constant (Corner) |
| $R$ | $L^{2} T^{\sim 20} K^{-1}$ | Gas Constant |
| $P$ | $M L^{-1} T^{-2}$ | Mean gas pressure at any time |
| $\chi$ | - | Factor for heat loss |
| $\xi$ | - | $N T^{1}-\phi$ |
| $\zeta$ | - | $\phi-N$ |
| $\Psi$ | - | Nozzle discharge co-efficient |
| $U_{0}$ | $L^{3}$ | Total internal capacity of the gun |
| Subscript | SS | Refers to condition at Shot start |
|  | NS | Refers to condition at Nozzle start |
|  | E | Refers to condition at Shot exit |

## INTERNALBALLISTIC EQUATIONS

The ballistic equations to be considered are given below for the five possible conditions.
(a) Upto shot or nozzle start whichever is earlier the solution is that of a closed vessel and the equations are :

$$
\begin{gather*}
P\left[U-\frac{C}{\delta}-\left(\eta-\frac{1}{\delta}\right) C \phi\right]=C \phi R T_{0}  \tag{1}\\
\phi=(1-f)(1+\theta f) \tag{2}
\end{gather*}
$$

(b) If $P_{N S}>P_{S S}$, from shot start to nozzle start the system behaves as a conventional gun and equations are :

$$
\begin{equation*}
P\left[U+A x-\frac{C}{\delta}-\left(\eta-\frac{1}{\delta}\right) C \phi\right]=C \phi R T_{0}-\frac{1}{2}(\gamma-1)\left(W+\frac{C}{3}\right) V^{2} \tag{3}
\end{equation*}
$$

where

$$
\begin{gather*}
\bar{\gamma}=1+(\gamma-1)(1+\chi) \\
\left(W+\frac{C}{3}\right) \frac{d V}{d t}=A P  \tag{4}\\
-D \frac{d f}{d t}=\beta P^{\alpha} \tag{5}
\end{gather*}
$$

$\phi$ is given as before by (2)
(c) If $P_{N S}<P_{S S}$, the equations from nozzle start to shot start are :

$$
\begin{gather*}
P\left[U-\frac{C(1-\phi)}{\delta}-N C \eta\right]=C N R T_{0} T^{2}\left[1+\frac{k C N}{6 W}\right]  \tag{6}\\
\frac{d N}{d t}=\frac{d \phi}{d t}-\frac{\psi S_{t} P\left(T^{1}\right)^{-1 / 2}}{C\left(R T_{0}\right)^{1 / 2}} \tag{7}
\end{gather*}
$$

$$
\begin{equation*}
\frac{d}{d t}\left(N T^{1}\right)=\frac{d \phi}{d t}-\frac{\gamma^{\prime} \psi S_{t} P\left(T^{1}\right)^{1 / 2}}{C\left(R T_{0}\right)^{1 / 2}} \tag{8}
\end{equation*}
$$

where ${ }^{1}$

$$
\gamma^{\prime}=\gamma+(\gamma-1) \epsilon
$$

or

$$
\begin{equation*}
=1+(\gamma-1)\left[1+\frac{\eta P}{R T_{0} T^{1}}\right] \tag{9}
\end{equation*}
$$

(d) If $P_{N S}=P_{S S}$ or after shot as well as nozzle have started and propellant is still burning the equations are :

$$
\begin{align*}
& P\left[U+A x-\frac{C(1-\phi)}{\delta}-N C_{\eta}\right]=C N R T_{0} T^{1}\left(1+\frac{k C N}{6 W}\right)  \tag{10}\\
& \left(W+\frac{1}{2} k C N\right) \frac{d V}{d t}=A P  \tag{11}\\
& \frac{d}{d t}\left(N T^{1}\right)=-(\gamma-1)\left(\frac{A P}{C R T_{0}} \frac{d x}{d t}\right)+\frac{d \phi}{d t}-\frac{\gamma^{\prime} \psi S_{t} P\left(T^{1}\right)^{1 / 2}}{C\left(R T_{0}\right)^{1 / 2}} \tag{12}
\end{align*}
$$

