ESTABLISHMENT OF A SPRING DESIGN FORMULA FOR LOW TEMPERATURE USE OF SPRING

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Normally the springs are designed for room temperature conditions and the equations of elasticity are used to correlate the spring parameters. However, when the springs are required for use at low temperatures, these equations do not hold fairly good for design purposes. In this paper the establishment of new formula, that can be used for designing the springs for use at low temperature conditions has been discussed. Tests have been carried out at 20°C and —40°C to check the reliability of this formula. The formula has given satisfactory results.

NOTATIONS

W = Spring Load, lb.

δ = Deflection, in.

G = Shear modulus, psi.

D = Mean dia. of a spring, in.

d = Dia. of spring wire, in.

n = Number of active coils of spring.

K = Curvature correction factor.
$= \dfrac{(4C-1)}{(4C-4)} + \dfrac{(0.615)}{(C)}$

C = Spring index D/d.

G_{t_1} = Shear modulus at t_1 temperature, psi.

G_{t_2} = Shear modulus at t_2 temperature, psi.

t_1 = Room temperature (20°C).

t_2 = Subzero temperature (—40°C).

Z = A constant.

X = Correction factor, psi.
= $(Gt_2 - Gt_1)$, psi.

τ = Uncorrected shear stress, psi.

τ' = Corrected shear stress, psi.
= $\tau \times K$.

There is a wide application of springs in a number of equipments to act as an elastic link between two machine elements[1] or to act as an isolator[2]. These springs find their applications in stores like mechanical fuzes, small arms, switches, mountings, carriages, instruments, spring release mechanisms, many automobile parts, electronic appliances and aero engine parts. Under ordinary circumstances most of the springs are designed for room temperature conditions and the equations of theory for helical springs[3] are used to correlate the design parameters. The springs gave satisfactory performance at room temperature or slightly above room temperature conditions. However, these springs are put to use at low temperature, the machine parts including spring operated by them show considerable variations in their functioning and performance as compared to the performance at room temperature (20°C).

To investigate the variations in functioning and performance, the design parameters (W & G) were studied experimentally. The formula for designing a spring at room temperature conditions is given below:

$$\frac{W}{\delta} = \frac{Gd^4}{8D^3n} \tag{1}$$

From (1) it is seen that 'W' and 'δ' are the primary conditions of spring design, because 'W' is the force exerted over the spring and deflection 'δ' is constant to achieve a definite displacement of machine part operated by the spring. Since force W which is obtained from the machine is always kept constant at all temperature conditions, hence 'W' and 'δ' both are constants in equation (1). Diameter of spring wire 'd', number of active coils of the spring 'n' and mean spring diameter 'D' are dependent on the shear modulus 'G' of the spring material. This shear modulus of the spring material is obtained from the specifications by the manufacturer. Since the manufacturers determine the shear modulus at room temperature only, the value of shear modulus 'G' from the specification holds good for designing the spring for room temperature conditions using the formula given in (1). In view of the above, when the springs are required for use at sub-zero temperatures the value of 'G' in (1) must be known at sub-zero temperature. Thus it indicates that equation (1) can not be used for designing springs required to function satisfactorily at low temperature. A new formula has been established by carrying out the study at normal temperature (20°C) and (—40°C).

EXPERIMENTAL PROCEDURES

The schematic diagram of the experimental arrangements is shown in Fig. 1. For minimising the friction between mating surfaces, powdered graphite was used as lubricant and dimensions of test rig were selected with sufficient care. The test rig consists of a body which houses the spring rod over which the test specimen of spring is assembled. The spring rests between the flange of spring rod and top of the lower cavity of the body. The neck of the spring rod and the sleeve are held together by means of two steel balls. The head of the sleeve is provided with an eye in which the neck of a precision spring balance is engaged for pulling the sleeve towards the top edge of the body.

