ON INTERNAL BALLISTICS OF H/L GUN DURING BURNING WITH COMPOSITE CHARGES

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An attempt has been made to solve the internal ballistics of H/L gun with composite charge having two components assuring constant pressures in both the chambers during the second stage of burning i.e. after the burning of the first component charge.

The internal ballistics of high-low pressure gun was discussed by Kapur[1], Aggarwal[2] and Corner[3,4] for the general form function. Recently author[5] had discussed the attainment of constant pressures in both the chambers in an H/L gun with moderated charges during the second stage of burning. In the present paper, the author tries to solve the internal ballistics of H/L gun with composite charges having two components, Pressures in both the first and second chamber in the H/L gun have been assumed to remain constant during the second stage of burning. The constant pressures being equal to the pressures at burnt during the first stage. The internal ballistics during the first stage of burning are assumed to be known and those for the second stage of burning are determined. The conditions for the attainment of constant pressures determine two relations between the four characteristics of the second propellant component of which two may be known from the physical properties of the propellant so that the other two may be calculated.

FIRST STAGE OF BURNING

The ballistic equations in the non-isothermal model during the first stage of burning in an H/L gun are as follows:

The equation of state for the gases in the first and second chamber are.

$$P_1 \left[U_1 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} + \frac{C_1 \phi_1}{\delta_1} + \frac{C_2 \phi_2}{\delta_2} - (C_1 + C_2) N \eta \right] = (C_1 + C_2) N R T_1 \tag{1}$$

and

$$P_2 [U_2 + Ax - (C_1 \phi_1 + C_2 \phi_2) \eta + (C_1 + C_2) N \eta] = [C_1 \phi_1 + C_2 \phi_2 - (C_1 + C_2) N] R T_2 \tag{2}$$

The equation of continuity (when $\omega < \omega^*$) is

$$(C_1 + C_2) \frac{dN}{dt} = C_1 \frac{d\phi_1}{dt} + C_2 \frac{d\phi_2}{dt} - \frac{\Psi S P_1}{\sqrt{RT_1}} \tag{3}$$

Where $\Psi = \left(\frac{2\gamma}{\gamma - 1}\right)^{\gamma_2}$ and N is the fraction of the total charge turned into gas

We take the law of burning and the form function as

$$D_i \frac{df_i}{dt} = \beta_i P_1 \tag{4}$$

and

$$\phi_i = (1 - f_i)(1 + \theta_i f_i) \quad (i = 1, 2) \tag{5}$$

The equations of energy for the first and second chamber are

$$\frac{d}{dt}\left[(C_1 + C_2) N T_1 \right] = T_0 \left(C_1 \frac{d\phi_1}{dt} + C_2 \frac{d\phi_2}{dt} \right) - \gamma T_1 \frac{d}{dt}\left[C_1 \phi_1 + C_2 \phi_2 - (C_1 + C_2) N \right] \tag{6}$$

and

$$\frac{d}{dt}\left[\left\{ C_1 \phi_1 + C_2 \phi_2 - (C_1 + C_2) N \right\} T_2 \right] = \gamma T_1 \frac{d}{dt}\left[C_1 \phi_1 + C_2 \phi_2 - (C_1 + C_2) N \right] \tag{7}$$

The equations of motion of the shot

$$W \frac{dv}{dt} = A\, P_2 \tag{8}$$

These equations are solved with initial conditions

$$P_1 = P_2 = T_1 = T_2 = N = 0 = v \quad \text{when} \quad f_1 = f_2 = 1 \text{ i.e. } \phi_1 = \phi_2 = 0$$

We suppose that this solution gives

$$P_1 = P_{1B1},\ P_2 = P_{2B1},\ T_1 = T_{1B1},\ T_2 = T_{2B1},\ N = N_{B1},\ v = v_{B1} \text{ and } x = x_{B1}$$

When $f_1 = 0$ i.e. when the first component is completely burnt.