Equations (2), (5) \& (7) hold good.
(e) From the propellant 'all burnt' upto shot ejection, the equations are :

$$
\begin{gather*}
P[U+A x-N C \eta]=C N R T_{0} T^{1}\left[1+\frac{k C N}{6 W}\right]  \tag{13}\\
\frac{d N}{d t}=-\frac{\psi S_{t} P}{C\left(R T_{0}\right)^{1 / 2}}\left(T^{1}\right)^{-1 / 2}  \tag{14}\\
\frac{d\left(N T^{1}\right)}{d t}=(\bar{\gamma}-1) \frac{A P}{C R T_{0}} \frac{d x}{d t}-\frac{\gamma^{\prime} \psi S_{t} P\left(T^{1}\right)^{1 / 2}}{C\left(R T_{0}\right)^{1 / 2}} \tag{15}
\end{gather*}
$$

and equation (11)

$$
\left(W+\frac{1}{2} k C N\right) \frac{d V}{d t}=A P
$$

(f) After the shot ejection upto complete gas exhaustion the equations are :

$$
\begin{gather*}
P\left[U_{0}-N C \eta\right]=C N R T_{0} T^{1}  \tag{16}\\
\frac{d N}{d t}=-\frac{\psi\left(S_{t}+A\right) P}{C\left(R T_{0}\right)^{1 / 2}}\left(T^{1}\right)^{-1 / 2}  \tag{17}\\
\frac{d}{d t}\left(N T^{1}\right)=-\gamma^{\prime} \frac{\psi\left(S_{t}+A\right) P\left(T^{1}\right)^{1 / 2}}{C\left(R T_{0}\right)^{1 / 2}} \tag{18}
\end{gather*}
$$

RECOIL MOMENTUM AND RECOIL ENERGY OF,RCL GUNS
The thrust Co-efficient 1,6 of a nozzle is defined as :

$$
\xi=\frac{F}{P_{f} S_{t}}
$$

Where $P_{r}$ is the reservoir pressure taken as the maximum pressure in the gun at that instant. The recoil momentum can be calculated as follows :
Case A:-

$$
P_{N S}>P_{S S}
$$

Forward momentum from shot start to nozzle start :

$$
-A \int_{S S}^{N S} P d t=-A \int_{S S}^{N S}-\frac{D}{\beta} \frac{d f}{d t}=\frac{A D}{\beta}\left(f_{N S}-f_{S S}\right)
$$

From nozzte start to 'all burnt'

$$
=\left(\xi S_{t}-A\right) \int_{N S}^{\text {Burnt }} P d t=\left(\xi S_{t}-A\right) \int_{N S}^{\text {Bumt }}-\frac{D}{\beta} \frac{d f}{d t}
$$

from all burnt to shot ejection

$$
=\left(\xi S_{t}-A\right) \int_{\text {Burnt }}{ }^{E} P d t
$$

from shot ejection to complete gas exhaustion

$$
=\left(\xi S_{t}-A\right) \int_{E}^{\infty} P d t
$$

$\therefore$ Total forward momentum imparted to the gun

$$
=\frac{D}{\beta}\left[\xi S_{t} f_{N S}-A f_{S S}\right]+\left(\xi S_{t}-A\right) \int_{B u r n t}^{\infty} P d t
$$

Similarly for Case B : $\left(P_{N S}<P_{S S}\right)$ and Case C : $\left(P_{N S}=P_{S S}\right)$.
It can be shown that the expression for recoil momentum is the same. \{ Pdt from all burnt to gas exhaustion is determined by simple summation of $\boldsymbol{P} \times d t$.

## METHODOFSOLUTION

(a) From ignition to sho or nozzle start whiohever is earlier :-

Taking: $P=P_{S S}$ or $P_{\text {NSS }}$ (Lower of the two) the value of ' $f$ ' is calculated from equation (1) using the value of ' $\phi$ ' in terms of ' $f$ ' from (2). This value of ' $f$ ' gives $f_{s s}$ or $f_{N g}$. In the case $P_{N S}=P_{S S}, f_{S S}=f_{N S}$. At the end of this stage the ballistic parameters are $\nabla=0, x=0, P=P_{N S}$ or $P_{S S}, f=f_{N S}$ or $f_{S \varphi}$ and $t=0$.
(b) Case A: $\boldsymbol{P}_{N s}>P_{s s}$. From shot start to nozzle start, the equations (3), (4), \& (5) can be put as follows :