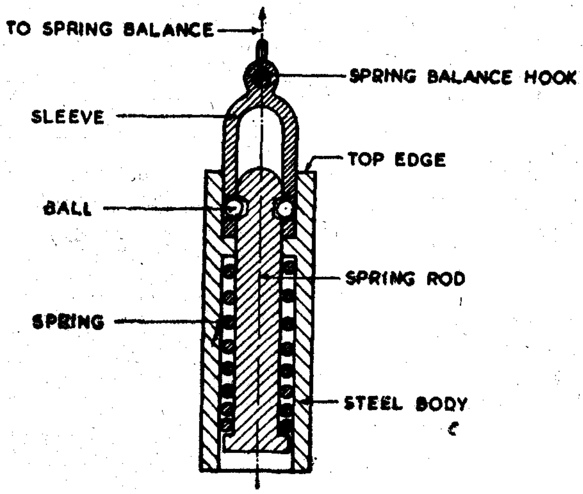

Fig. 1—Test set-up.

When the spring is pulled towards the top edge of the body the spring rod travels up and thus deflects (free length of the spring diminishes) the spring under test. As soon as the steel balls reach the top edge of the body, they roll out of the spring rod neck. This releases the rod and the spring reasserts itself and goes to its original position. From the inspection of the figure 1 it is clear that deflection 'δ' of the spring is always constant for one type of spring because the position of the top edge is fixed in the body i.e., 'δ' is maintained constant for one type of specimen. Four different test rigs, one for each type of spring specimens were used.

Ten springs of each types A, B, C and D, whose chemical compositions are given in Table 1, were selected for the test. Their relevant design parameters viz.-wire dia 'd', mean spring dia 'D' number of active coils 'n' and deflection 'δ' (from the measurement of body top edge and the ball diameter) were recorded. These springs were then assembled in their respective test rigs. The test rigs, one at a time were placed in a chamber at 20°C for a period of 72 hours. Each rig was fastened to a vice inside the chamber. From outside the chamber and with the help of a precision spring balance the sleeve of the rig was pulled

Table 1

Chemical composition and condition of spring

Spring Type	Composition (%)							Condition
	C	*Si*	*Mn*	*Mo*	*S*	*P*	*Fe*	
A	0·93	0·37	0·35	Nil	0·04	0·04	98·27	The spring wire was coated with a thick layer of copper. The spring was hardened and tempered after coiling.
B	0·95	0·38	0·40	0·01	0·03	0·03	97·9	The spring wire was coated with cadmium, hardened and tempered after coiling.
C	0·85	0·30	0·75	Nil	0·04	0·04	98·02	The spring wire was coated with tin and the spring was made from hard drawn steel wire.
D	0·80	0·37	0·46	Nil	0·04	0·04	98·10	The spring was coated with cadmium and made from cold drawn wire.

TABLE 2

VALUES OF SPRING PARAMETERS

Spring Type	Parameters					
	n	d (inch)	D (inch)	F (inch)	W at 20°C (lb)	W at —40°C (lb)
					23·0	24·0
					22·0	24·0
					22·5	24·0
					22·8	24·2
A	9	0·042	0·20	0·35	21·5	23·5
					24·0	25·0
					23·5	24·8
					24·0	25·0
					23·6	25·5
					23·0	24·2
					36·0	37·2
					36·5	37·5
					35·0	37·1
					34·8	37·0
B	7	0·061	0·307	0·364	34·9	37·0
					35·0	37·5
					35·2	37·6
					35·6	37·9
					35·8	38·9
					36·8	39·5
					7·0	8·05
					7·3	8·00
					7·5	9·00
					6·8	8·60
C	4	0·048	0·403	0·30	7·0	9·00
					8·0	9·50
					7·5	9·00
					8·2	9·50
					6·9	8·05
					8·0	9·60
					0·4400	0·4500
					0·4410	0·4500
					0·4410	0·4510
					0·4400	0·4500
D	6	0·042	0·683	0·275	0·4410	0·4512
					0·4410	0·4513
					0·4415	0·4520
					0·4405	0·4521
					0·4400	0·4502
					0·4700	0·4750

out of the body and the load required to disengage the spring rod from the sleeve. The test was carried out with 10 specimens of each type of the springs A, B, C and D. The spring was kept at one temperature for 72 hours to represent the time which was normally required for commissioning and truing a machine tool equipment, before starting the actual work and also to have full effect of temperature on the spring material.

