UNSTEADY FLOW OF A DUSTY VISCOUS LIQUID IN A CHANNEL BOUNDED BY TWO PARALLEL FLAT PLATES

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This paper describes the unsteady motion of a dusty viscous liquid in a channel bounded by two parallel flat plates. The differential equations describing the problem are given. The liquid and the dust velocity profiles are obtained exactly and discussed in terms of the parameters of the problem.

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Work in this field has been carried out by Saffman[1], Michael[2], Rao[3], Reddy[4], Verma and Mathur[5]. Situations which occur frequently are concerned with the motion of a liquid or gas which contains a distribution of dust particles. In the present paper a dusty viscous liquid has been considered in a channel bounded by two parallel flat plates. Initially the liquid and the dust particles are at rest. At $t = 0$, a constant pressure gradient is impressed on the system. The change in the velocity profiles with time is determined and shown in Fig. 1 and 2. It is interesting to note that the velocity of dust particles is plausible only when $t > 2$. This, of course, is due to the inertia of the dust particles.

EQUATIONS OF MOTIONS

The equations of motion of a dusty viscous liquid are

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\frac{1}{\rho}\,\text{grad}\,p + \nu\nabla^2\vec{u} + \frac{KN}{\rho}(\vec{v} - \vec{u}), \tag{1}$$

$$m\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}\right] = K(\vec{u} - \vec{v}), \tag{2}$$

$$\text{div}\,\vec{u} = 0, \tag{3}$$

$$\frac{\partial N}{\partial t} + \text{div}(N\vec{v}) = 0, \tag{4}$$

where $\vec{u}, \vec{v}$ are the velocity vectors of liquid and dust particles respectively, p the pressure, m the mass of a dust particle, N the number density of the dust particles, K the Stokes resistance coefficient which for spherical particles of radius E is $6\pi\mu E$, μ the viscosity of the liquid, ρ the density and $\nu\left(=\frac{\mu}{\rho}\right)$ the kinematic coefficient of viscosity.

FORMULATION OF THE PROBLEM

For the present problem the velocity distributions of liquid and dust particles are defined respectively as :

$$u'_x = u'_x(y', t'), \quad u'_y = 0, \quad u'_z = 0; \tag{5}$$

$$v'_x = v'_x(y', t'), \quad v'_y = 0, \quad v'_z = 0, \tag{6}$$

where (u'_x, u'_y, u'_z) and (v'_x, v'_y, v'_z) are velocity components of the liquid and dust particles.

Further, for $N = N_\circ$ (a constant), the equation (4) is satisfied throughout the flow field.

The equations of motion then reduce to

$$\frac{\partial u'_x}{\partial t'} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + \nu\frac{\partial^2 u'_x}{\partial y'^2} + \frac{KN_\circ}{\rho}(v'_x - u'_x), \tag{7}$$

$$m \frac{\partial v'_x}{\partial t'} = K\,(u'_x - v'_x) \tag{8}$$

We now introduce the following non-dimensional quantities :

$$y = \frac{y'}{l'},\quad x = \frac{x'}{l'},\ \bar{p} = \frac{p' l'^2}{\rho \nu^2},\ t = \frac{\nu t'}{l'^2},\ u = \frac{u'_x\, l'}{\nu},\ v = \frac{v'_x\, l'}{\nu},$$

$$l = \frac{m\, N_0}{\rho},\ \sigma = \frac{m\nu}{K\, l'^2}.$$

Equations (7) and (8) then become

$$\frac{\partial u}{\partial t} = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}\,(v - u)\,, \tag{9}$$

$$\sigma \frac{\partial v}{\partial t} = (u - v) \tag{10}$$

Eliminating v from (9) and (10) we get

$$\sigma \frac{\partial^2 u}{\partial t^2} + (l + 1)\frac{\partial u}{\partial t} - \sigma \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial^2 u}{\partial y^2} \tag{11}$$

The applied pressure gradient, say C, is a constant, for $t > 0$, so that $-\frac{\partial \bar{p}}{\partial x} = C$ (for $t > 0$). The equation (11) then becomes

$$\sigma \frac{\partial^2 u}{\partial t^2} + (l + 1)\frac{\partial u}{\partial t} - \sigma \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) = C + \frac{\partial^2 u}{\partial y^2}\,, \tag{12}$$

Let $\bar{u} = \int_0^{\infty} e^{-st}\, u\, dt$ be the Laplace transform of u and let $u_0 = 0$ be the initial value of u.