$$
\begin{aligned}
\frac{d x}{d f} & =-\frac{D V}{\beta P^{\alpha}} \\
\phi & =(1-f)(1+\theta f) \\
\frac{d V}{d f} & =-\frac{A D P^{1-\alpha}}{\beta(W+C / 3)}=\Psi_{w}(V, P) \\
\frac{d t}{d f} & =-\frac{D}{\beta P^{\alpha}}-\Psi_{v}(P) \\
P & =\frac{C \phi R T_{0}-\frac{1}{2}(\gamma-1)(W+C / 3) V^{2}}{u+A x-C / \delta-(\eta-1 / \delta) \cdot C \phi}=\Psi_{P}(\phi, V, x)
\end{aligned}
$$

for oonvenienoe the functions have boon dofinad as above. The Runge-Kutta steps for the solution of above equations are :
(i) Initial conditions : $x=x_{0}=0, V=V_{0}=0, t=t_{0}=0, \quad P=P_{S S}, f=f_{0}=f_{S S}$

$$
\begin{array}{ll}
k_{1}=\Psi_{x}\left(V_{0}, P\right)^{2} & V=V_{0}+m_{1} / 2 \\
l_{1}=\Psi_{t}(P) \cdot h & f=f_{0}-h / 2
\end{array}
$$

$$
\begin{array}{cr}
m_{1}=\Psi_{v}(P) \cdot h & \phi=(1-f)(1+\theta f) \\
x=x_{0}+k_{1} / 2 & P=\Psi_{P}(\phi, \nabla, f) \\
{[h=-d f \text { is the step length }] .}
\end{array}
$$

Using new values of $V$ and $P, l_{2}, l_{2}$ and $m_{2}$ are again determined. Similarly $k_{3}, l_{3}, m_{3}$ and $k_{6}, l_{4}$ and $m_{4}$ are calculated in the usual way of Runge-Kutta steps ${ }^{7}$ giving:

$$
\begin{aligned}
& x=x_{0}+\left(l_{1}+2 k_{2}+2 k_{3}+k_{4}\right) / 6 \\
& V=V_{0}+\left(m_{1}+2 m_{2}+2 m_{3}+m_{4}\right) / 6 \\
& t=t_{0}+\left(l_{1}+2 l_{2}+2 l_{3}+l_{4}\right) / 6
\end{aligned}
$$

The value of ballistic parameters after the first step, i.e, $f=f_{0}-h$ and $V, P, x$ and $t$ as calculated above are taken as the initial conditions for the second step. The process is repeated till $P=P_{N S}$. The parameters at $P=P_{N S}$ can be found by inter-polation.
(c) Case B:- $P_{N S}<P_{S S}$. From nozzle start, to shot start the equations (6), (7), (8) \& (9) can be put as follows :

$$
P=\frac{C N T_{0} T^{1}\left(1+\frac{k C N}{6 W}\right)}{U-C(1-\phi) / \delta-N C \eta}=\Psi_{P}\left(\phi, N, T^{1}\right) \text { say }
$$

$$
\begin{aligned}
& \frac{d(\zeta)}{d f}=\frac{\Psi S_{t} P^{1-\alpha} D\left(T^{1}\right)^{-1 / 2}}{\beta C\left(R T_{0}\right)^{1 / 2}}=\Psi_{\zeta}\left(P T^{1}\right) \text { say } \\
& \frac{d(\xi)}{d f}=\frac{\gamma \Psi S_{t} P\left(T^{1}\right)^{1 / 2} D}{\beta C\left(R T_{0}\right)^{1 / 2}}=\Psi_{\xi\left(P, T^{1}\right) \text { say }}=1
\end{aligned}
$$

$\gamma^{\prime}$ has been kept equal to $\gamma$ in view of Corner's comment ${ }^{1}$

$$
\phi=(1-f)(1+\theta f)
$$

Here the initial conditions are $P=P_{N S,} f=f_{N S}, N=\phi_{N S}, \xi=0, \zeta=0$ and $T^{1}=1$, the RungeKutta method is used to give the value of $\xi$ and $\zeta, N$ and $T^{1}$ as follows:

$$
\begin{aligned}
& k_{1}=\Psi_{\zeta}\left(P, T^{1}\right) \cdot h \\
& \left.l_{1}=\Psi_{\xi} T P, T^{1}\right) \cdot h \\
& f=f_{0}-h / 2 \\
& \phi=(1-f)(1+\theta f) \\
& \zeta=\zeta_{0}+k_{1} / 2 \\
& \xi=\xi_{0}+l_{1} / 2 \\
& N=\phi-\zeta \\
& T^{1}=(\xi+\phi) / N \\
& P=\Psi_{P}\left(P, T^{1}\right)
\end{aligned}
$$

