ON THE FLOW OF A VISCO-ELASTIC FLUID BETWEEN TWO ROTATING NON-CONCENTRIC CYLINDERS FOR SMALL ECCENTRICITY

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The motion of a visco-elastic fluid contained between two rotating circular cylinders whose axes are set slightly apart is considered. An approximate solution of the Navier-Stokes equations is obtained by a perturbation method for the case of small eccentricity. The expansions contain a perturbation parameter ϵ related to the eccentricity e which is assumed to be small. The method entails the employment of asymptotic expansions of Bessel functions and the results thus obtained are valid for finite gap between the two cylinders. The leading terms of the expansions are the exact solution to the Couette flow of Newtonian fluid between concentric rotating cylinders. The transverse velocity profiles are presented for small eccentricity and gap, when both the cylinders are rotating with the same angular velocities in the same direction and compared with the corresponding transverse velocity profile in Newtonian flow.

NOMENCLATURE

A, B	Constants in the concentric problem basic flow
d	gap ratio $(R_2 - R_1)$
e	eccentricity
h	radial distance between inner and outer cylinders
R_1, R_2	radii of inner and outer cylinders respectively
R_0	mean radius $(R_1 + R_2)/2$
r, θ, z	polar cylinderical coordinates
u_0, v_0	concentric problem velocity components
u_1, v_1	first order velocity components
u_2, v_2	second order velocity components
v_r, v_θ, v_z	velocity components
x	non-dimensional independent variable
J_λ	Bessel function of first kind and order λ
Y_λ	Bessel function of second kind and order λ
ω	imaginary argument of Bessel functions
δ	gap ratio (d/R_0)
ϵ	eccentricity ratio (e/d)
η	radii ratio (R_1/R_2)
μ	speed ratio (Ω_2/Ω_1)
ρ	density of the fluid
ν_1	kinematic viscosity (ϕ_1/ρ)

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ν_2	coefficient of visco-elasticity (ϕ_2/ρ)
ν_3	coefficient of cross-viscosity (ϕ_3/ρ)
Ω	angular velocity
Ω_1, Ω_2	angular velocity of inner and outer cylinders respectively
Ω_0	concentric problem angular velocity (v_0/r)
R	Reynolds number $(R_1\,\Omega_1\,d/\nu_1)$

During the last two decades a number of papers have been published (namely Nikitin[1], Wood[2], Segel[3], Kulinski and Ostrach[4] and Urban[5]) concerned with approximate solution of the basic flow for small eccentricity. The variations in view points taken in these papers are concerned with the coordinate system employed, the distance between the inner and outer cylinders, the eccentricity, the rotation of the cylinders and the Reynolds number. A perturbation method was employed in the work of these authors and the expansions employed a perturbation parameter related to the eccentricity e which is assumed to be small. They used modified bipolar and polar coordinate systems. In the small gap analysis all of them have assumed the outer cylinder to be fixed.

Recently, Urban considered the flow of a viscous incompressible fluid between eccentric cylinders fo- small eccentricity. An approximate solution of the Navier-Stokes equations is obtained by a perturbation method. He obtained a second solution of the basic flow by imposing the additional geometric restriction at small gap between the two cylinders and employing the asymptotic expansion of Bessel functions by Meissel's series. This second solution is also examined by formulating a small gap boundary value problem. He also used polar coordinate system. He employs a perturbation parameter $\epsilon = (e/\delta)$. The outer boundary condition is also not satisfied in his case. In his investigation the transverse velocity profiles are presented for the case of small eccentricity and when the gap δ tends to zero. He also assumed the outer cylinder to be fixed.

In our analysis the two dimensional flow of an incompressible visco-elastic fluid between two rotating non-concentric cylinders is examined. It is assumed that there is no flow in the axial direction of the cylinders. An approximate solution to the basic flow is developed by a perturbation method for the case of small eccentricity, as suggested by Urban, and valid for any gap. The method entails the employment of asymptotic expansions of Bessel functions. In our investigation a polar coordinate system is used although it is recognized there exists an inherent disadvantage of handling the outer boundary condition since the outer boundary is not a coordinate curve. However, it will be shown that this boundary condition can be handled adequately.