The similar experiment tests were repeated at -40 ± 1 °C by lowering the temperature of the conditioning chamber. The conditioning chamber was working as a hot oven as well as a refrigerator using R 13 and R 22 coolants.

Details of spring parameters of types A, B, C and D and the recorded loads of ten samples of each type of springs are given in Table 2.

CALCULATION OF STRESS AND PLOT OF GRAPH

Shear stresses for different types of spring specimens were calculated using the formula[4] $\tau' = \dfrac{8\,W\,D\,K}{\pi\, d^3}$ and the two values of 'W' at 20°C and —40°C) from Table 2. Graph showing variations of shear stress with load were plotted. These are given at Figs 2 & 3, 4 & 5, 6 & 7 and 8 & 9 illustrate these variations for the spring types A, B, C and D respectively.

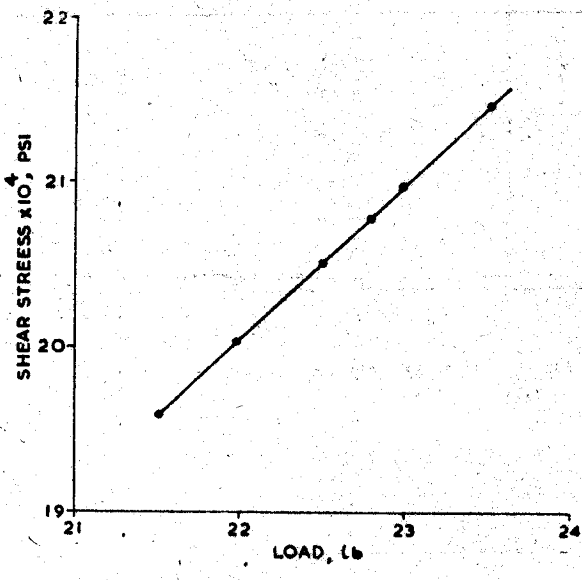

Fig. 2—Variation of shear stress with load at 20°C

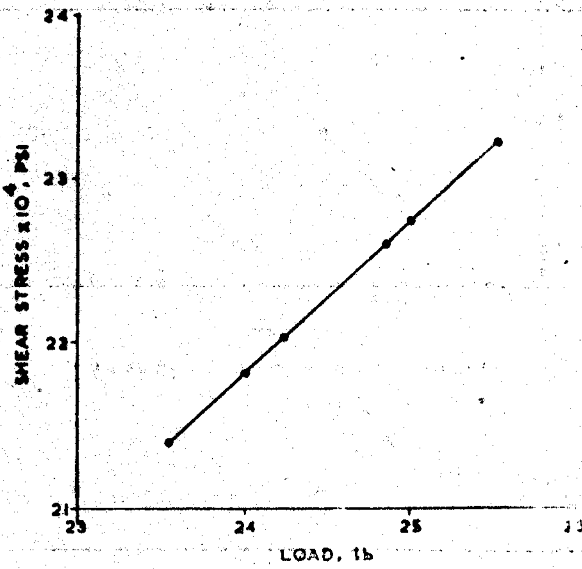

Fig. 3—Variation of shear stress with load at—40°C

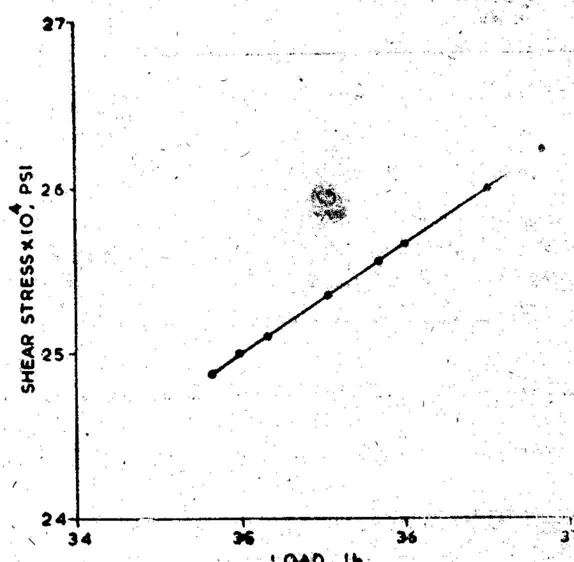

Fig. 4—Variation of shear stress with load at 20° C

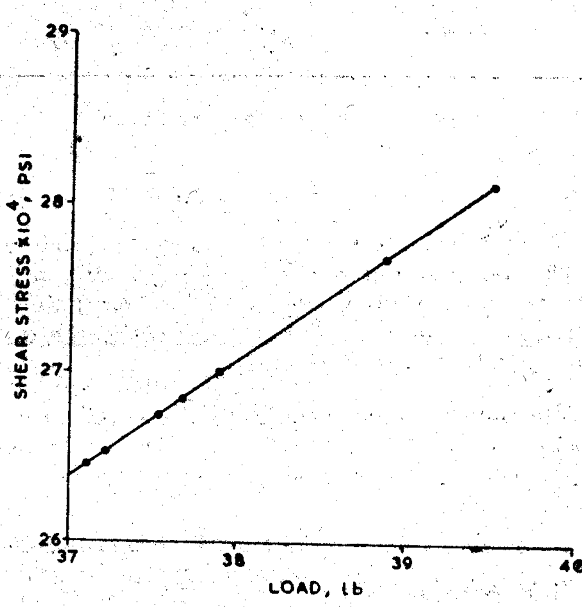

Fig. 5—Variation of shear stress with load at—40° C

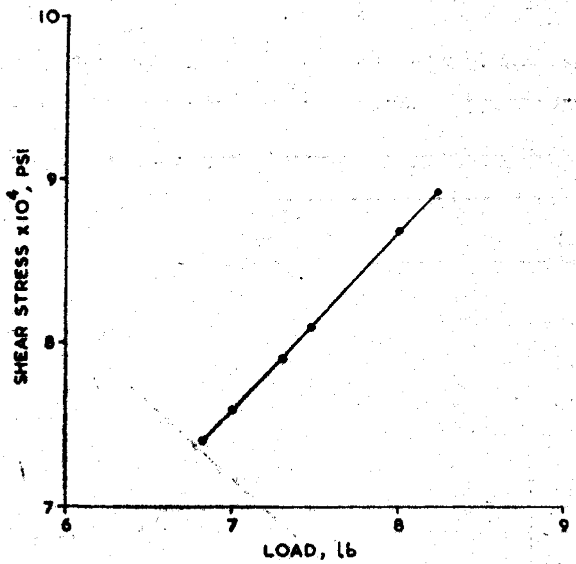

Fig. 6—Variation of shear stress with load at 20° C

