

# THE STABILITY OF NON-NEWTONIAN FLUID BETWEEN THE TWO ROTATING POROUS CYLINDERS (WIDE GAP CASE)

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This paper is primarily concerned with the stability of the non-Newtonian fluid between two porous cylinders in the case of wide gap. The problem is discussed for  $\mu > 0$ ,  $\mu = 0$  and  $\mu < 0$ . The results show that the Taylor number depends on the gap size in the case of non-Newtonian fluids and the presence of suction stabilizes the flow whereas the injection destabilizes the flow. It is found that the stability of the fluid decreases when the gap increases. The non-Newtonian fluid is less stable when compared to the Newtonian fluid in the case of wide gap. It is also found in the case of wide gap (for non-Newtonian fluid and the cylinders are counter rotating equally) the application of injection at the outer cylinder disturbs the radial velocity and no change when suction is applied. It is also concluded that the presence of suction or injection will not effect any appreciable change or disturbance in the vortex cell pattern at the onset of instability.

The stability of the Newtonian fluid between two concentric cylinders has been extensively studied by many authors like Taylor<sup>1</sup>, Jeffreys<sup>2</sup>, Meksyn<sup>3</sup> etc. in the case of narrow gap approximation. Bhaskarao<sup>4</sup>, Iyenger<sup>5,6,7</sup> respectively studied the stability of the second order fluid and non-Newtonian fluid with the narrow gap approximation. Recently Chan Man Fong<sup>8</sup> studied the stability of the visco elastic fluid in the case of wide gap approximation and found that the stability of visco elastic fluid depends on the size of the gap between the two cylinders. Also Westbrook<sup>9</sup>, in his research paper No. 58 studied the stability of the convective flow in a porous medium. Reddy<sup>10,11</sup> studied the problem of porous cylinders for narrow gap case. This problem is the extension of Reddy's problem with wide gap. The results are flexible for slightly wide gap and the fluid is more stable between the cylinders in the present case. In the case of wide gap parameter  $\frac{d}{R_0}$  is zero, the results are coinciding with Reddy's results.

The present paper mainly concerns the stability of the non-Newtonian fluid between two porous cylinders (in the presence of suction or injection) when the gap is wide. The stability of fluid is discussed for different values of  $\mu$ , i.e., when the cylinders are co-rotating ( $\mu > 0$ ), the outer cylinder is at rest ( $\mu = 0$ ), the cylinders are counter rotating ( $\mu < 0$ ). In all cases the presence of suction or injection disturbs the non-Newtonian fluid appreciably when the cylinders are counter rotating (for the wide gap approximation). Also it is noted that the application of suction does not disturb the radial velocity. When the cylinders are counter rotating equally ( $\mu = -1.0$ ) whereas the presence of injection disturbs the radial velocity. Also we can find little disturbance in radial velocity for the application of suction or injection when the cylinders are co-rotating or the outer cylinder is at rest. Also it is found that the effect of suction or injection in wide gap case when the non-Newtonian fluid is between the two cylinders is considerably less on the vortex cells at the outset of instability.

## EQUATION OF MOTION

The equations of motion and continuity in the cylindrical coordinates ( $r, \theta, z$ ) with axial symmetry are (Navier-Stokes equation)

$$\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right] = \frac{\partial}{\partial r} \tau_{rr} + \frac{\partial}{\partial z} \tau_{rz} + \frac{\tau_{r\theta} - \tau_{\theta\theta}}{r} \quad (1)$$

$$\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right] = \frac{\partial}{\partial r} \tau_{\theta r} + \frac{\partial}{\partial z} \tau_{\theta z} + \frac{2}{r} \tau_{\theta\theta} \quad (2)$$

$$\rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = \frac{\partial}{\partial r} \tau_{zr} + \frac{\partial}{\partial z} \tau_{zz} + \frac{\tau_{z\theta}}{r} \quad (3)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