The constitutive equations of an incompressible visco-elastic second order fluid as suggested by Rivlin and Ericksen[6] are

$$\tau_{ij} = -p\,g_{ij} + \phi_1 A_{ij} + \phi_2 B_{ij} + \phi_3 A_{ik} A_{kj}, \tag{1}$$

where

$$A_{ij} = v_{i,j} + v_{j,i} \tag{2}$$

and

$$B_{ij} = a_{i,j} + a_{j,i} + 2v_{m,i}\,v_{m,j} \tag{3}$$

where τ_{ij} is the stress tensor, g_{ij} the metric tensor, v_i the velocity vector, a_i the acceleration vector, p the pressure, comma denotes covariant differentiation and ϕ_1, ϕ_2, ϕ_3 are the fluid parameters.

Two dimensional flows between moving nearly concentric circular cylinders currently are of great interest in connection with the design of control mechanism for aircrafts and rockets, with experiments on Helium II (detection of single quanta of circulation in rotating Helium II[7]), rotor stability and the hydrodynamic stability of the lubricant film.

BASIC FLOW SOLUTION BY A PERTURBATION METHOD

We shall use cylinderical polar coordinates (r, θ, z) with the z-axis coinciding with the inner cylinder axis. Let e be the distance between the centers of the inner O_i and outer O_0 cylinders and is called as eccentricity. The radial distance between the inner and outer cylinders is denoted by h and can be considered as a variable gap.

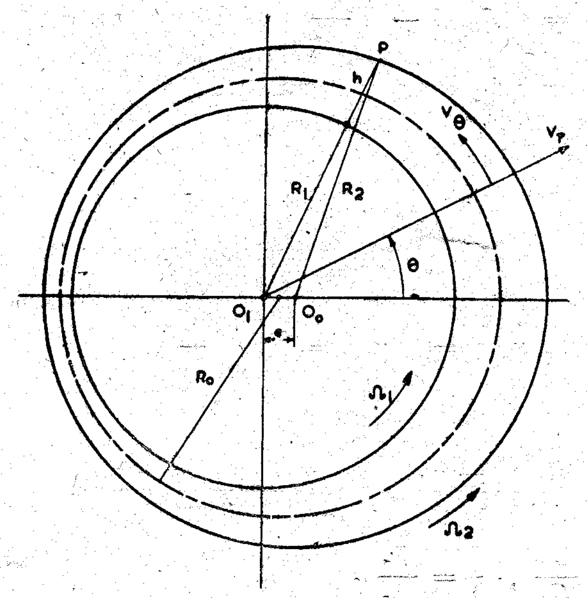

Fig. 1—Non-concentric cylinders.

The eccentricity ratio ϵ is a non-dimensional parameter defined as

$$\epsilon = \frac{e}{d}, \text{ where } 0 < \epsilon < 1$$

Another dimensionless parameter introduced is the gap ratio δ defined as

$$\delta = \frac{d}{R_0}, \text{ where } 0 < \delta < 2.$$

In the triangle $O_i P O_0$ of figure 1, we employ the cosine law and get

$$h = -R_1 + \epsilon d \cos\theta + R_2 \left(1 - \frac{4\epsilon^2 \delta^2}{(2+\delta)^2} \sin^2\theta\right)^{1/2} \tag{4}$$

The momentum equation for the incompressible flow are

$$\rho v_j v_{i,j} = \tau_{ij,j} \tag{5}$$

and the equation of continuity is

$$v_{i,i} = 0 \tag{6}$$

Since it is assumed that there is no flow in the axial direction (i.e. $v_z = 0$), therefore the velocity and pressure are the function of r and θ only. We transfer equations (1), (2), (3), (5) and (6) in cylinderical polar coordinate system. The flow is governed by the steady state two dimensional equations of motion and equation of continuity in polar cylinderical coordinates. The boundary conditions for the inner cylinder are

$$\left.\begin{array}{l} (i)\ v_r = 0 \quad \text{at } r = R_1 \\ (ii)\ v_\theta = R_1 \Omega_1 \quad \text{at } r = R_1 \end{array}\right\} \tag{7}$$

We note that the velocity vector at point P on the outer cylinder, in the figure 1, is not orthogonal to the position vector of point P. Therefore, it is resolved into a radial and transverse component. Then the boundary conditions for the outer cylinder are

$$\left.\begin{aligned} &(iii)\ v_r = -\ \Omega_2\, \epsilon\, d \sin\theta \text{ at } r = R_1 + h \\ &(iv)\ v_\theta = R_2\, \Omega_2 \left(1 - \left(\frac{\epsilon d}{R_2} \right)^2 \sin^2\theta \right)^{1/2} \text{ at } r = R_1 + h \end{aligned}\right\} \quad (8)$$