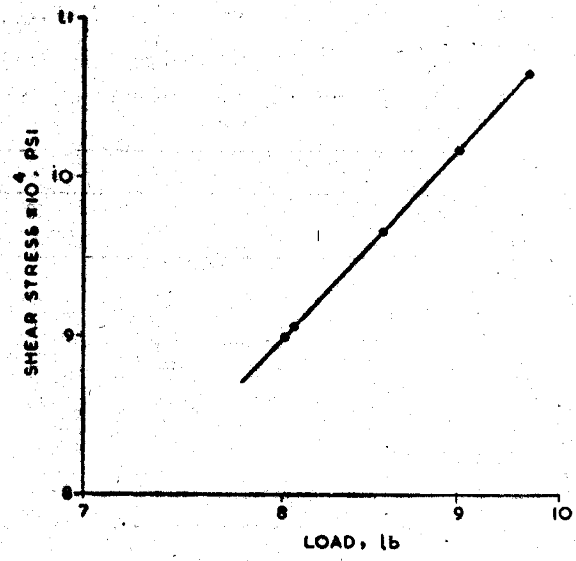

Fig. 7—Variation of shear stress with load at—40° C

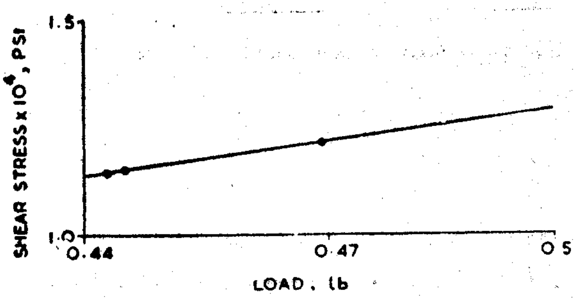

Fig.8 —Variation of shear stress with load at 20°C

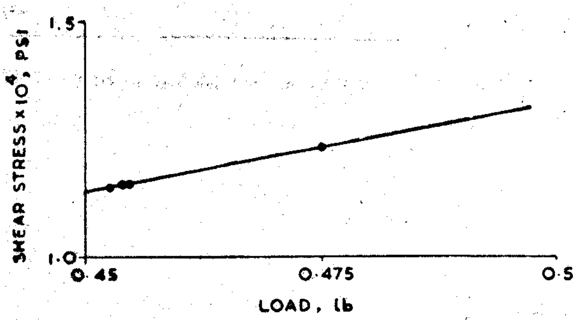

Fig. 9—Variation of shear stress with load at—40°C

COMPUTATION OF SHEAR MODULUS

To determine the shear modulus of the spring material at 20°C and — 40°C the formula given at (1) and the data of spring load 'W' recorded at 20°C and — 40°C were utilised. Since the major effect of temperature was on shear modulus 'G', the computed value of shear modulus using the data from Table 2 is justified, because the shear modulus is the ratio of shear stress to shear strain [5] and in this case the strain automatically remained constant.

The values of shear modulus 'G' obtained by computation using the data from Table 2 and equation 1 are given in Table 3.

RESULTS AND DISCUSSION

It is seen from Table 2 that the load required to deflect the spring at — 40°C is considerably more than that required at normal temperature (20°C). The increase in operational load on the spring increases the induced stress and in many cases puts the machine, out of order [6] by increased loading, as the machines are normally designed and manufactured at normal temperature conditions. It can be