From (4) the value of f_2 at burnt of the first component is,

$$f_{2B1} = 1 - \frac{D_1/\beta_1}{D_2/\beta_2} = 1 - \frac{1}{\alpha_0}$$

where,

$$\alpha_0 = \frac{D_2/\beta_2}{D_1/\beta_1} \tag{9}$$

SECOND STAGE OF BURNING

The equation of state for the gases in the first and second chamber are

$$P_1 \left[U_1 - \frac{C_2}{\delta_2} + \frac{C_2}{\delta_2}\, \phi_2 - (C_1 + C_2)\, N\, \eta \right] = (C_1 + C_2) N\, R\, T_1 \tag{10}$$

and

$$P_2\, [U_2 + Ax - (C_1 + C_2\, \phi_2)\, \eta + (C_1 + C_2)\, N\eta] = [C_1 + C_2\, \phi_2 - (C_1 + C_2)\, N]\, RT_2 \tag{11}$$

The equation of continuity (when $\omega < \omega^*$) is

$$(C_1 + C_2)\, \frac{dN}{dt} = C_2\, \frac{d\phi_2}{dt} - \frac{\psi S P_1}{\sqrt{R T_1}} \tag{12}$$

Further, we have the equation of burning

$$D_2\, \frac{df_2}{dt} = -\, \beta_2\, P_1 \tag{13}$$

The form function

$$\phi_2 = (1 - f_2)\, (1 + \theta_2 f_2) \tag{14}$$

and the equation of energy for the first and second chamber

$$\frac{d}{dt} \left[(C_1 + C_2)\, N T_1 \right] = T_0\, C_2\, \frac{d\phi_2}{dt} - \gamma\, T_1\, \frac{d}{dt} \left[C_2\, \phi_2 - (C_1 + C_2)\, N \right] \tag{15}$$

and

$$\frac{d}{dt} \left[\left\{ C_1 + C_2\, \phi_2 - (C_1 + C_2)\, N \right\} T_2 \right] = \gamma\, T_1\, \frac{d}{dt} \left[C_2\, \phi_2 - (C_1 + C_2)\, N \right] \tag{16}$$

The equation of motion

$$w\, \frac{dv}{dt} = AP_2 \tag{17}$$

Let us assume that the solutions of the above equations are possible with

$$P_1 = P_{1B1}\ ,\quad P_2 = P_{2B1} \tag{18}$$

and seek conditions so that this solution may give

$$x = x_{B1}\ ,\ v = v_{B1},\ N = N_{B1}\ ,\ T_1 = T_{1B1}$$

and

$$T_2 = T_{2B1}\ ,\ \text{at } f_2 = f_{2B1}$$

and the systems of (10) to (17) may remain consistent for the solutions $P_1 = P_{1B1}$ and $P_2 = P_{2B1}$. with (18), (13) and (17)

we have
$$\frac{dv}{df_2} = -\frac{A}{\omega}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}} \tag{19}$$

which by (13) and (18) reduces to
$$\frac{d^2x}{df_2^2} = \frac{AD_2^2}{\beta_2^2\,\omega\,P_{1B1}}\cdot\frac{P_{2B1}}{P_{1B1}} \tag{20}$$

Integrating (19) with the condition $v = v_{B1}$ at $f_2 = f_{2B1}$

we have
$$v = v_{B1} + \frac{A}{W}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}}\,(f_{2B1} - f_2) \tag{21}$$

now we impose the condition that (10) and (11) with (18) gives,

$$X = X_{B1},\ T_1 = T_{1B1},\ T_2 = T_{2B1}$$

and $N = N_{B1}$ at $f_2 = f_{2B1}$ and further (10) and (11) are consistent with (20).

Now $x = x_{B1}$, $T_2 = T_{2B1}$ $N = N_{B1}$ and $\phi_2 = \phi_{2B1}$ will satisfy (11) if

$$P_{2B1}\,[U_2 + Ax_{B1} - (C_1 + C_2\phi_{2B1})\,\eta + (C_1 + C_2)\,N_{B1}\,\eta] = [C_1 + C_2\,\phi_{2B1} - (C_1 + C_2)\,N_{B1}].\ R\,T_{2B1} \tag{22}$$

which is true, for (22) is obtained from (2) by considering values when the first component burns out.

Also $N = N_{B1}$, $T_1 = T_{1B1}$ and $\phi_2 = \phi_{2B1}$ will sastify (10) if

$$P_{1B1}\left[U_1 - \frac{C_2}{\delta_2} + \frac{C_2}{\delta_2}\,\phi_{2B1} - (C_1 + C_2)\,N_{B1}\,\eta\right] = (C_1 + C_2)\,N_{B1}\,R\,T_{1B1} \tag{23}$$

which is true, for (23) is obtained from (1) by considering values when the first component burns out.