Now, we assume that the velocity components can be expanded by a perturbation series in terms of ϵ

$$\left.\begin{aligned} v_r(r,\theta) &= u_0(r) + \epsilon\, u_1(r,\theta) + \epsilon^2 u_2(r,\theta) + \dots \\ v_\theta(r,\theta) &= v_0(r) + \epsilon\, v_1(r,\theta) + \epsilon^2 v_2(r,\theta) + \dots \end{aligned}\right\} \quad (9)$$

It is noted that the leading terms in (9) are the exact solution to the concentric problem and are given by

$$u_0(r) = 0,\ v_0(r) = Ar + \frac{B}{r}, \quad (10)$$

where

$$\left.\begin{aligned} A &= \frac{\Omega_1 (\eta^2 - \mu)}{\eta^2 - 1}, \quad B = \frac{\Omega_1 R_1^2 (\mu - 1)}{\eta^2 - 1} \\ \mu &= \frac{\Omega_2}{\Omega_1}, \quad \eta = \frac{R_1}{R_2}. \end{aligned}\right\} \quad (11)$$

We place a geometric restriction upon the problem, that is, we asume e to be small in comparison to d ($e << d$) which implies that $\epsilon << 1$.

From equations of motion, continuity and (9), the differential equations for the first order solutions are

$$\begin{aligned} \Omega_0 \left(\frac{\partial^2 u_1}{\partial\theta} \right) - \frac{\partial v_1}{\partial\theta} - \nu \frac{\partial^2 v_1}{\partial r \partial\theta} \Big) &= v_1 \left(\frac{\partial^3 u_1}{\partial r^2 \partial\theta} + \frac{1}{r^2} \frac{\partial^3 u_1}{\partial\theta^3} + \right. \\ &+ \frac{2}{r^2} \frac{\partial u_1}{\partial\theta} + \frac{1}{r} \frac{\partial v_1}{\partial r} - 2 \frac{\partial^2 v_1}{\partial r^2} - r \frac{\partial^3 v_1}{\partial r^3} - \frac{1}{r} \frac{\partial^3 v_1}{\partial r \partial\theta^2} - \left. \frac{v_1}{r} \right) + \\ &+ v_2 \left\{ A \left(\frac{\partial^4 u_1}{\partial r^2 \partial\theta^2} - \frac{1}{r} \frac{\partial^3 u_1}{\partial r \partial\theta^2} + \frac{1}{r^2} \frac{\partial^2 u_1}{\partial\theta^2} + \frac{1}{r^2} \frac{\partial^4 u_1}{\partial\theta^4} - \right. \right. \\ &- \frac{1}{r} \frac{\partial^4 v_1}{\partial r \partial\theta^3} - r \frac{\partial^4 v_1}{\partial\theta \partial r^3} - 2 \frac{\partial^3 v_1}{\partial\theta \partial r^2} - \frac{1}{r^2} \frac{\partial^3 v_1}{\partial\theta^2} + \frac{1}{r} \frac{\partial^2 v_1}{\partial\theta \partial r} - \left. \frac{1}{r^2} \frac{\partial v_1}{\partial\theta} \right) + \\ &+ B \left(\frac{1}{r^2} \frac{\partial^4 u_1}{\partial r^2 \partial\theta^2} + \frac{1}{r^4} \frac{\partial^4 u_1}{\partial\theta^4} + \frac{2}{r} \frac{\partial^3 u_1}{\partial r^3} + \frac{2}{r^4} \frac{\partial^2 u_1}{\partial\theta^2} - \frac{6}{r^3} \frac{\partial u_1}{\partial r} + \right. \\ &+ \frac{6}{r^4} u_1 + \frac{1}{r^3} \frac{\partial^4 v_1}{\partial r \partial\theta^3} - \frac{1}{r} \frac{\partial^4 v_1}{\partial\theta \partial r^3} - \frac{3}{r^3} \frac{\partial^2 v_1}{\partial\theta \partial r} + \left. \left. \frac{5}{r^4} \frac{\partial v_1}{\partial\theta} \right) \right\} + \\ &+ v_3 \left\{ \frac{4B}{r} \left(- \frac{3}{r^2} \frac{\partial^2 v_1}{\partial r \partial\theta} - \frac{6}{r^2} \frac{\partial u_1}{\partial r^3} + \frac{1}{r} \frac{\partial^3 v_1}{\partial r^2 \partial\theta} + \frac{\partial^3 u_1}{\partial r^3} \right) \right\} \end{aligned} \quad (12)$$

and

$$\frac{\partial (r u_1)}{\partial r} + \frac{\partial v_1}{\partial\theta} = 0 \quad (13)$$