TABLE 3

COMPUTED VALUE OF SHEAR MODULUS ,G'

Spring A				Spring B			
At 20°C		At —40°C		At 20°C		At —40°C	
W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)
23·0	12·00	24·0	12·60	36·0	11·50	37·2	12·00
22·0	11·50	24·0	12·60	36·5	11·70	37·5	12·10
22·5	11·80	24·0	12·60	35·0	11·30	37·1	12·00
22·8	11·90	24·2	12·60	34·8	11·10	37·0	12·00
21·5	11·25	23·5	12·15	34·9	11·20	37·0	12·00
24·0	12·55	25·0	13·00	35·0	11·30	37·5	12·10
23·5	12·30	24·8	12·90	35·2	11·35	37·6	12·10
24·0	12·55	25·0	13·00	35·6	11·50	37·9	12·20
23·6	12·35	25·5	13·25	35·8	11·60	38·9	12·50
23·0	12·00	22·2	12·00	36·8	11·80	39·5	12·80

Spring C				Spring D			
At 20°C		At —40°C		At 20°C		At —40°C	
W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)	W (lb)	G×10⁶ (psi)
7·0	9·00	8·05	10·47	0·4400	7·81	0·4500	7·90
7·3	9·60	8·00	10·50	0·4410	7·86	0·4500	7·90
7·5	9·85	9·00	11·75	0·4410	7·86	0·4510	8·10
6·8	8·95	8·60	11·30	0·4400	7·81	0·4500	7·90
7·0	9·20	9·00	11·74	0·4410	7·65	0·4512	8·20
8·0	10·50	9·50	12·40	0·4410	7·85	0·4513	8·20
7·5	9·85	9·00	11·74	0·4415	7·87	0·4500	8·30
8·2	10·80	9·50	12·40	0·4405	7·84	0·4521	8·30
6·9	9·05	8·05	12·47	0·4400	7·82	0·4502	8·00
8·0	10·50	9·60	12·60	0·4400	8·42	0·4502	8·60