and the stress tensor for an incompressible non-Newtonian fluids is given by

$$\begin{aligned} \begin{bmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{bmatrix} &= -p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \zeta_1 \begin{bmatrix} e_{rr} & e_{r\theta} & e_{rz} \\ e_{\theta r} & e_{\theta\theta} & e_{\theta z} \\ e_{zr} & e_{z\theta} & e_{zz} \end{bmatrix} + \\ &+ \zeta_2 \begin{bmatrix} e_{rr} & e_{r\theta} & e_{rz} \\ e_{\theta r} & e_{\theta\theta} & e_{\theta z} \\ e_{zr} & e_{z\theta} & e_{zz} \end{bmatrix} \end{bmatrix} \quad (5)$$

where  $p$  is the pressure,  $\rho$  the density,  $u, v$  and  $w$  are the velocity components in the direction of  $r, \theta$  and  $z$  respectively.  $\zeta_1$  and  $\zeta_2$  are the coefficients of viscosity and cross viscosity.

$$\left. \begin{aligned} e_{rr} &= 2 \frac{\lambda u}{\partial r} & e_{rz} &= e_{zr} = \frac{\lambda w}{\lambda r} + \frac{\partial u}{\partial z} \\ e_{\theta\theta} &= 2 \left( \frac{\partial v}{\partial r} + \frac{u}{r} \right) & e_{\theta z} &= e_{z\theta} = \frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial v}{\partial \theta} \\ e_{zz} &= 2 \frac{\partial v}{\partial r} & e_{r\theta} &= e_{\theta r} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \end{aligned} \right\} \quad (6)$$

The Navier-Stokes equations (1)-(4) admit the steady state solution of the form

$$\left. \begin{aligned} u &= U(r) = \frac{c}{r} = \frac{R_1 u_1}{r} \\ v &= v(r) = Ar^{\lambda+1} + \frac{B}{r} \\ w &= 0 \end{aligned} \right\} \quad (7)$$

where  $\lambda = \frac{R_1 u_1}{\alpha}$  the suction parameter  $u_1$  the radial velocity of the fluid and  $\alpha$  the co-efficient of kinematic viscosity and  $R_1, R_2, \Omega_1$  and  $\Omega_2$  are the radii and angular velocities of the inner and outer cylinders and  $R_2 - R_1 = d$ . Also it is assumed  $R_0 = R_2 - \frac{d}{2} = R_1 + \frac{d}{2}$  and  $d \ll (R_1 + R_2)/2$ . The arbitrary constant  $A$  and  $B$  can be determined by no-slip condition and are given by

$$A = -\Omega_1 \eta^2 \frac{(1 - \mu/\eta^2)}{R_2^\lambda (1 - \eta^\lambda + 2)}, \quad B = \frac{R_1^2 \Omega_1 (1 - \mu/\eta^\lambda)}{(1 - \eta^\lambda + 2)} \quad (8)$$

where

$$\mu = \frac{\Omega_2}{\Omega_1} \quad \text{and} \quad \eta = \frac{R_1}{R_2}$$

By considering the symmetric perturbation of solution (7) by a periodic disturbance in the direction parallel to the axis of rotation, the equations governing the marginal stability are given by

$$\begin{aligned} (DD_* - p^2)^2 u + \frac{p^2 \lambda}{r} Du - \lambda D \left( \frac{1}{r} DD_* u \right) &= 2 \frac{V}{r} v \frac{p^2}{\alpha} + \\ + \gamma \frac{p^2}{\alpha} \left[ D^2 v - \frac{1}{r} Dv - p^2 + \frac{2}{r^2} v \right] &\left( \lambda A r^\lambda - \frac{2B}{r^2} \right) + \\ + \frac{2}{r} \left( Dv - \frac{v}{r} \right) &\left( \lambda(\lambda - 1) Ar^\lambda + \frac{4B}{r^2} \right) \end{aligned} \quad (9)$$