The modified boundary conditions are

$$\left.\begin{array}{ll}(i)\ u_1 = 0 & \text{at } r = R_1 \\ (ii)\ v_1 = 0 & \text{at } r = R_1 \\ (iii)\ u_1 = -\Omega_2\, d \sin\theta & \text{at } r = R_2 \\ (iv)\ v_1 = -d\left(A - \dfrac{B}{R_2^{\,2}}\right)\cos\theta & \text{at } r = R_2\end{array}\right\} \tag{14}$$

Substituting (9) into boundary condition (*iii*) of (8), we get

$$\epsilon u_1 (R_1 + h, \theta) + \epsilon^2 u_2 (R_1 + h, \theta) + \ldots = -\Omega_2 \epsilon l \sin\theta$$

Now it is noted with the help of (4) that the perturbation parameter ϵ also appears implicitly in the argument of the function $u_1, u_2 \ldots$ Hence it is not possible to equate coefficients of ϵ and ϵ^2 directly. This difficulty is resolved by expanding u_1 and u_2 in Taylor series about r equal to R_2. This expansion will exhibit explicitly the dependence on ϵ of u_1 and u_2 and thus making it possible to equate coefficients of ϵ and ϵ^2 directly. In the same manner the boundary condition (*iv*) of (8) is handled. This procedure is referred to as 'transfer of boundary conditions' by Van Dyke[8]. The boundary conditions are shifted from the perturbed boundary to the basic boundary which corresponds to $\epsilon = 0$.

First Order Solution for any Gap

Equation (12) will have a solution of the type

$$u_1 = U(r)\, i e^{i\theta}, \quad v_1 = -\frac{d(rU)}{dr} e^{i\theta}. \tag{15}$$

Substituting (15) into (12) gives the following ordinary differential equation in U

$$U^{IV} + \frac{6}{r} U''' + \frac{3}{r^2} U'' - \frac{3}{r^3} U' - \frac{\Omega_{0i}}{\nu_1 + \Omega_{0i}\, v_2}\left(U'' + \frac{3}{r} U'\right) = 0. \tag{16}$$

Again, since Ω_0 is a real function, U is supposed to be a real function. We further assume that both the cylinders are rotating with the same angular velocity and in the same direction. Hence from (11) we have $B=0$. In (16) both the real and imaginary parts must separately be equal to zero. Hence, the two particular solutions that will render the real and imaginary parts of (16) to be zero are $1/r^2$ and a constant. Since (16) is a fourth order equation two other solutions are further required. When two solutions are known, the method of reduction of order is employed to determine the remaining solutions which are integrals of Bessel functions. The arbitrary constants of integration arising are evaluated from the boundary conditions (14).

Thus the first order solutions of the basic flow are

$$u_1(r, \theta) = \text{Re}\left\{\left[C_1 I_1(r) + D_1 I_2(r)\right] i e^{i\theta}\right\} \tag{17}$$

$$v_1(r, \theta) = \text{Re}\left\{-\left[C_1 I_3(r) + D_1 I_4(r)\right] e^{i\theta}\right\} \tag{18}$$

where Re denotes the real part and

$$\left.\begin{aligned} I_1(r) &= \int_{R_1}^{r} J_1(\omega)\, dr - \frac{1}{r^2} \int_{R_1}^{r} r^2 J_1(\omega)\, dr \\ I_2(r) &= \int_{R_1}^{r} Y_1(\omega)\, dr - \frac{1}{r^2} \int_{R_1}^{r} r^2 Y_1(\omega)\, dr \\ I_3(r) &= \int_{R_1}^{r} J_1(\omega)\, dr + \frac{1}{r^2} \int_{R_1}^{r} r^2 J_1(\omega)\, dr \\ I_4(r) &= \int_{R_1}^{r} J_1(\omega)\, dr + \frac{1}{r^2} \int_{R_1}^{r} r^2 Y_1(\omega)\, dr \end{aligned}\right\} \qquad (19)$$

where

$$\omega = \left(- \frac{i\Omega_0}{v_1 + i v_2 \Omega_0} \right)^{1/2} r \qquad (20)$$

$$\left.\begin{aligned} C_1 &= \frac{\Omega_2\, dI_4(R_2) - dA I_2(R_2)}{I_1(R_2)\, I_4(R_2) - I_2(R_2)\, I_3(R_2)} \\ D_1 &= \frac{dA I_1(R_2) - \Omega_2\, dI_3(R_2)}{I_1(R_2)\, I_4(R_2) - I_2(R_2)\, I_3(R_2)} \end{aligned}\right\} \qquad (21)$$

$J_1(\omega)$ and $Y_1(\omega)$ are the Bessel functions of the first and second kind respectively with an imaginary argument ω and of order 1.