seen from the principles of heat treatment of alloys that when an alloy is heated to a suitable temperature and then quenched in a liquid, the structure of the alloy is changed to a definite pattern and the alloy becomes harder[7]. Similarly as the spring material, from its raw form to the finished form, undergoes so many hot and cold operations, the stresses during the process of manufacture cause dislocations[8] and set the atoms of the material in a particular array at room temperature. When the machines designed to work at room temperature conditions are put to work in sub-zero temperature (— 40°C in this case), a definite cooling of the machine elements takes place. Although the sub-zero temperature may not be as high as the difference between heating temperatures and water temperature during the heat treatment but the sub-zero temperature is supported by the duration of constant low temperature and thus helps the material to undergo full effect of cooling. Due to this cooling the material again undergoes rearrangements of atoms, formation of new dislocations and changes in its structure of harder nature[9]. On account of hardening of the material, the magnitude of applied load 'W' has to be increased to get the same deflection as obtained at 20°C.

It is observed from the graphs illustrated in Fig. (2) to (9) that the variations of stress with load at 20°C and — 40°C show the same trend. The stress increases with the increase in the spring load and generally follows a linear relationship. However, it is seen that the magnitudes of the stress as well as load for — 40°C are more than those for 20°C. This is due to cooling effects on the spring materials caused by the combined effects of time duration and low temperature [10].

It is seen from Table 3 that the value of shear modulus at low temperature (— 40°C), is more than that obtained at room temperature (20°C)[11]. This is explained with the help of (1) as follows :

$$\frac{W}{\delta} = \frac{Gd^4}{8D^3n}$$

$$\text{Or} \qquad W = \frac{GD^4\,\delta}{8D^3n} \tag{2}$$

As the value of the 'd' and 'D'do not change due to change in temperature and the values of 'δ' and 'n' are known, the equation 2 can be written as $W = Z\,G$

Where Z is a constant

$$\text{or} \quad W \propto G \tag{3}$$

From (3) it is evident that any change in spring load "W" is directly proportional to shear modulus 'G'. This proves that the increased load recorded at low temperature for the spring was due to increase in shear modulus as a result of structural change of the material. Due to this increase in shear modulus at low temperature many machine elements like rifle bolts, pistons and other dynamic parts of an equipment fail due to higher induced stresses.

This suggests that the formula given at equation (1) cannot be utilised for efficient and accurate functioning and performance of machines, machine elements and the equipments, unless the value of shear modulus 'G' obtained at the low temperature, is substituted in that equation.

Alternatively a correction factor which is obtained below may be incorporated in the formula and the equation thus obtained can be used to correlate the spring parameters at the temperature.

The shear modulus at temperatures, t_1 is Gt_1
and at t_2 is Gt_2

Therefore $(G_{t2} - G_{t1}) = X$, is the difference in the two values (at — 40°C and 20°C) of the shear modulus 'G'. From Table 3, it is seen that 'X' is a positive number, hence it is proved that the value of 'G' in equation (1) which is substituted from the specifications of the spring material, for designing a spring for use at low temperature, must be reduced by 'X', so that during the actual service life of the spring at sub-zero temperature, the increase in the value of shear modulus compensates it and the machine gives the same performance as that at room temperature. Now the corrected formula for low temperature use of springs therefore becomes :

$$\frac{W}{\delta} = (G - X) \quad \frac{d^4}{8\,D^3\,n} \tag{4}$$

The dimensions of correction factor 'X' are the same as that of 'G' (psi), because 'X' is only the difference of two values of shear modulus 'G'.

The values of 'X' obtained by the present tests for springs types A, B, C and D are given in Table 4.

TABLE 4

VALUE OF CORRECTION FACTOR 'X'

Type of Spring	Value of 'X' (psi)
A	$0{\cdot}80 \times 10^6$
B	$0{\cdot}75 \times 10^6$
C	$1{\cdot}98 \times 10^6$
D	$0{\cdot}20 \times 10^6$

CONCLUSION

The formula given at equation (1) does not fit into the experimental data, whereas the equation (4) fit more appropriately which can be useful in the design and development of springs for machines, equipments, instruments, weapons, ammunition and miscellaneous mechanisation stores. These are required to be operated at sub-zero temperature and the value of correction factor 'X' determined at different low temperatures is to be used in equation (4).

Also the result of this experiment is a good guide for future research and developments of springs for use in the fields of Mechanical Engineering design and steel industry for developing mechanisms and material for use at sub-zero environment.

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