$$\begin{aligned}
 (DD_* - p^2)v &= \frac{1}{\alpha} u D_* r + \frac{\lambda}{r} D_* v - \frac{\gamma}{\alpha} \left[ \frac{2}{r} D_* u \left( \lambda^2 A r \lambda + \frac{4B}{r^2} \right) + \right. \\
 &+ 2 \left( \lambda A r \lambda - \frac{2B}{r^2} \right) DD_* u + \frac{2}{r} D_* u \left( 2\lambda A r \lambda - \frac{4B}{r^2} \right) - \\
 &\left. - \frac{2\lambda\alpha}{r} p^2 v + \left( \lambda A r \lambda - \frac{2B}{r^2} \right) \left( DD_* - p^2 \right) u \right] \quad (10)
 \end{aligned}$$

where

$$\left. \begin{aligned}
 DD_* &= D_* D - \frac{1}{r^2} = \frac{d}{dr} \left( \frac{d}{dr} + \frac{1}{r} \right) \\
 D &= \frac{d}{dr}, \quad D_* = \frac{d}{dr} + \frac{1}{r}, \quad \alpha = \frac{\varphi_1}{\rho}, \quad \gamma = \frac{\varphi_2}{\rho}
 \end{aligned} \right\} \quad (11)$$

with the boundary conditions

$$u = D_* u = v = 0 \text{ at } r = R_1 \text{ and } r = R_2 \quad (12)$$

Here the wide gap approximation is considered and neglected the terms of  $\frac{d^2}{R_0^2}$  and higher powers.

Since we are considering the terms upto  $\frac{d}{R_0}$  only, we cannot find the difference between  $D$  and  $D_*$ . Using this approximation in the equations (11) and (12) can be written as by using the transformation

$$u \rightarrow \frac{2a^2 d^2}{\alpha} \Omega_1 u$$

$$\begin{aligned}
 \left[ (D^2 - a^2)^2 - \lambda \frac{d}{R_0} D (D^2 - a^2) \right] u &= \left[ 1 + (1 - \mu)\zeta + \right. \\
 + \frac{\lambda - 3}{2} \left\{ (1 - \mu)\zeta - \zeta^2 \right\} \frac{d}{R_0} + \zeta(\mu - 1) \left\{ 1 + (\lambda - 2)(\zeta - 2) \frac{d}{R_0} \right\} (D^2 - a^2) \Big] v \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 \left[ D^2 - a^2 - \lambda \frac{d}{R_0} D \right] v &= -T a^2 \left\{ 1 + \frac{(\mu - 1)(\lambda(\zeta - \frac{1}{2}) + 1)}{(\mu - 1)\frac{R_0}{d} + 2} - \frac{d}{R_0} \left\{ \lambda + \right. \right. \\
 &+ \frac{(1 - \mu)(5\lambda^2 - 4\lambda + 8)}{12} - \frac{\lambda(1 + 3\mu)\zeta}{2} \\
 &\left. \left. - \frac{\lambda^2}{2} (1 - \mu)\zeta + \frac{\lambda(\lambda - 1)(1 - \mu)}{2} \zeta^2 \right\} \frac{1}{(\mu - 1)\frac{R_0}{d} + 2} \right\} u \quad (14)
 \end{aligned}$$

where

$$T = -\frac{2\Omega_1^2 d^4}{\alpha^2} \left\{ (\mu - 1) \frac{R_0}{d} + 2 \right\}$$

$$S = \frac{\gamma R_1}{2d^3}$$

$$\begin{aligned}
 Ar\lambda &= \frac{\Omega_1}{\lambda + 2} \left[ (\mu - 1) \frac{R_0}{d} - \left\{ \lambda(1 - \mu) \left( \zeta - \frac{1}{2} \right) - 2 + (1 - \mu) \right\} - \right. \\
 &- \frac{d}{R_0} \left\{ \lambda + 2 + \frac{(\lambda^2 - \lambda)(1 - \mu)}{8} - \frac{\lambda}{2} \left( (1 + \mu) + \lambda + 2 - \mu\lambda \right) \zeta + \right. \\
 &\left. \left. + \frac{(\lambda^2 - \lambda)(1 - \mu)}{2} \zeta^2 + \lambda(1 - \mu)\zeta - 2 - \frac{\lambda}{2} (1 - \mu) + (1 - \mu) \frac{7\lambda^2 + 7\lambda + 6}{24} \right\} \right]
 \end{aligned}$$

