LIFE TESTS WITH PERIODIC CHANGE IN FAILURE RATE

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In this paper two life testing procedures viz., the progressively censored samples and Bartholomew's experiment have been discussed under the assumption that the failure rate of an item is constant, though different under the two different conditions of usage at regular intervals. The estimates of the two failure rates have been derived alongwith their asymptotic variances for both types of data, i.e., when the failure times are recorded and when only the numbers of items failing in each interval are recorded. Numerical examples have been worked out to illustrate the type of data and relevant calculations.

Aroian[1] and Srivastava[2,3] have studied life test experiments, where the failure rate of an item changes, in steps, with time. The failure rate of an item may change due to change in conditions of usage. They have assumed the life distribution to be exponential. Gajjar & Khatri[4] have discussed life test experiments where the life distributions assumed are Log Normal and Logistic. They assumed that the life distribution parameters undergo change at specified times. However, we can visualise situations where an item will be used only under two different conditions of usage one after the other in a cycle.

In this paper it has been assumed that the lite of an item follows an exponential distribution. Thus its failure rate is constant, though it changes periodically under the two conditions of usage at regular intervals of time, i.e. if the failure rate at the beginning of the experiment (at time $t = 0$) is λ_1, it changes to λ_2, after time T_1 when the usage condition changes. However, after another interval of time T_2 when the usage condition changes back to previous one, the failure rate again changes to λ_1 and so on. Thus in each cycle of duration $T_1 + T_2$ the failure rate is λ_1 in the first part of duration T_1 and λ_2 in the second part of duration T_2.

Two life test experiments namely the progressively censored samples envisaged by Cohen[5] and Bartholomew's[6] experiment have been considered. The data is assumed to be available in either of the following forms:

1. The failure times of items are recorded.
2. Only the number of items that fail in each part of a cycle is recorded.

MODEL

The probability density function of the random variable t representing the life of an item, having a negative exponential distribution with a single parameter λ, is given by

$$f(t\,;\lambda) = \lambda e^{-\lambda t}\;;\quad \lambda > 0\,, t > 0\,. \tag{1}$$

In life testing situations the parameter λ represents the failure rate of an item while $1/\lambda$ represents the mean life of an item.

For the situations under consideration, the probability density function of t can be written as

$$f(t) = \begin{cases} a_{1,j} f_1(t)\,; (j-1)\,(T_1+T_2) < t \leqslant (j-1)\,(T_1+T_2) + T_1 \\ a_{2,j} f_2(t)\,; (j-1)\,(T_1+T_2) + T_1 < t \leqslant j\,(T_1+T_2) \end{cases} \tag{2}$$

where

$$a_{1,j} = e^{(j-1)\,(\lambda_1-\lambda_2)\,T_2}$$
$$a_{2,j} = e^{-j\,(\lambda_1-\lambda_2)\,T_1}$$
$$f_i(t) = \lambda_i\, e^{-\lambda_i t}\quad;\quad \lambda > 0,\; t > 0,\; i = 1, 2\,.$$

and j (= 1, 2, ...) represents the jth cycle.

The corresponding distribution function of t is given by

$$1 - F(t) = \begin{cases} a_{1,j}\,[1 - F_1(t)] & ; \quad (j-1)\,(T_1+T_2) < t \leqslant (j-1)\,(T_1+T_2) + T_1 \\ a_{2,j}\,[1 - F_2(t)] & ; \quad (j-1)\,(T_1+T_2) + T_1 < t \leqslant j\,(T_1+T_2) \end{cases} \tag{3}$$

where

$$F_i(t) = \int_0^t f_i(t)\,dt \qquad i = 1, 2\,,$$

PROGRESSIVELY CENSORED SAMPLES

Life testing experiments involving progressively censored samples were envisaged by Cohen[5]. In such an experiment certain known number of items are placed on test at start. However, because of the need for them at another place some of these are removed from the experiment at some predetermined times. Such type of censoring has been referred to as Type I by Cohen[5].