With the help of (13), equations (12), (15) and (16) can be written as,

$$\left(C_1 + C_2\right)\frac{dN}{df_2} = C_2\,\frac{d\phi_2}{df_2} + \frac{D_2}{\beta_2}\,\frac{\psi S}{\sqrt{RT_1}} \tag{24}$$

$$\frac{d}{df_2}\left[(C_1 + C_2)\,NT_1\right] = T_0\,C_2\,\frac{d\phi_2}{df_2} - \gamma\,T_1\,\frac{d}{df_2}\left[C_2\,\phi_2 - (C_1 + C_2)\,N\right] \tag{25}$$

and
$$\frac{d}{df_2}\left[\left\{C_1 + C_2\,\phi_2 - (C_1 + C_2)\,N\right\}T_2\right] = \gamma\,T_1\,\frac{d}{df_2}\left[C_2\,\phi_2 - (C_1 + C_2)\,N\right] \tag{26}$$

On integrating (6), (7), (25) and (26), we get

$$\{C_1 + C_2\,\phi_2 - (C_1 + C_2)\,N\}\,T_2 + (C_1 + C_2)\,N\,T_1 = T_0\,(C_1 + C_2\,\phi_2) \tag{27}$$

Adding (10) and (11) to (18) and (27), we get

$$P_{1B1}\left[U_1 - \frac{C_2}{\delta_2} + \frac{C_2}{\delta_2}\phi_2 - (C_1 + C_2)\,N\,\eta\right] + P_{2B1}\left[U_1 + Ax - (C_1 + C_2\phi_2)\eta + (C_1 + C_2)N\,\eta\right] = $$
$$= R\,T_0\,(C_1 + C_2\,\phi_2) \tag{28}$$

On differentiating (28) and (14) with respect to f_2, with the help of (13), (18) and (24), we get

$$\frac{-AP_{2B1}\,v\,D_2}{\beta_2\,P_{1B1}} + \eta\,\frac{P_{2B1}\,\psi\,S\,D_2}{\beta_2\sqrt{RT_1}} + \frac{C_2}{\delta_2}\,P_{1B1}\left\{(1 - f_2)\,\theta_2 - (1 + \theta_2 f_2)\right\} -$$
$$-\eta\,P_{1B1}\left\{C_2\,(\theta_2 - 1 - 2\theta_2 f_2) + \frac{\psi\,S\,D_2}{\beta_2\sqrt{RT_1}}\right\} = RT_0\,C_2\,(\theta_2 - 1 - 2\theta_2 f_2) \tag{29}$$

Now $v = v_{B1}$, $T_1 = T_{1B1}$ and $f_2 = f_{2B1}$ will satisfy (29) if

$$-Av_{B1}\frac{D_2}{\beta_2}\frac{P_{2B1}}{P_{1B1}} + (1-\theta_2+2\theta_2 f_{2B1}).$$

$$\left[RT_0\,C_2 + C_2\,P_{1B1}\left(\eta - \frac{1}{\delta_2}\right)\right] = \frac{D_2}{\beta_2}\;\frac{\psi\eta S}{\sqrt{RT_{1B1}}}\;(P_{1B1} - P_{2\;\;1}) \tag{30}$$

Introducing the following dimensionless constants

$$\left.\begin{aligned}
&\frac{C_2}{C_1} = \beta_0, \quad \frac{D_2/\beta_2}{D_1/\beta_1} = \alpha_0, \; \Psi = \frac{\psi S D_1}{\beta_1 C_1\, RT_0} \\
&\eta_{B1} = \frac{v_{B1}/AD_1}{C_1\,\beta_1\,RT_0}, \quad \frac{P_{2B1}}{P_{1B1}} = \omega_{B1}, \; \frac{T_{1B1}}{T_0} = T_0', \\
&\eta\;\frac{P_{1B1}}{RT_0} = \nu_0, \quad \left(\eta - \frac{1}{\delta_2}\right)\frac{P_{1B1}}{RT_0} = \delta_0
\end{aligned}\right\} \tag{31}$$

we have

$$1-\theta_2+2\,\theta_2 f_{2B1} = \frac{\alpha_0}{\beta_0}\cdot\frac{\gamma_0\,\Psi\,(1-\omega_{B1})/\sqrt{T_0'} + \eta_{B1}\,\omega_{B1}}{1+\delta_0} \tag{32}$$