$$Ar\lambda + \frac{B}{r^2} = \Omega_1 \left[ 1 + (\mu - 1)\zeta + \frac{\lambda - 3}{2} \frac{d}{R_0} \left\{ (1 - \mu)\zeta - \zeta^2 \right\} \right]$$

with the boundary conditions

$$u = Du = v = 0 \text{ at } \zeta \pm \frac{1}{2} \tag{15}$$

The equations (13) — (15) determine the eigenvalue problem for  $T$  as a function of the parameters  $\mu, a, \lambda, \frac{d}{R_0}$  and  $S$ .

METHOD OF SOLUTION

The eigenvalue problem given in equations (13) and (14) with the boundary conditions (15) can be solved by using the Galerkin's method which has been employed in several studies by Diprima<sup>12</sup> and Kurzeg<sup>13</sup>. This method consists of expanding the solution of the problem in series of a complete set of orthonormal functions each of which satisfy the boundary conditions imposed on the solution. The trial functions satisfying the boundary conditions (15) are given by

$$\left. \begin{aligned} u &= a_1 (1 - 4\zeta^2)^2 + a_2 \zeta (1 - 4\zeta^2)^2 \\ v &= b_1 (1 - 4\zeta^2) + b_2 \zeta (1 - 4\zeta^2) \end{aligned} \right\} \tag{16}$$

where  $a_1, a_2, b_1$  and  $b_2$  are constants. Applying the orthogonal condition<sup>13</sup>

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ u, v \right] L(u, v) \zeta = 0 \tag{17}$$

where  $L(u, v)$  is the error matrix obtained on substituting (15) in (16) and (17) we get the secular equation as

$$\left\{ \begin{array}{cccc} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & 0 \\ G_{41} & G_{42} & 0 & G_{44} \end{array} \right\} = 0 \tag{18}$$

where

$$G_{11} = 16a^4 + 384a^2 + 8064$$

$$G_{22} = 8a^4 + 704a^2 + 31680$$

$$G_{33} = \left( 7a^2 + 10 + \lambda \frac{d}{R_0} \right)$$

$$G_{44} = \left( 3a^2 + 42 + \lambda \frac{d}{R_0} \right)$$

$$G_{12} = 8\lambda(a^2 + 36) \frac{d}{R_0}$$

$$G_{13} = 18(\mu + 1) - \frac{\lambda - 3}{2} \frac{d}{R_0} - 6S(\mu - 1)(3a^2 + 28) \left( 2 - (\lambda - 2) \frac{d}{R_0} \right)$$

$$G_{14} = (\mu - 1) + \frac{17}{16} (\lambda - 3) (\mu - 1) \frac{d}{R_0} - S(\mu - 1)(\lambda - 2)(a^2 + 36) \frac{d}{R_0}$$

$$G_{21} = -176\lambda(a^2 + 36) \frac{d}{R_0}$$

$$G_{23} = 22(\mu - 1) - 11(\lambda - 3)(\mu - 1) \frac{d}{R_0} - 22S(\mu - 1)(\lambda - 2)(a^2 + 12) \frac{d}{R_0}$$

$$G_{24} = 11(\mu + 1) - \frac{3}{4}(\lambda - 3) \frac{d}{R_o} - 11(a^2 + 36)(\mu - 1) \left( (2 - (\lambda - 2) \frac{d}{R_o}) \right)$$

$$G_{31} = a^2 \left[ 3 \left( 1 - \frac{K_1}{2} + K_2 + K_3 \frac{d}{R_o} \right) + \frac{4}{3} K_5 \frac{d}{R_o} \right]$$

$$G_{32} = \frac{a^2}{12} \left( K_1 + K_4 \frac{d}{R_o} \right)$$

$$G_{41} = a^2 \left( K_1 + K_4 \frac{d}{R_o} \right)$$

$$G_{42} = \left( 1 - \frac{K_1}{2} + K_2 + K_3 \frac{d}{R_o} + \frac{3}{44} K_5 \frac{d}{R_o} \right) a^2$$

