STRESS DISTRIBUTION IN A THIN CIRCULAR ELASTIC PLATE CONTAINING A GRIFFITH CRACK EMBEDDED IN AN INFINITE ELASTIC MEDIUM

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In this paper the problem of determining stress distribution in the neighbourhood of a Griffith crack, opened by a thin symmetric wedge, in a thin circular elastic plate embedded in an infinite elastic medium containing a circular hole of same radius as that of the plate is considered. The elastic properties of the plate and infinite elastic medium are different. Using the known solutions of elastic equilibrium equations, the mixed boundary value problem is reduced to a set of triple integral equations involving cosine kernels. Finite Hilbert transform is used to reduce this set to a Fredholm integral equation of the second kind, this integral equation is solved by well-known iterative procedure. Stress intensity factor is found for the case of a rectangular wedge and some numerical calculations have also been done. Results arrived at are shown graphically.

Markuzon[1] was first to consider distribution of stress in the neighbourhood of a Griffith crack opened by a thin symmetric wedge. He extended the complex variable technique of Muskhilishvili to solve this problem. Smith[2] solved the same problem by the theory of Fourier transforms and dual series relations. Tweed[3] has solved the same problem by an alternative approach of Lowengrub and Srivastava[4,5]. The authors[6] have determined the stress distribution in a circular plate containing a Griffith crack opened by a symmetric wedge.

In this paper, we consider the problem of determining stress distribution in a thin circular plate containing a Griffith crack opened by a thin symmetric wedge and the plate is embedded in the infinite elastic medium containing a circular hole of same radius as that of the plate. The elastic properties of the plate and the medium are different. The corresponding mixed boundary value problem is reduced to a set of triple integral equations involving cosine kernels. Following[4,5], the set of triple integral equations is reduced to a Fredholm integral equation of the second kind, which is solved by the well-known iterative method. Expressions for the quantities of physical interest are derived from the iterative solution.

FORMULATION OF THE PROBLEM

We consider a thin circular plate of radius $\rho > 1$ and the material of the plate is supposed to be homogeneous and isotropic. This plate contains a symmetrical Griffith crack on a diameter. This circular plate is embedded in an infinite elastic medium containing a circular hole of the same radius as that of the plate. The elastic properties of the plate and the infinite elastic medium are different. In plane polar co-ordinates (r, θ), the circular disc is supposed to occupy the region $0 \leqslant r \leqslant \rho$, $\theta = 0, \pi$ (hereafter known as region 1) while the infinite medium with circular hole is supposed to occupy the region $\rho < r < \infty$, $\theta = 0, \pi$ (call it region 2). The non vanishing components of displacement and stresses in the two regions are denoted by $(u_\theta^{(1)}, u_r^{(1)}, \sigma_\theta^{(1)}, \sigma_r^{(1)}, \sigma_{r\theta}^{(1)})$ and $(u_\theta^{(2)}, u_r^{(2)}, \sigma_\theta^{(2)}, \sigma_r^{(2)}, \sigma_{r\theta}^{(2)})$ respectively. The crack is supposed to occupy the region $0 \leqslant r \leqslant 1$, $\theta = 0, \pi$. It is opened by a thin symmetric wedge which makes contact with a part of the crack surface defined by $0 \leqslant r \leqslant k$ $(k < 1)$, $\theta = 0, \pi$, where k is unknown and depends on shape of the wedge. The remaining part of the crack surface is opened by a known pressure $p(r)$, $k < r < 1$, $\theta = 0, \pi$. The boundary conditions on $\theta = 0$ are

$$\sigma_\theta^{(1)}(r, 0) = -p(r) \quad ; \quad k < r < 1, \tag{1}$$

$$u_\theta^{(1)}(r, 0) = \frac{\pi f(r)}{2\mu_1} \quad ; \quad 0 \leqslant r < k, \tag{2}$$

$$u_\theta^{(1)}(r, 0) = 0 \quad ; \quad 1 \leqslant r \leqslant \rho, \tag{3}$$

$$\sigma_\theta^{(1)}(r, 0) = 0 \quad ; \quad 0 \leqslant r \leqslant \rho. \tag{4}$$