Here, we shall derive the maximum likelihood estimates of the failure rates λ_1 and λ_2 alongwith their asymptotic variances for the above experimental situation.

Let n items be placed on test.

In the jth cycle let,

n_{1j} be the number of items that failed by time T_1,

n_{2j} be the number of items that failed between times T_1 and $T_1 + T_2$,

r_{1j} be the number of items removed after time T_1,

r_{2j} be the number of items removed after time $T_1 + T_2$,

$n_1 = \sum_{j=1}^{k} n_{1j}$, and $n_2 = \sum_{j=1}^{k} n_{2j}$.

Let the experiment be finally terminated after kth cycle.

(*i*) *Failure Times Known*

The likelihood function of the sample arising as a result of the above experiment is given by

$$P\,(S) = \prod_{j=1}^{k} \prod_{i=1}^{2} \left[\left\{ \prod_{p=1}^{n_{ij}} f\,(t^{j}_{pi}) \right\} (a_{i,j})^{r_{ij}} \left\{ 1 - F_i\,(\overline{j-1}\;\overline{T_1+T_2} + T_1 + \overline{i-1}\;T_2) \right\}^{r_{ij}} \right]$$

$$= \prod_{j=1}^{k} \left[\left\{ \lambda_1^{n_{1j}} \exp\left(n_{1j}\,(j-1)\,(\lambda_1 - \lambda_2)\,T_2 - \lambda_1 \sum_{p=1}^{n_{1j}} t^{j}_{p1} \right) \right\} \cdot \right.$$

$$\cdot \left\{ \lambda_2^{n_{2j}} \exp\left(-j\,n_{2j}(\lambda_1 - \lambda_2)\,T_1 - \lambda_2 \sum_{p=2}^{n_{2j}} t^{j}_{p2} \right) \right\} \cdot$$

$$\left. \cdot \left\{ \exp\left(-r_{1j}\,\lambda_1 T_1 + (j-1)\,(\lambda_1 T_1 + \lambda_2 T_2) - j\,r_{2j}\,(\lambda_1 T_1 + \lambda_2 T_2) \right) \right\} \right]$$

where t^{j}_{pi} are the times at which the pth item ($p = 1, 2, \ldots\ldots\ldots\ldots, n_{ij}$) fail in the ith part ($i = 1, 2.$) of the jth cycle ($j = 1, 2, \ldots\ldots\ldots\ldots\ldots, k$).

It may be noted that $\prod_{p=1}^{n_{1j}}$ and $\sum_{p=1}^{n_{1j}}$ refer to product and summation taken over n_{1j} items that failed in the first part of the jth cycle while $\prod_{p=1}^{n_{2j}}$ and $\sum_{p=1}^{n_{2j}}$ refer to product and summation taken over n_{2j} items that failed in the second part of the jth cycle.

Usual method leads to the following maximum likelihood estimates of λ_1 and λ_2,

$$\hat{\lambda}_1 = \frac{\sum_{j=1}^{k} n_{1j}}{\sum_{j=1}^{k} \left[\sum_{p=1}^{n_{1j}} t^{j}_{p1} + j\,(n_{2j} + r_{1j} + r_{2j})\,T_1 - (j-1)\,n_{1j}\,T_2 \right]}$$

$$\hat{\lambda}_2 = \frac{\sum_{j=1}^{k} n_{2j}}{\sum_{j=1}^{k} \left[\sum_{p=1}^{n_{2j}} t^{j}_{p2} + T_2\,(j-1)\,(n_{1j} + r_{1j}) + j\,r_{2j} - j\,n_{2j}\,T_1 \right]}$$

To derive the asymptotic variances of $\hat{\lambda}_1$ and $\hat{\lambda}_2$, we note that

$$E\left\{\frac{\partial^2 \log P(S)}{\partial \lambda_1 \, \partial \lambda_2}\right\} = 0 .$$