and

$$K_1 = \frac{(\mu - 1)\lambda}{(\mu - 1) \frac{R_o}{d} + 2}$$

$$K_2 = (\mu - 1) / \left( (\mu - 1) \frac{R_o}{d} + 2 \right)$$

$$K_3 = \left\{ (\mu - 1)(5\lambda^2 - 4\lambda + 8) - 12\lambda \right\} / 12 \left\{ (\mu - 1) \frac{R_o}{d} + 2 \right\}$$

$$K_4 = \left\{ \lambda(1 + 3\mu) - (\mu - 1)\lambda^2 \right\} / 2 \left\{ (\mu - 1) \frac{R_o}{d} + 2 \right\}$$

$$K_5 = \lambda(\lambda - 1)(\mu - 1) / 2 \left\{ (\mu - 1) \frac{R_o}{d} + 2 \right\}$$

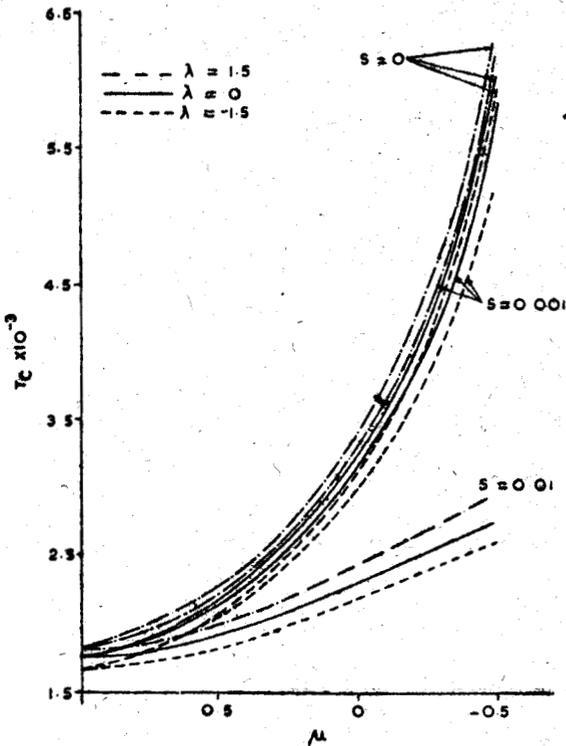


Fig. 1—The behaviour of critical Taylor number with  $\mu$  at  $\frac{d}{R_o} = 0.05$ .

With respect to the gap size and irrespective of the fluid the Taylor number does not differ when the cylinders are co-rotating equally, when suction or injection is made at the outer cylinder (Fig. 1)

For constant values of  $d$  and  $\frac{d}{R_o}$ ,  $\mu$ ,  $S$ ; the variation of the critical Taylor number when the suction or injection is made at the outer cylinder shows that the fluid is stable in the case of suction (The result has been concluded after calculation of Taylor number for different values of suction and injection in the case of narrow gap and wide gap). It is also noteworthy that Taylor number depends on the gap size both for Newtonian and non-Newtonian fluids, which satisfy the results of Chan Man Fong<sup>8</sup> in the absence of suction or injection.

In the case of wide gap, we can find from Fig. 1 and Fig. 2 that the presence of suction or injection, effects more in the case of non-Newtonian fluids than in the case of Newtonian fluids. The Fig. 3 is drawn from the critical wave number  $a_c$  against  $\mu$ . In the Fig. 3, the curves for suction or injection in different cases of  $S$  are not shown because the change in wave number is little (negligibly small). The curves are highly damped even in wide gap is noted in the case of Newtonian

fluids for suction and injection (Fig. 2), which differs for non-Newtonian fluids (Fig. 2 and 3). The results obtained here from Fig. 2 and 3 are satisfying the results of Harris and Reid<sup>14</sup> (the curve in the Galerkin method is used where Reid use some of the Chandrasekhar's results).