In the region of bond, the continuity conditions are

$$\sigma_r^{(1)}(\rho, \theta) = \sigma_r^{(2)}(\rho, \theta) \quad , \quad 0 \leqslant \theta \leqslant 2\pi$$

$$\sigma_{r\theta}^{(1)}(\rho, \theta) = \sigma_{r\theta}^{(2)}(\rho, \theta) \quad , \quad 0 \leqslant \theta \leqslant 2\pi$$

$$u_\theta^{(1)}(\rho,\theta) = u_\theta^{(2)}(\rho,\theta) \quad , \quad 0 \leqslant \theta \leqslant 2\pi$$
$$u_r^{(1)}(\rho,\theta) = u_r^{(2)}(\rho,\theta) \quad , \quad 0 \leqslant \theta \leqslant 2\pi \tag{5}$$

The appropriate expressions for components of displacement and stress for region 1 are quoted from Srivastava, Kumar & Jha[6]

$$\sigma_\theta^{(1)}(r,\theta) = -\int_0^\infty \xi\, A(\xi)\, e^{-\xi r \sin\theta} \Big[\cos(\xi r\cos\theta) + \xi r \sin\theta \cos(2\theta - \xi r\cos\theta)\Big] d\xi + $$
$$+ \sum_{n=0}^{\infty} \Big[(n+1)(n+2)\, b_n r^n + n(n-1)\, a_n r^{n-2}\Big] \cos n\theta \tag{6}$$

$$\sigma_r^{(1)}(r,\theta) = -\int_0^\infty \xi\, A(\xi)\, e^{-\xi r \sin\theta} \Big[\cos(\xi r\cos\theta) - \xi r \sin\theta \cos(2\theta + \xi r\cos\theta)\Big] d\xi - $$
$$- \sum_{n=0}^{\infty} \Big[n(n-1)\, a_n r^{n-2} + (n+1)(n-2)\, b_n r^n\Big] \cos n\theta \tag{7}$$

$$\tau_{r\theta}^{(1)}(r,\theta) = -\int_0^\infty \xi\, A(\xi)\, e^{-\xi r \sin\theta}\, \xi r \sin\theta \sin(2\theta - \xi r\cos\theta)\, d\xi + $$
$$+ \sum_{n=0}^{\infty} \Big[n(n-1)\, a_n r^{n-2} + n(n+1)\, b_n r^n\Big] \sin n\theta \tag{8}$$

$$2\mu_1 u_\theta^{(1)}(r,\theta) = \int_0^\infty A(\xi)\, e^{-\xi r\sin\theta} \Big[(2 - 2\eta_1 + \xi r\sin\theta)\cos\theta\cos(\xi r\cos\theta) + $$
$$+ (1 - 2\eta_1 - \xi r\sin\theta)\sin\theta\sin(\xi r\cos\theta)\Big] d\xi + \sum_{n=0}^{\infty}\Big[n\, a_n r^{n-1} + $$
$$+ (n - 4\eta_1 + 4)\, b_n r^{n+1}\Big] \sin n\theta \tag{9}$$

$$2\mu_1 u_r^{(1)}(r,\theta) = \int_0^\infty A(\xi)\, e^{-\xi r\sin\theta} \Big[(2 - 2\eta_1 + \xi r\sin\theta)\sin\theta\cos(\xi r\cos\theta) - (1 - 2\eta_1 - $$
$$- \xi r\sin\theta)\cos\theta\sin(\xi r\cos\theta)\Big] d\xi - \sum_{n=0}^{\infty}\Big[n\, a_n r^{n-1} + $$
$$+ (n + 4\eta_1 - 2)\, b_n r^{n+1}\Big] \cos n\theta \tag{10}$$

The expressions for components of displacement and stress for region 2 are quoted from Srivastava & Kumar[7]

$$\sigma_\theta^{(2)}(r,\theta) = -c'_0 r^{-2} + \sum_{n=1}^{\infty}\Big[n(n+1)\, c'_n r^{-n-2} + (n-1)(n-2)\, d'_n r^{-n}\Big] \cos n\theta \tag{11}$$

$$\sigma_r^{(2)}(r,\theta) = c'_0 r^{-2} - \sum_{n=1}^{\infty} \left[n(n+1) c'_n r^{-n-2} + (n-1)(n+2) d'_n r^{-n} \right] \cos n\theta \tag{12}$$

$$\sigma_{r\theta}^{2}(r,\theta) = - \sum_{n=1}^{\infty} \left[n(n+1) c'_n r^{-n-2} + n(n-1) d'_n r^{-n} \right] \sin n\theta \tag{13}$$

$$2\mu_2 u_\theta^{(2)}(r,\theta) = c'_1 r^{-2} \sin\theta + \sum_{n=2}^{\infty} \left[n c'_n r^{-n-1} + (n + 4\eta_2 - 4) d'_n r^{-n+1} \right] \sin n\theta \tag{14}$$

$$2\mu_2 u_r^{(2)}(r,\theta) = -c'_0 r^{-1} + c'_1 r^{-2} \cos\theta + \sum_{n=2}^{\infty} \left[n c'_n r^{-n-1} + (n + 2 - 4\eta_2) d'_n r^{-n+1} \right] \cos n\theta \tag{15}$$

where μ_1, η_1 and μ_2, η_2 are modulus of rigidity and Poission's ratio for the two regions.