Hence the asymptotic variances of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are given as

$$V(\hat{\lambda}_1) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_1^2}\right\}\right]^{-1} = \frac{\lambda_1^2}{\sum_{j=1}^{k} E(n_{1j})}$$

$$V(\hat{\lambda}_2) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_2^2}\right\}\right]^{-1} = \frac{\lambda_2^2}{\sum_{j=1}^{k} E(n_{2j})}$$

where

$$E(n_{11}) = n\,(1 - e^{-\lambda_1 T_1})$$

$$E(n_{21}) = \left\{n \,.\, \exp(-\lambda_1 T_1) - r_{11}\right\}\left(1 - \exp(-\lambda_2 T_2)\right)$$

$$E(n_{1j}) = \left[n - \sum_{m=1}^{j-1}\left\{r_{1m} + r_{2m} + E(n_{1m} + n_{2m})\right\}\right] .$$

$$\cdot \frac{a_{2,(j-1)}\left[1 - F_2\left\{(j-1)(T_1+T_2)\right\}\right] - a_{1,j}\left[1 - F_1\left\{(j-1)(T_1+T_2)+T_1\right\}\right]}{a_{2,(j-1)}\left[1 - F_2\left\{(j-1)(T_1+T_2)\right\}\right]}$$

$$= \exp\left(-(j-1)(\lambda_1 T_1 + \lambda_2 T_2)\right)\left[n - \sum_{m=1}^{j-1} \exp\left(m(\lambda_1 T_1 + \lambda_2 T_2)\right) .\right.$$

$$\left. \cdot \left(r_{1m} \exp(-\lambda_2 T_2) + r_{2m}\right)\right]\left(1 - \exp(-\lambda_1 T_1)\right) \; ; \quad (j = 2, 3, \ldots\ldots\ldots, k) .$$

$$E(n_{2j}) = \left[n - \sum_{m=1}^{j-1}\left\{r_{1m} + r_{2m} + E(n_{1m} + n_{2m})\right\} - r_{1j} - E(n_{1j})\right] .$$

$$\cdot \frac{a_{1,j}\left[1 - F_1\left\{(j-1)(T_1+T_2)+T_1\right\}\right] - a_{2,j}\left[1 - F_2\left\{j\,(T_1+T_2)\right\}\right]}{a_{1,j}\left[1 - F_1\left\{(j-1)(T_1+T_2)+T_1\right\}\right]}$$

$$= \exp\left(-(j-1)(\lambda_1 T_1 + \lambda_2 T_2)\right)\left[n \,.\, \exp(-\lambda_1 T_1) - \sum_{m=1}^{j-1} \exp\left(m(\lambda_1 T_1 + \lambda_2 T_2)\right) .\right.$$

$$\cdot \left\{r_{1m} \,.\, \exp\left(-(\lambda_1 T_1 + \lambda_2 T_2)\right) + r_{2m} \,.\, \exp\left(-\lambda_1 T_1\right)\right\} - r_{1j} \,.\, \exp\left((j-1)(\lambda_1 T_1 +\right.$$

$$\left.\left. + \lambda_2 T_2)\right)\right]\left(1 - \exp(-\lambda_2 T_2)\right) \; ; \qquad (j = 2, 3, \ldots\ldots\ldots, k) .$$

(ii) Failure Times Unknown

In such a situation the likelihood function of the sample is given by

$$P(S) = \prod_{j=1}^{k} \prod_{i=1}^{2} \left(P_{ij}\right)^{n_{ij}} \left(a_{i,j}\right)^{r_{ij}} \left[1 - F_i\left\{(j-1)(T_1+T_2) + T_1 + (i-1)\,T_2\right\}\right]^{r_{ij}}$$

where P_{ij} is the probability for an item to fail in the ith part of the jth cycle and is given by

$$P_{ij} = \exp.\left\{-(j-1)(\lambda_1 T_1 + \lambda_2 T_2) - (i-1)\,\lambda_1 T_1\right\}\left(1 - \exp(-\lambda_i T_i)\right)$$