The Runge-Kutta process is repeated to get $k_{2}, l_{2}, k_{3}, l_{3}$ and $k_{4}, l_{4}$ to give the final value of all these variables upto $P=P_{S S}$.
(d) This section deals with the solution upto the all burnt of the propellant. The equations (10), (11) \& (12) neglecting heat losses through barrel can be put as follows:

$$
\begin{aligned}
P & =\frac{C N R T_{0} T^{1}\left(1+\frac{k C N}{6 W}\right)}{\left[U+A x-\frac{C(1-\phi)}{\delta}-N C_{\eta}\right]}=\Psi_{P}\left(x, \phi, N, T^{1}\right) \\
\frac{d V}{d f} & =-\frac{A D P^{1}-\alpha}{\beta\left(W+\frac{1}{2} k C N\right)}-\Psi_{v}(P, N) \\
\phi & =(1-f)(1+\theta f) \\
\frac{d t}{d f} & =-\frac{\beta}{D} P-\alpha=\Psi ;(P)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d \zeta}{d f}=-\frac{\psi S_{t} P^{1-\alpha}\left(T^{1}\right)^{-1 / 2} D}{\beta C\left(R T_{0}\right)^{1 / 2}}=\Psi_{\zeta}\left(P, T^{1}\right) \\
& \frac{d \xi}{d f}=\frac{(\gamma-1) A P^{1-\alpha} D V}{\beta C R T_{0}}+\frac{\gamma \psi S_{t} P^{1-\alpha} D\left(T^{1}\right)^{1 / 2}}{\beta\left(R T_{0}\right)^{1 / 2}}=\Psi_{\zeta}\left(P, V, T^{1}\right) \\
& N=\phi-\zeta \\
& T^{1}=(\xi+\phi) / N \\
& \frac{d x}{d f}=-\frac{V D}{\beta P^{\alpha}}=\Psi(V, P)
\end{aligned}
$$

The last equation has been obtained by writing $d x / d t=V$ and then dividing by equation (5). It may be noted that in the above set of equations there are five differential equations all with ' $f$ ' as independant variable connected with four simple equations giving $P, \phi, N$ and $T^{1}$. The initial conditions are :
(i) Case A :-P $=P_{N S}, f=f_{N S}, V=V_{N S} t=t_{N S}$ (given at the end of section $5(b), T^{\mathbf{1}}=1$, $\phi=\phi_{N S}, N=\phi_{N S}$ and $\xi=\zeta=0$.
(ii) Case B :-P=P $P_{S S}, f=f_{S S}, V=0, \phi=\phi_{S S}, t=0, x=0, N, \xi, \zeta$ and $T^{1}$ are given at the end of section $5(a)$.
(iii) Case C :- when $P_{N S}=P_{S S}:-P=P_{N S}=P_{S S,}, f=f_{N S,}$ (Calculated) in section $5(a)$, $\phi=\phi_{N S}, t, x, V, \xi, \zeta=0 ; T^{1}=1$. The suffix ' ${ }_{0}$ ' is used for initial conditions.
The steps of Runge-Kutta method followed are as below:

$$
\begin{array}{rlrl}
k_{1} & =\Psi_{\xi}\left(P_{0}, V_{0}, T_{0}{ }^{1}\right)(-h) & \xi & =\xi_{0}+k_{1} / 2 \\
l_{1} & =\Psi_{\xi}\left(P_{0}, T_{0}{ }^{1}\right)(-h) & \zeta & =\zeta_{0}+l_{1} / 2 \\
m_{1} & =\Psi_{x}\left(V_{0}, P_{0}\right)(-h) & N & =\phi-\zeta \\
n_{1} & =\Psi_{v}\left(P_{0}, N_{0}\right)(-h) & T^{1} & =(\xi+\phi) / N \\
O_{1} & =\Psi_{i}\left(p_{0}\right)(-h) & x & =x_{0}+m_{1} / 2 \\
f & =f_{0}-h / 2 & V & =V_{0}+n_{1} / 2 \\
\phi & =(1-f)(1+\theta f) & P & =\Psi_{p}\left(x, \phi, N, T^{1}\right)
\end{array}
$$