Multiplying equation (12) by e^{-st} and then integrating between the limits 0 to ∞, we get

$$\frac{\partial^2 \bar{u}}{\partial y^2} - p^2 \bar{u} = -\frac{C}{s\,(1 + \sigma s)}\,, \tag{13}$$

where

$$p^2 = \frac{s\,(l + 1) + \sigma s^2}{1 + \sigma s}$$

The boundary conditions for $\bar{u}$ are

$$\bar{u} = 0 \quad \text{when} \quad y = -1\,,$$

and

$$\bar{u} = 0 \quad \text{when} \quad y = 1\,.$$

The solution of equation (13) under the above boundary conditions is

$$\bar{u} = \frac{C}{s^2\,[(l + 1) + \sigma s]}\left[1 - \frac{\cosh py}{\cosh p}\right] \tag{14}$$

Now applying Laplace inversion theorem, we get

$$u = \frac{C}{2}(1 - y^2) + \frac{4c}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n \exp(-\alpha_1 t)\,.\,\cos\left[\frac{(2n+1)\pi y}{2}\right]}{(2n+1)\,\alpha_1\left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}}$$

$$-\frac{4c}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n \exp(-\alpha_2 t)\,.\,\cos\left[\frac{(2n+1)\pi y}{2}\right]}{(2n+1)\,\alpha_2\left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}}\,, \tag{15}$$

where

$$2\,\sigma\,\alpha_1 = (l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma + \left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}$$

and

$$2\,\sigma\,\alpha_2 = (l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma - \left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}$$

Now using Laplace transform method it is easy to show that

$$v = \frac{C}{2}(1-y^2) + \frac{4C}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n.\exp(-\alpha_1 t).\cos\left[\frac{(2n+1)\pi y}{2}\right]}{(1-\sigma\alpha_1)(2n+1)\alpha_1\left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}}$$

$$-\frac{4C}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n.\exp(-\alpha_2 t).\cos\left[\frac{(2n+1)\pi y}{2}\right]}{(1-\sigma\alpha_2)(2n+1)\alpha_2\left[\left\{(l+1) + \frac{(2n+1)^2\pi^2}{4}\sigma\right\}^2 - (2n+1)^2\pi^2\sigma\right]^{\frac{1}{2}}}$$

NUMERICAL DISCUSSION

The velocity profiles for the liquid and the dust particles are shown in Fig. 1 and 2 respectively. From these figures, the following points are observed :

(*i*) The velocity of dust particles is plausible only when $t > 2$. Clearly, this is due to the inertia of the dust particles.

(*ii*) The velocity of liquid particles at any point in the flow field is greater than the velocity of dust particles at that point. It means that the liquid particles will reach the steady state earlier than the dust particles.

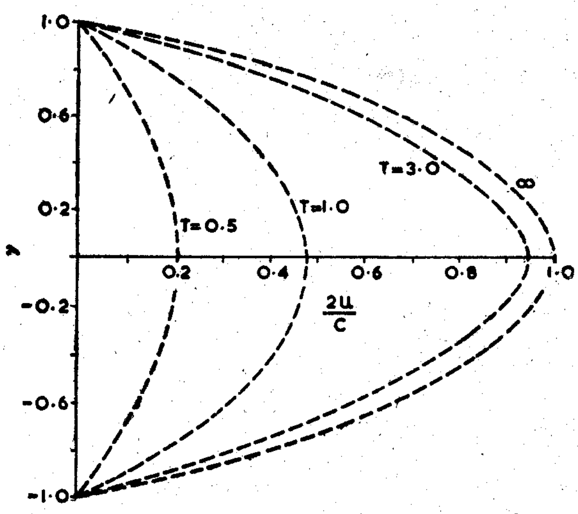

Fig. 1—Velocity profiles for liquid at different times ($l = 0{\cdot}2, \sigma = 0{\cdot}8$)

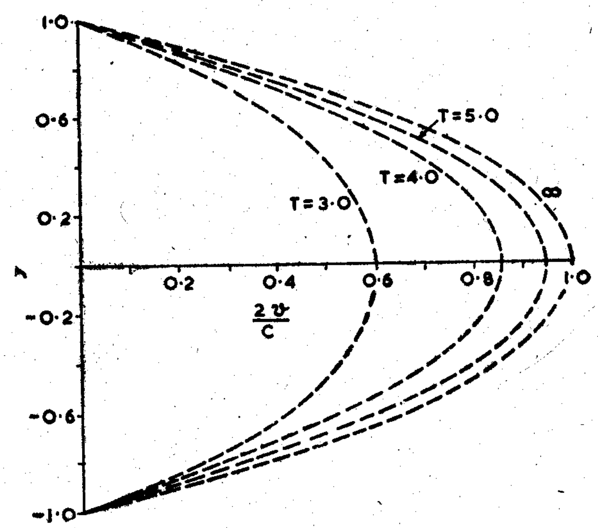

Fig. 2—Velocity profiles for dust particles at different times ($l = 0{\cdot}2, \sigma = 0{\cdot}8$)

(*iii*) The velocities of the liquid and the dust particles become the same when $\sigma \to 0$, *i.e.*, when the dust particles become very fine.

(*iv*) The dusty liquid's velocity will take more time to reach the steady state than the clean liquid's velocity. It means that the presence of dust in the liquid reduces the flow field.

(*v*) It is also clear that u and v increase with t for fixed y and for any t, u and v decrease with the increase of y and are maximum when $y = 0$. It means that both the liquid and the dust particles which are nearer to the axis of the channel move with greater velocity.

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