To satisfy that (20) and (28) should be consistent, we differentiate (28) twice w.r.t. f_2 and with the help of the equations (14), (23) and (20), we get

$$\frac{A^2 D_2^2 P_2{}^2{}_{B1}}{\beta_2^2\,\omega P_{1B1}{}^2} - \frac{\eta\,P_{2B1}\,D_2\,\psi\,S}{2\beta_2\sqrt{R}\,T_1{}^{3/2}}\;\frac{dT_1}{df_2} - 2\theta_2\;\frac{C_2}{\delta_2}\;P_{1B1} -$$

$$-\eta\,P_{1B1}\left\{-2\,\theta_2\,C_2 - \frac{D_2\,\psi\,S}{2\beta_2\sqrt{R_1}\;T_1{}^{3/2}}\;\frac{dT_1}{df_2}\right\} = -2\,\theta_2\,R\,T_0\,C_2$$

or

$$-2\,\theta_2\,C_2\left[R\,T_0 + \left(\eta - \frac{1}{\delta_2}\right)P_{1B1}\right] = \frac{A^2 D_2^2 P_{2B1}{}^2}{\beta_2^2\,\omega\,P_1{}^2{}_{B1}} + \frac{\eta D_2\psi S}{2\beta_2\sqrt{R}\,T_1{}^{3/2}}\,(P_{1B1} - P_{2B1})\,\frac{dT_1}{df_2} \tag{33}$$

Again from (23) and (24), we have

$$\frac{dT_1}{df_2} = \frac{1}{(C_1+C_2)\,N}\left[C_2\,(T_0 - T_4)\,(\theta_2 - 1 - 2\theta_2 f_2) + \frac{D_2}{\beta_2}\,\frac{\psi\,S\,(\gamma-1)}{\sqrt{R}}\,(T_1)^{\frac{1}{2}}\right. \tag{34}$$

Since $T_1 = T_{1B1}$, $N = N_{B1}$ and $f_2 = f_{2B1}$ will satisfy (33) and (34), we get

$$-2\theta_2\,C_2\left[R\,T_0 + \left(\eta - \frac{1}{\delta_2}\right)P_{1B1}\right] = \frac{A^2\,D_2^2\,P_2{}^2{}_{B1}}{\beta_2^2\,\omega\,P_{1B1}{}^2} +$$

$$+\frac{\eta\,D_2\,\psi\,S\,(P_{1B1} - P_{2B1})}{2\,\beta_2\sqrt{R}\;T_{1B1}{}^{3/2}(C_1+C_2)N_{B1}}\left[\frac{D_2}{\beta_2}\;\frac{\psi\,S\,(r-1)}{\sqrt{R}}\;(T_{B1})^{\frac{1}{2}} -\right.$$

$$\left. - C_2\,(T_0 - T_{1B1}).\;(1-\theta_2+2\theta_2 f_{2B1})\right] \tag{35}$$

We introduce the central ballistic parameter $M = \dfrac{A^2 D_1^2}{\beta_1^2\,\omega\,C_1\,RT_0}$

From (31), the equation (35) can be written in the non-dimensional form as;

$$-2\theta_2(1+\delta_0) = M_1 \frac{2\alpha_0^2\omega_{B1}^2}{\beta_0} + \frac{\gamma_0 \Psi\alpha_0(1-\omega_{B1})}{2(1+\beta_0)\sqrt{T_0'}\,N_B)} \cdot \left[(1-\theta_2+2\theta_2 f_{2B1})\left(1-\frac{1}{T_0'}\right)+\frac{\Psi\alpha_0(\gamma-1)}{\beta_0\sqrt{T_0'}}\right] \tag{36}$$

The simultaneous satisfaction of (32) and (36) gives the condition that $P_1 = P_{1B1}$ and $P_2 = P_{2B1}$ may be the solution of (10) to (17).

The equations(32) and (36) connect four parameters α_0, β_0, θ_2 and δ_0 defining the second propellant component.

The properties and mass of the first component have been assumed to be known. δ_0 involves η and $1/\delta_2$ and β_0 involves C_2 which are supposed to be known in the integration of equation for the first stage of burning.

Thus (32) and (36) give the size and shape of the second propellant component provided α_0 is positive and θ_2 satisfies $-1 < \theta_2 \leqslant 1$ for practical values of constants involved.