In the same way $k_{2}, k_{3}, k_{4}, l_{2}, l_{3}, l_{4}, m_{2}, m_{3}, m_{4}, n_{2}, n_{3}, n_{4}$ and $O_{2}, O_{3}, O_{4}$ are determined giving :

$$
\begin{aligned}
& \xi=\xi_{0}+\left(k_{1}+2 k_{2}+2 k_{3}+k_{4}\right) / 6 \\
& \zeta=\zeta_{0}+\left(l_{1}+2 l_{2}+2 l_{3}+l_{4}\right) / 6 \\
& x=x_{0}+\left(m_{1}+2 m_{2}+2 m_{3}+m_{4}\right) / 6
\end{aligned}
$$

$$
V=V_{0}+\left(n_{1}+2 n_{2}+2 n_{3}+n_{4}\right) / 6
$$

$$
\zeta=\zeta_{0}+\left(l_{1}+2 l_{2}+2 l_{3}+l_{4}\right) / 6 \quad t=t_{0}+\left(O_{1}+2 O_{2}+2 O_{3}+O_{4}\right) / 6
$$

The values of $P, N, T^{1}$, and $\phi$ are also calculated from the above equations the values of all the nine variables at the end of 1st step, i.e., at $f=f-h$ and taken as the initial values for the second step and the process is repeated till $f=0$. If at any stage $(f-h)<0$, by interpolation the variables at $f=0$ are calculated.
(e) From 'all burnt' to shot ejection, the set of equation (13), (14) \& (15) can be written as follows :

$$
\begin{aligned}
P & =\frac{C N R T_{0} T^{1}\left(1+\frac{k C N}{6 W}\right)}{(U+A x-N C \eta)} \\
\frac{d t}{d N} & =-\frac{C\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2}}{\psi S_{t} P} \\
\frac{d \xi}{d N} & =\frac{(\gamma-1) A\left(T^{1}\right)^{1 / 2} V}{\psi S_{t}\left(R T_{0}\right)^{1 / 2}}+\gamma T^{1} \\
\frac{d V}{d N} & =\frac{A C\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2}}{\left(W+\frac{1}{2} k C N\right) \psi S_{t}} \\
\frac{d x}{d N} & =\frac{C\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2}}{\psi S_{t} P}
\end{aligned}
$$

Taking the values of $P, t, \xi, \nabla, x, N, T^{1}(\phi=1)$ from the end of last section ( $d$ ) the Runge-Kutta method is applied as before, $N$ being the independent variable in this case, the step length is taken as $d N=-h^{1}$.

This process is continued till $x=x_{3}$, i.e., the total shot travel. The values of parameters $p, t, V$, $x, N, T^{1}$ are calculated at $x=x_{3}$.
( $f$ ) From shot ejection upto complete gas exhaustion :
Writing

$$
\Delta=\frac{C}{U_{0}}
$$

Equations (16), (17) \& (18) can be written as

$$
\begin{gather*}
P=\frac{R T_{0} \Delta N T^{1}}{1-\Delta N \eta}  \tag{19}\\
\frac{d N}{d t}=-\frac{\psi}{C} \frac{\left(S_{t}+A\right) P}{\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2}}  \tag{20}\\
\frac{d}{d N}\left(N T^{1}\right)=\gamma^{1} T^{1} \tag{21}
\end{gather*}
$$

or

$$
N \frac{d T^{1}}{d N}=\left(\gamma^{1}-1\right) T^{1}
$$

giving

$$
\begin{aligned}
\frac{d T^{1}}{T^{1}} & =\left(\gamma^{1}-1\right) \frac{d N}{N} \\
& =(\gamma-1)\left[\frac{1}{N}+\frac{\triangle \eta}{1-N \triangle \eta}\right] d N
\end{aligned}
$$

integrating from shot exit.

$$
\left[\begin{array}{c}
T_{E}^{1} \\
T_{E} \\
\log T^{1}
\end{array}\right]=(\gamma-1)\left[(\log N)_{N_{E}}^{N}-\log \left(N_{N_{E}}^{N} 1-N \triangle \eta\right)\right]
$$

or

$$
\begin{equation*}
\frac{T^{\mathbf{1}}}{\overline{T_{E}}}=\left[\frac{N}{N_{E}} \cdot \frac{1-N_{E} \triangle \eta}{1-N_{\triangle \eta}}\right]^{\gamma-1} \tag{22}
\end{equation*}
$$