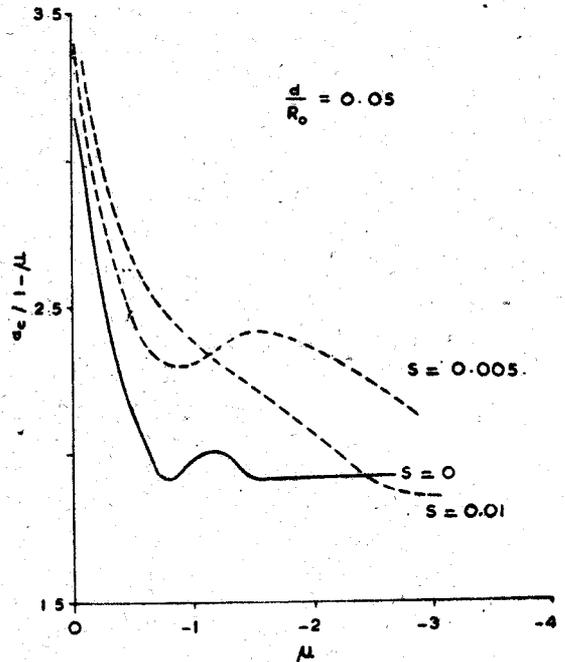
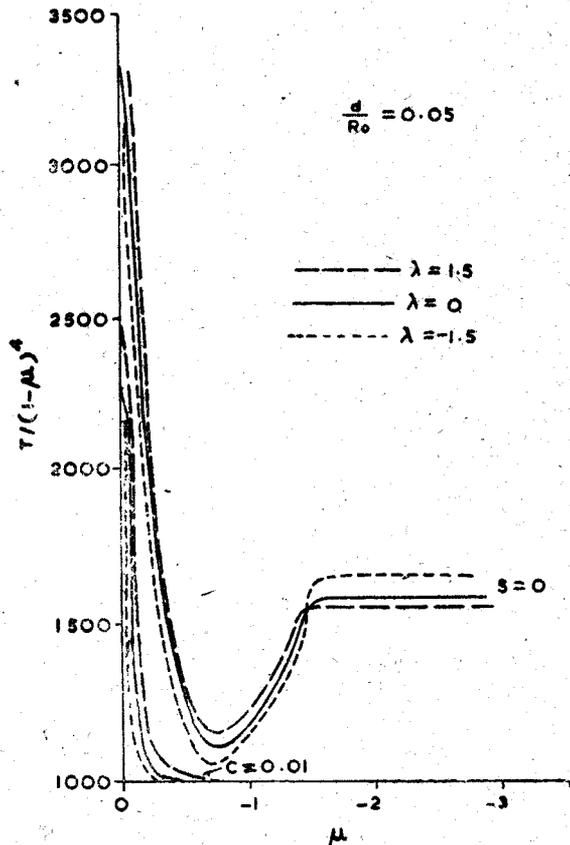


Fig. 2—The behaviour of critical Taylor number with  $\mu$ .

Fig. 3—The behaviour of critical wave number with  $\mu$ .

The disturbance in radial velocity is calculated for  $\mu > 0$ ,  $\mu = 0$  and  $\mu < 0$  and found that the disturbance in the radial velocity is negligibly small when they are co-rotating or the outer cylinder is at rest. The author has calculated for many values of  $\mu$  when  $\mu > 0$  and  $\mu < 0$  but it is given here in Table 1 only when  $\mu = -1.0$ . The results are coinciding with Reddy's<sup>10,11</sup> results in the absence of wide gap case.

The disturbance in the case of wide gap approximation for non-Newtonian fluid is more when compared to the narrow gap approximation. When the cylinders are counter rotating equally (i.e.  $\mu = -1.0$ ) the presence of injection disturbs the radial velocity whereas with suction there will not be any change in the radial velocity (in the case of narrow gap approximation or wide gap approximation and non-Newtonian fluid). But in the case of Newtonian fluid there is equal disturbance in the case of wide gap approximation. This can be observed clearly from Table 1. The results are coinciding with Bhaskar Rao's<sup>4</sup> results in the absence of wide gap and suction parameter.