The solutions given by (17) and (18) are valid for any gap and are often referred to as the finite gap solution. If we take $v_2 = 0$ in the argument ω the solutions are in full agreement with Urban[5].

SOLUTIONS BY ASYMPTOTIC EXPANSIONS

We will evaluate the integrals (19) arising in the solution under the assumption that both the cylinders are rotating with the same speed i.e. $B = 0$. Hence in this case the argument ω becomes

$$\omega = \left(\frac{Ai}{v_1 + Ai\, v_2} \right)^{1/2} r.$$

This argument of the Bessel functions can be expressed as a function of δ by substituting A from (11) and employing a geometric relations between η and δ given as $\delta = 2(1 - \eta)/(\eta + 1)$, $\eta = (2 - \delta)/(2 + \delta)$.

Thus, we have

$$|\omega| = \left[2R \left\{ 4R^2 v_2^2 + (2 - \delta)^2 R_0^4 \delta^2 \right\} \right]^{1/2} \qquad (22)$$

It is observed from (22) that for small δ and for given values of R, v_2 and R_0, $|\omega|$ is large. Therefore, we can employ the following asymptotic expansions which are valid for large values of ω and $|\arg \omega| < \pi$.

From Watson[9], we have

$$J_1(\omega) \sim \left(\frac{2}{\pi\omega} \right)^{1/2} \left[\cos\left(\omega - \frac{3}{4}\pi \right) \sum_{m=0}^{\infty} \frac{(-)^m (1, 2m)}{(2\omega)^{2m}} - \right.$$

$$\left. - \sin\left(\omega - \frac{3}{4}\pi \right) \sum_{m=0}^{\infty} \frac{(-)^m (1, 2m+1)}{(2\omega)^{2m+1}} \right]. \qquad (23)$$

$$Y_1(\omega) \sim \left(\frac{2}{\pi\omega}\right)^{1/2} \left[\sin\left(\omega - \frac{3}{4}\pi\right) \sum_{m=0}^{\infty} \frac{(-)^m (1, 2m)}{(2\omega)^{2m}} + \right.$$

$$\left. + \cos\left(\omega - \frac{3}{4}\pi\right) \sum_{m=0}^{\infty} \frac{(-)^m (1, 2m+1)}{(2\omega)^{2m+1}} \right] \tag{24}$$

where

$$\sum_{m=0}^{\infty} \frac{(-)^m (1, 2m)}{(2\omega)^{2m}} = 1 - \frac{(4-1^2)(4-3^2)}{\underline{|2} \, (8\omega)^2} + \ldots$$

and

$$\sum_{m=0}^{\infty} \frac{(-)^m (1, 2m+1)}{(2\omega)^{2m+1}} = \frac{4-1^2}{(8\omega)} - \frac{(4-1^2)(4-3^2)(4-5^2)}{\underline{|3} \, (8\omega)^3} + \ldots$$

Substituting (21) in (18) and using (11) and the geometric relation between η and δ gives the following expression

$$\tilde{v}_1(r, \theta) = \frac{v_1(r,\theta)}{R_1 \Omega_1} \text{ Re} \left\{ \left[\frac{4\delta}{2(2-\delta)\{I_1(R_2) I_4(R_2) - I_2(R_2) I_3(R_2)\}} \right. \right. \cdot$$

$$\left. \left. \cdot \left\{ (I_3(R_2) I_4(r) - I_4(R_2) I_3(r)) - (I_1(R_2) I_4(r) - I_2(R_2) I_3(r)) \right\} \right] e^{i\theta} \right\} \tag{25}$$

Now with the help of (19), (23), (24) and (25) can be evaluated which gives first order transverse velocity.

First Order Radial Velocity

The analysis of the first order radial velocity is parallel to that of the first order transverse velocity. Here we introduce a characteristic velocity of $d\Omega_1$.