Thus

$$P(S) = \prod_{j=1}^{k} \left[\exp\left\{-(j-1)(\lambda_1 T_1 + \lambda_2 T_2)(n_{1j} + r_{1j} + n_{2j} + r_{2j})\right\} \left\{\exp\left(-(r_{1j} + r_{2j} + n_{2j})\,\lambda_1 T_1 - r_{2j}\,\lambda_2 T_2\right)\left(1 - \exp(-\lambda_1 T_1)\right)^{n_{1j}}\left(1 - \exp(-\lambda_2 T_2)\right)^{n_{2j}}\right\}\right]$$

Following the usual procedure we obtain the maximum likelihood estimates and their asymptotic variances as follows :

$$\hat{\lambda}_1 = \frac{1}{T_1}\log\left[\frac{\sum_{j=1}^{k} j\,(n_{1j} + n_{2j} + r_{1j} + r_{2j})}{\sum_{j=1}^{k} j\,(n_{1j} + n_{2j} + r_{1j} + r_{2j}) - n_1}\right],$$

$$\hat{\lambda}_2 = \frac{1}{T_2}\log\left[\frac{\sum_{j=1}^{k} (j-1)(n_{1j} + n_{2j} + r_{1j} + r_{2j}) + n_2 + r_2}{\sum_{j=1}^{k} (j-1)(n_{1j} + n_{2j} + r_{1j} + r_{2j}) + r_2}\right],$$

$$V(\hat{\lambda}_1) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_1^2}\right\}\right]^{-1} = \frac{\left(1 - \exp(-\lambda_1 T_1)\right)^2}{T_1^2\left\{\sum_{j=1}^{k} E(n_{1j})\right\}\exp(-\lambda_1 T_1)}$$

$$V(\hat{\lambda}_2) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_2^2}\right\}\right]^{-1} = \frac{\left(1 - \exp(-\lambda_2 T_2)\right)^2}{T_2^2\left\{\sum_{j=1}^{k} E(n_{2j})\right\}\exp(-\lambda_2 T_2)}$$

where $E(n_{1j})$ and $E(n_{2j})$ are the same as in section (i).

NUMERICAL EXAMPLE

Table 1 gives data pertaining to the situation involving progressively censored samples. Each cycle is of 24 hour duration—the first part lasting 16 hours and the second part 8 hours.

NUMERICAL CALCULATIONS

For data given in Table I

$$n_1 = \sum_{j=1}^{8} n_{1j} = 397\,; \qquad \sum_{j=1}^{8} r_{1j} = 80\,;$$

$$n_2 = \sum_{j=1}^{8} n_{2j} = 343\,; \qquad \sum_{j=1}^{8} r_{2j} = 180\,;$$

$$E(n_1) = \sum_{j=1}^{8} E(n_{1j}) = 401{\cdot}3\,;$$

TABLE 1

FREQUENCY DISTRIBUTION OF LIVES OF VACUUM TUBES
(Total number of tubes placed on test=1000)

Time (hrs)	Cycle no.	First 16 hrs of cycle: No. of tubes failed (n_{1j})	First 16 hrs of cycle: No. of tubes removed (r_{1j})	Last 8 hrs of cycle: No. of tubes failed (n_{2j})	Last 8 hrs of cycle: No. of tubes removed (r_{2j})
0	1	92	10	82	5
24	2	68	10	70	5
48	3	60	10	55	5
72	4	53	10	46	5
96	5	40	10	35	5
120	6	38	10	25	5
144	7	30	10	18	5
168	8	16	10	12	145

$$E(n_2) = \sum_{j=1}^{8} E(n_{2j}) = 341 \cdot 8 .$$

Failure Times Known

$$\sum_{j=1}^{8} \sum_{p=1}^{n_{1j}} t_{p_1}^{\,j} = 26778 ; \qquad \sum_{j=1}^{8} \sum_{p=1}^{n_{2j}} t_{p_2}^{\,j} = 25304 .$$