For non-Newtonian fluids in the case of wide gap approximation when the cylinders are co-rotating, for application of suction or injection the variation in vortex cells is little (negligibly small i.e. we cannot show the difference on the graph clearly). When the other cylinder is at rest (i.e.  $\mu = 0$ ) the effect of suction or injection have still less disturbance on the vortex cell pattern.

TABLE I

THE RADIAL VELOCITY DISTRIBUTION FOR  $v=1.0$ 

$\theta/\lambda$	$S = 0$						$S = 0.005$					
	$d/R_0 = 0$			$d/R_0 = 0.05$			$d/R_0 = 0$			$d/R_0 = 0.05$		
	-1.5	0	1.5	-1.5	0	1.5	-1.5	0	1.5	-1.5	0	1.5
-0.4500	0.0811	0.0812	0.0812	0.0848	0.0834	0.0822	0.0620	0.0620	0.0620	0.0637	0.0629	0.0629
-0.4000	0.2688	0.2689	0.2689	0.2797	0.2756	0.2719	0.2110	0.2108	0.2108	0.2161	0.2137	0.2137
-0.3500	0.4944	0.4944	0.4945	0.5116	0.5052	0.4992	0.3999	0.3996	0.3996	0.4085	0.4040	0.4044
-0.3000	0.7074	0.7075	0.7075	0.7273	0.7199	0.7130	0.5927	0.5924	0.5924	0.6035	0.5983	0.5983
-0.2500	0.8739	0.8739	0.8740	0.8912	0.8849	0.8788	0.7630	0.7627	0.7627	0.7741	0.7688	0.7688
-0.2000	0.9737	0.9737	0.9738	0.9830	0.9797	0.9765	0.8933	0.8930	0.8930	0.9025	0.8981	0.8981
-0.1500	0.9990	0.9990	0.9990	0.9953	0.9970	0.9983	0.9735	0.9733	0.9733	0.9787	0.9762	0.9762
-0.1000	0.9518	0.9518	0.9518	0.9314	0.9395	0.9465	1.0000	1.0000	1.0000	0.9996	0.9999	0.9999
-0.0500	0.8421	0.8421	0.8420	0.8031	0.8183	0.8317	0.9748	0.9750	0.9750	0.9678	0.9713	0.9713
0.0000	0.6857	0.6855	0.6854	0.6281	0.6503	0.6701	0.9041	0.9045	0.9045	0.8903	0.8971	0.8971
0.0500	0.5019	0.5017	0.5016	0.4282	0.4565	0.4819	0.7975	0.7980	0.7980	0.7773	0.7872	0.7872
0.1000	0.3120	0.3118	0.3116	0.2264	0.2592	0.2886	0.6665	0.6672	0.6672	0.6414	0.6536	0.6536
0.1500	0.1366	0.1364	0.1362	0.0451	0.0800	0.1116	0.5240	0.5248	0.5248	0.4950	0.5095	0.5095
0.2000	-0.0061	-0.0063	-0.0065	-0.0965	-0.0620	-0.0308	0.3826	0.3834	0.3834	0.3539	0.3679	0.3679
0.2500	-0.1025	-0.1027	-0.1029	-0.1845	-0.1533	-0.1250	0.2541	0.2549	0.2549	0.2275	0.2404	0.2404
0.3000	-0.1457	-0.1459	-0.1460	-0.2127	-0.1872	-0.1641	0.1480	0.1486	0.1486	0.1258	0.1366	0.1366
0.3500	-0.1377	-0.1378	-0.1379	-0.1848	-0.1669	-0.1506	0.0705	0.0709	0.0709	0.0546	0.0623	0.0623
0.4000	-0.0911	-0.0912	-0.0912	-0.1169	-0.1071	-0.0982	0.0234	0.0236	0.0236	0.0146	0.0189	0.0189
0.4500	-0.0316	-0.0317	-0.0317	-0.0395	-0.0365	-0.0338	0.0032	0.0033	0.0033	0.0006	0.0019	0.0019