Putting values of $P$ and $T^{1}$ from equations (20) and (22), (21) can be written as :

$$
\frac{d t}{d N}=\frac{-C\left\{\frac{1}{N_{A}}-\triangle \eta\right\}^{\frac{1-\gamma}{2}}}{\psi\left(S_{t}+A\right)\left(R T_{0}\right)^{1 / 2}\left(T^{1} E\right)^{1 / 2} \triangle}\left[\frac{1}{N}-\Delta \eta\right]^{\frac{\gamma+1}{2}}
$$

giving ${ }^{2}$

$$
t=t_{E}-\frac{C\left\{\frac{1}{N_{R}}-\triangle \eta\right\}^{\frac{1-\gamma}{2}}}{\psi\left(S_{t}+A\right)\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2} \triangle} \int_{N_{E}}^{N}\left(\frac{1}{N}-\Delta \eta\right)^{\frac{\gamma+1}{2}} d N
$$

After the shot ejection the parameters of interest are $P, t$ (for calculating $\int P d t$ and time for complete gas exhaustion). Initial conditions are $P_{E}, N_{E}, T_{1}^{1}$ and $t_{E}$ (these values at the end of previous section).

The following functions are defined

$$
\left.\begin{array}{rl}
\Psi_{P}\left(N, T^{1}\right) & =\frac{R T_{0} \Delta N T^{1}}{1-\triangle N \eta} \\
\Psi_{T^{1}}(N) & =T^{1}\left[\frac{N}{N_{E}} \frac{1-N_{E} \triangle \eta}{1-N \triangle \eta}\right]^{\gamma-1} \\
\Psi_{t}(N) & =\frac{C\left\{\frac{1}{N_{E}}-\triangle \eta\right\}^{\frac{1-\gamma}{2}}}{\psi(S+A)\left(R T_{0}\right)^{1 / 2}\left(T^{1}\right)^{1 / 2}} \Delta
\end{array} \frac{1}{N_{E}}-\triangle \eta\right]^{\frac{\gamma-1}{2}},
$$

$$
\begin{aligned}
& N=N_{\mathbb{Z}}-h^{\prime} \\
& P=\Psi_{P}\left(N, T^{1}\right) \\
& T^{1}=\Psi_{T^{1}}(N) \\
& d t=\left[\Psi_{t}\left(N_{Z}\right)+4 \Psi_{t}\left(N_{\mathbb{Z}}-\frac{h^{\prime}}{2}\right)+\Psi_{t}\left(N_{E}-h^{\prime}\right)\right] \cdot\left(-\frac{h^{\prime}}{6}\right) \\
& N_{\mathbb{Z}}-h^{\prime} \\
& \int_{N_{Z}} P d t=P_{*} d t \quad \text { (from above) }
\end{aligned}
$$

This process is repeated till $N \simeq 0$ the value of $\int P d t$ is obtained by summation.

## FLOW OHART FOR THE COMPUTERPROGRAM

For calculating the complete internal ballistics upto gas exhaustion, the flow chart for the Computer Program areshown in Appendix "A". Appendix "B"' gives the Pressure/Velocity/Space curves calculated by this method for two guns.

MAIN PROGRAMME


Fig. 1-Flow chart for the eomputer programme.

## SUBROUTINE 1



Fig. 2-Ballistio parameters from nozzle start to start.

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## SUBROUTINE 2



Fig. 3 -Ballistic parameters from shọt start to nozzle start.

SUBROUTINE 3


Fig. 4-Ballistic parameters from shot and nozzle start upto all burnt or shot exit whichever is earlier.

SUBROUTINE 4


Fig. 5-Ballistic parameter from all burnt to shot exit.

SUBROUTINE 5


Fig. 6-Ballistic parameter from shot exit to complete gas exhaustion.

APPENDIX B


Fig. 7-Pressure/Velocity-Space curve for RCL gun 'A'. As fired bailistics : Max. Pressure $=4.1$ Ton $I^{-2}{ }^{-2}$, Muzzle Velocity $=1010$ Ft. Sec $^{-1}$.


Fig. 8--Pressure/Velocity-Space curve for RCL gun 'B'. As fired ballistios : Max. Pressure $=4.1$ Ton $\mathrm{In}^{-2}$ Muzzle Velocity $=1673 \mathrm{Ft}$. $\mathrm{Sec}^{-1}$,