Appropriate calculations outlined in section (i) lead to the following results:

$$\hat{\lambda}_1 = 0 \cdot 00627 ; \qquad \hat{\lambda}_2 = 0 \cdot 01226$$

$$V(\hat{\lambda}_1) = 0 \cdot 98 \times 10^{-7} ; \qquad V(\hat{\lambda}_2) = 4 \cdot 4 \times 10^{-7} .$$

Failure Times Unknown

For such situation the appropriate calculations outlined in section (ii) lead to the following results:

$$\hat{\lambda}_1 = 0 \cdot 00626 ; \qquad \hat{\lambda}_2 = 0 \cdot 01220$$

$$V(\hat{\lambda}_1) = 0 \cdot 978 \times 10^{-7} ; \qquad V(\hat{\lambda}_2) = 4 \cdot 355 \times 10^{-7} .$$

BARTHOLOMEW'S EXPERIMENT

Bartholomew[6] envisaged a life testing experiment in which all the items are placed on test at different times depending on their availability. A generalised form of such an experiment could be as follows.

Let a sample of N_m items be placed on test after time $(j-1)(T_1+T_2)$ elapses from the start of the experiment ($j = 2, 3, \ldots\ldots\ldots\ldots, k$), and the experiment be terminated after time $k(T_1+T_2)$ elapses from the start of the experiment.

It may be noted that in its general form Bartholomew's experiment could be termed as "progressively added samples" corresponding to "progressively censored samples" introduced by Cohen.

In this section we shall derive the maximum likelihood estimates of the failure rates λ_1 and λ_2 alongwith their asymptotic variances for the above experimental situation.

Let n be the number of items that fail during the experiment and r the number of items that survive. Among the r items that survive, let r_m be from the mth sample of N_m items ($m = 1, 2, \ldots\ldots\ldots, k$). Further let,

n_{1j} be the number of items that fail in the first part of the jth cycle, i.e. between $(j-1)(T_1+T_2)$ to $(j-1)(T_1+T_2)+T_1$,

n_{2j} be the number of items that fail in the second part of the jth cycle, i.e. between $(j-1)(T_1+T_2)+T_1$ to $j(T_1+T_2)$,

n_{1jm} be the number of items among the n_{1j} items from the mth sample,

n_{2jm} be the number of items among the n_{2j} items from the mth sample,

$$n_1 = \sum_{j=1}^{k} n_{1j}, \text{ and } n_2 = \sum_{j=1}^{k} n_{2j}.$$

The following results are then evident,

$$n_{1j} = \sum_{m=1}^{j} n_{1jm}; \qquad n_{2j} = \sum_{m=1}^{j} n_{2jm}$$

$$\sum_{m=1}^{k} N_m = n_1 + n_2 + r = n + r = N \text{ (say)}.$$

(*iii*) *Failure Times Known*

The likelihood function of the sample arising as a result of the above experiment is given by

$$P(S) = \prod_{j=1}^{k}\left[\left(a_{2,j}\right)^{r_{k+1-j}}\left[1-F_2\left\{j(T_1+T_2)\right\}\right]^{r_{k+1-j}} \prod_{m=1}^{j}\prod_{i=1}^{2}\left[\prod_{p=1}^{n_{ijm}} f\left(t_{pi}^{(j+1-m)}\right)\right]\right]$$

$$= \prod_{j=1}^{k}\left[\exp\left(-jr_{k+1-j}(\lambda_1 T_1+\lambda_2 T_2)\right)\prod_{m=1}^{j}\left\{\lambda_1^{n_{1jm}}\cdot\lambda_2^{n_{2jm}}\cdot\exp\left(-\lambda_1\sum_{p=1}^{n_{1jm}} t_{p1}^{(j+1-m)}\right)\cdot\right.\right.$$

$$\left.\left.\cdot\exp\left(-\lambda_2\sum_{p=1}^{n_{2jm}} t_{p2}^{(j+1-m)}\right)\cdot\exp\left(n_{1jm}(j-m)(\lambda_1-\lambda_2)T_2-n_{2jm}(j+1-m)(\lambda_1-\lambda_2)T_1\right)\right\}\right]$$

where $t_{pi}^{(j+1-m)}$ are the times at which the pth item of mth sample failed in the ith part of jth cycle.