For non-Newtonian fluids ( $S=0.01$ ) the presence of suction (Fig. 4) does not disturb the vortex cells much (i.e. disturbance is negligible) for all values of  $\mu$  in the case of wide gap but in the case of injection we can find slight disturbance (Fig. 5). Also when the non-Newtonian fluid is between the porous walls the effect of suction or injection makes little disturbance either in the case of narrow gap approximation or in the case of wide gap approximation. Fig. 4 and Fig. 5 respectively show the case of wide gap approximation in the presence of suction, injection at the outer cylinder respectively. We do not find much disturbance either on Newtonian or on non-Newtonian fluids in the presence of suction or injection. Here in these figures (4 and 5) it is not shown because the disturbance is very less.

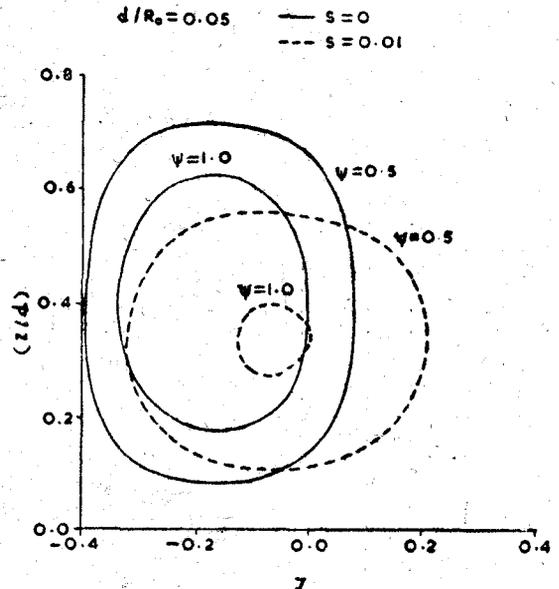
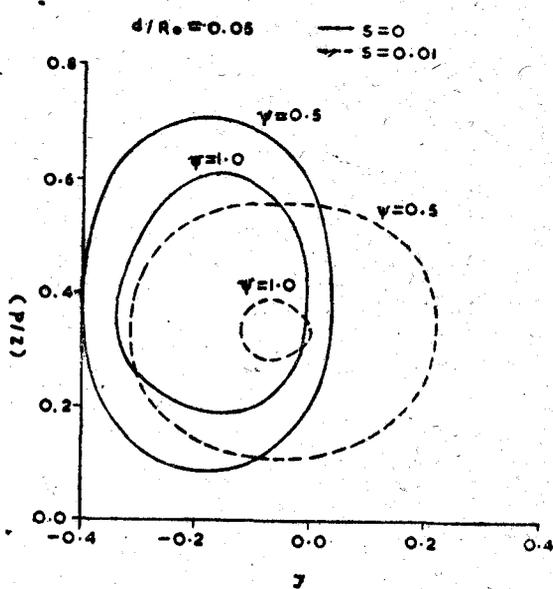


Fig. 4—The Vortex cells at the onset of instability  $\mu = -1.0$  and  $\lambda = 1.5$ .

Fig. 5—The Vortex cells at the onset of instability  $\mu = -1.0$  and  $\lambda = -1.5$ .

Here it can be concluded that when the non-Newtonian fluid is placed between two cylinders is at rest the application of suction or injection disturbs the vortex cells little (both in narrow gap approximation and wide gap approximation).

#### CONCLUSION

(i) The non-Newtonian fluid is more unstable in the wide gap case when compared to the narrow gap case. (ii) The fluid is more unstable in the wide gap approximation in the presence of injection, but suction stabilizes the flow. (iii) When the cylinders are co-rotating equally ( $\mu=1.0$ ) the Taylor number will not be changed for any fluid when suction or injection is applied at the outer cylinder. (iv) Irrespective of the gap size when the non-Newtonian fluid is kept between the two counter rotating cylinders equally ( $\mu = -1.0$ ) the presence of suction will not disturb the radial velocity. (v) In the case of non-Newtonian fluid between the two co-rotating or counter rotating cylinders (when the wide gap is considered) the presence of suction or injection makes little disturbance on the vortex cells at the onset of instability.

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