If $\eta = \frac{1}{\delta_2} \approx 0$ then, $\gamma_0 = \delta_0 = 0$, the (32) and (36) reduce to

$$1-\theta_2+2\theta_2 f_{2B1} = \frac{\alpha_0}{\beta_0}\eta_{B1}\,\omega_{B1}$$

and

$$-2\theta_2 = M_1 \frac{\alpha_0^2\omega_{B1}^2}{\beta_0}$$

Let $M_2 = \frac{A^2 D_2^2}{\beta_2^2\,\omega\, C_2 RT_0} = \frac{M_1\alpha_0^2}{\beta_0}$ be the central ballistic parameter corresponding for the second component charge.

Then

$$\theta_2 = -\frac{M_2}{2}\omega_{B1}^2$$

Thus within the practical range of values of M_2 and ω_{B1}, θ_2 is negative and greater than (-1) and also α_0 may be positive.

Now from (21)

$$v_{B2} = v_{B1} + \frac{A}{\omega}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}}\,f_{2B1} \tag{37}$$

Then

$$\frac{v_{B2}}{v_{B1}} = 1 + \frac{M_1\alpha_0}{\eta_{B1}}\,\omega_{B1} f_{2B1}$$

which is the velocity ratio and its value is less than the velocity ratio for the moderated charges.

From (20), (13) and (18), we get

$$-\frac{dx}{df_2}\,\frac{\beta_2 P_{1B1}}{D_2} = v_{B1} + \frac{A}{\omega}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}}\,(f_{2B1}-f_2) \tag{38}$$

On integrating (38) taking $x = x_{B1}$ at $f_2 = f_{2B1}$

$$\frac{\beta_2 P_{1B1}}{D_2}(x-x_{B1}) = v_{B1}(f_{2B1}-f_2) + \frac{AD_2 P_{2B1}}{2w\beta_2 P_{1B1}}(f_{2B1}-f_2)^2 \tag{39}$$

Let $x = x_{B2}$ when the second component burns out i.e. $f_2 = 0$.

Then, we get

$$x_{B2} - x_{B1} = \frac{D_2}{\beta_2} \frac{v_{B1}}{P_{1B1}} f_{2B1} + \frac{A D_2^2 P_{2B1}}{2w \beta_2^2 P_{1B1}^2} f_{2B1}^2 \tag{40}$$

From (31), we get

$$\frac{x_{B2} - x_{B1}}{x_{B1}} = \frac{\eta C_1}{\gamma_0 A x_{B1}} \left[\alpha_0 \eta_{B1} f_{2B1} + \frac{M_1 \alpha_0^2}{2} \omega_{B1} f_{2B1}^2 \right] \tag{41}$$

Let us introduce another dimensionless quantity

$$\xi_{B1} = \frac{\eta C_1}{A x_{B1}} .$$

Hence the travel ratio is

$$= \frac{\xi_{B1} f_{2B1}}{\gamma_0} \left[\alpha_0 \eta_{B1} + \frac{M_1 \alpha_0^2}{2} \omega_{B1} f_{2B1} \right] \tag{42}$$

which is also less than the ratio corresponding to the moderated charges.

Differentiating (10) w.r.t. f_2 and using (18), (24), (25) and (31), we get

$$(\theta_2 - 1 - 2\theta_2 f_2)(1 + \delta_0) = - \Psi \frac{\alpha_0}{\beta_0} \left[\frac{\gamma_0}{\sqrt{T'}} + \gamma \sqrt{T'} \right] \tag{43}$$

where $T' = \dfrac{T_1}{T_0}$. Thus (43) determines T′ as a function of f_2.

Now (24) can be writtten as,

$$\left(1 + \frac{1}{\beta_0}\right) \frac{dN}{df_2} = \frac{d\phi_2}{df_2} + \Psi \frac{\alpha_0}{\beta_0} \frac{1}{\sqrt{T'}} \tag{44}$$

Putting the value of T' from (43) and integrating (44), we get N as a functions of f_2

$$N = \frac{\beta_0}{1 + \beta_0} (1 - f_2)(1 + \theta_2 f_2) + \frac{\Psi \alpha_0}{1 + \beta_0} \int \frac{1}{\sqrt{T'}} df_2 + B \tag{45}$$

Where B is determined by the condition $N = N_{B1}$ and $f_2 = f_{2B1}$

As T_1 and N are determined from (43) and (45), T_2 is determined as a function of f_2 from the relation (27).

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