It may be noted that $\prod_{p=1}^{n_{1jm}}$ and $\sum_{p=1}^{n_{1jm}}$ refer to product and summation taken over n_{1jm} items of the mth sample that failed in the first part of the jth cycle while $\prod_{p=1}^{n_{2jm}}$ and $\sum_{p=1}^{n_{2jm}}$ refer to product and summation taken over n_{2jm} items that failed in second part of the jth cycle.

The usual method leads to the following maximum likelihood estimates and their asymptotic variances.

$$\hat{\lambda}_1 = \frac{n_1}{\sum_{j=1}^{k}\sum_{m=1}^{j}\sum_{p=1}^{n_{1jm}} t_{p_1}^{(j+1-m)} + T_1\sum_{j=1}^{k} jr_{k+1-j} - \sum_{j=1}^{k}\sum_{m=1}^{j}\left\{n_{1jm}(j-m)T_2 - n_{2jm}(j+1-m)T_1\right\}}$$

$$\hat{\lambda}_2 = \frac{n_2}{\sum_{j=1}^{k}\sum_{m=1}^{j}\sum_{p=1}^{n_{2jm}} t_{p_2}^{(j+1-m)} + T_2\sum_{j=1}^{k} jr_{k+1-j} + \sum_{j=1}^{k}\sum_{m=1}^{j}\left\{n_{1jm}(j-m)T_2 - n_{2jm}(j+1-m)T_1\right\}}$$

$$V(\hat{\lambda}_1) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_1^2}\right\}\right]^{-1} = \frac{\lambda_1^2}{E(n_1)}$$

$$V(\hat{\lambda}_2) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_2^2}\right\}\right]^{-1} = \frac{\lambda_2^2}{E(n_2)}$$

where

$$E(n_{1jm}) = N_m \exp.\left(-(j-m)(\lambda_1 T_1+\lambda_2 T_2)\right)\left(\left(1-\exp(-\lambda_1 T_1)\right)\right)$$

and

$$E(n_{2jm}) = N_m \exp.\left(-(j-m)(\lambda_1 T_1+\lambda_2 T_2)-\lambda_1 T_1\right)\left(1-\exp(-\lambda_2 T_2)\right).$$

(iv) Failure Times Unknown

In such a situation the likelihood function of the sample is given by

$$P(S) = \prod_{j=1}^{k}\left[\left(a_{2.j}\right)^{r_{k+1-j}}\left[1 - F_2\left\{j(T_1 + T_2)\right\}\right]^{r_{k+1-j}} \prod_{m=1}^{j}\prod_{i=1}^{2}\left(P_{i\,\overline{j+1-m}}\right)^{n_{ijm}}\right]$$

$$= \prod_{j=1}^{k}\left[\exp\left\{(-j r_{k+1-j})(\lambda_1 T_1 + \lambda_2 T_2)\right\}\prod_{m=1}^{j}\left\{\left(1 - \exp(-\lambda_1 T_1)\right)^{n_{1jm}} \cdot\right.\right.$$

$$\cdot \exp\left(-n_{1jm}(j-m)(\lambda_1 T_1 + \lambda_2 T_2)\right)\left(1 - \exp(-\lambda_2 T_2)\right)^{n_{2jm}} \cdot$$

$$\left.\left.\cdot \exp\left(-n_{2jm}\lambda_1 T_1\right) \cdot \exp\left(-(j-m)n_{2jm}(\lambda_1 T_1 + \lambda_2 T_2)\right)\right\}\right]$$

Following the usual procedure, we get the following maximum likelihood estimates and their asymptotic variances.