We get

$$\tilde{u}_1(r, \theta) = \frac{u_1(r, \theta)}{d\Omega_1} \tag{26}$$

Therefore the first order radial velocity component is

$$\tilde{u}_1(r, \theta) = \text{Re} \left\{ \left[\frac{1}{I_1(R_2) I_4(R_2) - I_2(R_2) I_3(R_2)} \cdot \left\{ (I_4(R_2) I_1(r) - \right. \right. \right.$$

$$\left. \left. \left. - I_2(R_2) I_1(r)) + (I_1(R_2) I_2(r) - I_3(R_2) I_2(r)) \right\} \right] ie^{i\theta} \right\} \tag{27}$$

RESULTS AND DISCUSSION

In figures the first order transverse velocity are plotted against x a new dimensionless independent variable defined as

$$r = R_1 + \frac{h}{2} + hx \text{ (from Fig. 1)} \tag{28}$$

where $h \sim d + e\cos\theta$, if $\epsilon << 1$ and/or $\delta << 1$. (29)

Now x has the range $-\dfrac{1}{2} < x < +\dfrac{1}{2}$. A point in the fluid domin can now be prescribed by x and θ. With the help of (28) and (29) we have

$$r \approx R_0 + x\,(d + e\cos\theta) + \frac{\epsilon}{2}\cos\theta$$

$$\frac{e}{R_0} = \frac{e}{d}\cdot\frac{d}{R_0} = \epsilon\,\delta$$

$$r \approx R_0\left[\,1 + x\delta\,(1 + \epsilon\cos\theta) + \frac{\epsilon\delta}{2}\cos\theta\,\right].$$

We have the transverse velocity from the following equation

$$\tilde{v}_\theta(x, \theta) = \tilde{v}_0(x) + \epsilon\tilde{v}_1(x, \theta) + \ldots \tag{30}$$

where $\tilde{v}_0(x)$ and $\tilde{v}_1(x, \theta)$ are given from (10) and (25) after transfoiming r into x.

The $\tilde{v}_\theta(x, \theta)$ is plotted when the both cylinders are rotating with the same speed (that is $\mu=1$). The profiles are shown at $\theta=0°$ and $\theta=180°$ and for eccentricity ratio 0·1 and 0·2. The values of δ and v_2 are taken as 0·05 and —0·1 respectively. It is noted that $x=-\frac{1}{2}$ corresponds to the inner boundary and $x=\frac{1}{2}$ corresponds to the outer boundary.

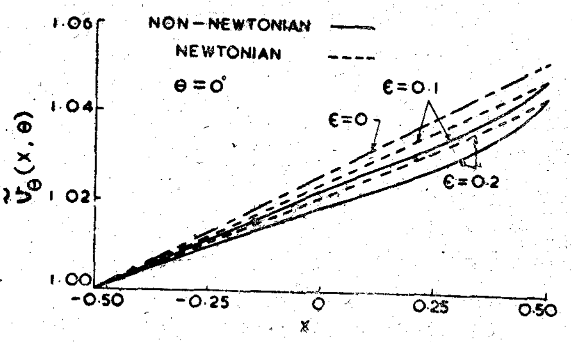

Fig. 2—Transverse velocity profile.

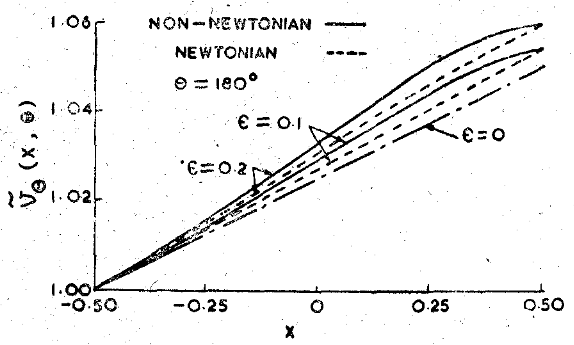

Fig. 3—Transverse velocity profile.

Figure 2 depicts the transverse velocity profiles at the location $\theta=0°$. Here we find that the non-Newtonian fluid parameter reduces the magnitude of the transverse velocity in comparison to Newtonian case. We further note that the transverse velocity in both the cases is less than the concentric case. Figure 3 reveals the velocity profiles at the location $\theta=180°$. It is interesting to note that the transverse velocity for the non-Newtonian fluid is greater in comparison to Newtonian case. This phenomenon is due to the gap between the boundaries which is reduced. Here we note that the boundary condition at the outer boundary is not satisfied this is due to the approximate method of handling the outer boundary condition. We also infer that the difference between the non-Newtonian and Newtonian velocity profiles decreases with the decrease of eccentricity ratio.

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