$$\hat{\lambda}_1 = \frac{1}{T_1}\log\left[\frac{n_1 + n_2 + \sum_{j=1}^{k} j r_{k+1-j} + \sum_{j=1}^{k}\sum_{m=1}^{j}(j-m)(n_{1jm} + n_{2jm})}{n_2 + \sum_{j=1}^{k} j r_{k+1-j} + \sum_{j=1}^{k}\sum_{m=1}^{j}(j-m)(n_{1jm} + n_{2jm})}\right]$$

$$\hat{\lambda}_2 = \frac{1}{T_2}\log\left[\frac{n_2 + \sum_{j=1}^{k} j r_{k+1-j} + \sum_{j=1}^{k}\sum_{m=1}^{j}(j-m)(n_{1jm} + n_{2jm})}{\sum_{j=1}^{k} j r_{k+1-j} + \sum_{j=1}^{k}\sum_{m=1}^{j}(j-m)(n_{1jm} + n_{2jm})}\right]$$

$$V(\hat{\lambda}_1) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_1^2}\right\}\right]^{-1} = \frac{(1 - e^{-\lambda_1 T_1})^2}{T_1^2 \exp(-\lambda_1 T_1)\, E(n_1)}$$

$$V(\hat{\lambda}_2) = \left[E\left\{-\frac{\partial^2 \log P(S)}{\partial \lambda_2^2}\right\}\right]^{-1} = \frac{(1 - e^{-\lambda_2 T_2})^2}{T_2^2 \exp(-\lambda_2 T_2)\, E(n_2)}$$

where $E(n_1)$ and $E(n_2)$ are same as in section *(iii)*.

NUMERICAL EXAMPLE

Table 2 gives data pertaining to the Bartholomew's experiment described. Each cycle is taken to be of 24 hour duration, the first part lasting 16 hours and second part 8 hours.

TABLE 2

FREQUENCY DISTRIBUTION OF LIVES OF VACUUM TUBES

(Total number of tubes placed on test=100)

Cycle no.	Description	Sample Number 1	2	3	4
1	No. of tubes that failed in first part	3	—	—	—
	No. of tubes that failed in second part	2	—	—	—
2	No. of tubes that failed in first part	2	2	—	—
	No. of tubes that failed in second part	2	2	—	—
3	No. of tubes that failed in first part	2	2	2	—
	No. of tubes that failed in second part	1	1	2	—
4	No. of tubes that failed in first part	1	2	2	2
	No. of tubes that failed in second part	1	1	2	2

NUMERICAL CALCULATIONS

For data given in Table 2:

$$N_1 = N_2 = N_3 = N_4 = 25, \qquad N = 100$$

$$n_1 = \sum_{j=1}^{4} \sum_{m=1}^{j} n_{1jm} = 20,$$

$$n_2 = \sum_{j=1}^{4} \sum_{m=1}^{j} n_{2jm} = 16,$$

$$r_1 = 11, \quad r_2 = 15, \quad r_3 = 17, \quad r_4 = 21.$$

Failure Times Known

$$\sum_{j=1}^{4} \sum_{m=1}^{j} \sum_{p=1}^{n_{1jm}} t_{p_1}{}^{(j+1-m)} = 532\cdot16$$

$$\sum_{j=1}^{4} \sum_{m=1}^{j} \sum_{p=1}^{n_{2jm}} t_{p_2}{}^{(j+1-m)} = 588\cdot00$$

The relevant calculations as outlined in section (*iii*) lead to following results :

$$\hat{\lambda}_1 = 0\cdot006353, \qquad \hat{\lambda}_2 = 0\cdot011204$$

$$V(\hat{\lambda}_1) = 0\cdot1988 \times 10^{-5}, \qquad V(\hat{\lambda}_2) = 0\cdot7062 \times 10^{-5}.$$

Failure Times Unknown

In such case the relevant calculations as outlined in section (*iv*) lead to the following results.

$$\hat{\lambda}_1 = 0\cdot00669, \qquad \hat{\lambda}_2 = 0\cdot01117$$

$$V(\hat{\lambda}_1) = 0\cdot2206 \times 10^{-5}, \qquad V(\hat{\lambda}_2) = 0\cdot7027 \times 10^{-5}.$$

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