TRIPLE INTEGRAL EQUATIONS AND THEIR SOLUTION

The boundary condition (4) is satisfied automatically provided the coefficients a_{2n+1} and b_{2n+1} are zero. The conditions (1) to (3) lead to the following set of triple integral equations

$$\int_0^\infty \xi A(\xi) \cos \xi r \, d\xi = F(r) \; ; \; k < r < 1 \tag{16}$$

$$\int_0^\infty A(\xi) \cos \xi r \, d\xi = \frac{\pi f(r)}{2(1-\eta_1)} \; ; \; 0 < r < k \tag{17}$$

$$\int_0^\infty A(\xi) \cos \xi r \, d\xi = 0 \quad ; \; 1 \leqslant r \leqslant \rho \tag{18}$$

where

$$F(r) = p(r) + \sum_{n=0}^{\infty} (2n+1)(2n+2)(a_{2n+2} + b_{2n}) r^{2n} \tag{19}$$

Following Srivastava and Lowengrub[5], we seek a solution of the above set in the form

$$A(\xi) = \xi^{-1} \int_0^1 h(t^2) \sin \xi t \, dt \tag{20}$$

where $h(t^2)$ is an unknown function with the property

$$h(t^2) = \left\{ \begin{array}{ll} m(t^2) \; ; & 0 < t < k, \\ g(t^2) \; ; & k < t < 1 \end{array} \right\} \tag{21}$$

The equations (17) and (18) are satisfied provided

$$\int_r^k m(t^2)\,dt + \int_k^1 g(t^2)\,dt = \frac{f(r)}{1-\eta_1}, \quad 0 \leqslant \quad k$$

i.e. if we choose

$$m(x^2) = -\frac{f'(x)}{1-\eta_1} \tag{22}$$

and

$$\int_k^1 g(x^2)\,dx = \frac{f(k)}{1-\eta_1} \tag{23}$$

The equation (16) is satisfied if $g(t^2)$ is the solution of the integral equation

$$\frac{2}{\pi}\int_k^1 \frac{t\,g(t^2)\,dt}{(t^2-r^2)} = \frac{2}{\pi}\,F(r) + \frac{2}{\pi(1-\eta_1)}\int_0^k \frac{t\,f'(t)}{t^2-r^2}\,dt, \quad k < t < 1 \tag{24}$$

The solution of the above integral equation as given by Srivastava and Lowengrub[5], is

$$g(t^2) = -\frac{2}{\pi}\,H\left[F(y) + \frac{1}{(1-\eta_1)}\int_0^k \frac{t\,f'(t)}{t^2-y^2}\,dt\right] + s/T(t^2) \tag{25}$$

where H is the finite Hilbert operator defined by

$$H[p(y)] = \frac{2}{\pi}\left(\frac{t^2-k^2}{1-t^2}\right)^{\frac{1}{2}}\int_k^1 \left(\frac{1-y^2}{y^2-t^2}\right)^{\frac{1}{2}} \frac{y\,p(y)}{y^2-t^2}\,dy \tag{26}$$

The arbitrary constant s in (25) is determined from the condition (23) and $T(t^2) = [(t^2-k^2)(1-t^2)]^{\frac{1}{2}}$.

We shall now utilize boundary conditions (5) to determine the unknown coefficients[6] a_{2n}, b_{2n}, c'_{2n} and d'_{2n}.

$$b_0 = \frac{\gamma}{(\alpha+\gamma)\,\rho^2}\int_0^1 x\,h(x^2)\,dx; \tag{27}$$

$$c'_0 = \frac{(\gamma-\alpha)}{(\alpha+\gamma)}\int_0^1 x\,h(x^2)\,dx, \quad c'_1 = 0 \tag{28}$$

Also for $n \geqslant 1$

$$2n(2n-1)\,a_{2n}\,\rho^{2n-1} = \int_0^1 h(x^2)\left[\frac{1}{2}\left\{\frac{\gamma}{\alpha}(4n^2-1) + \frac{(\beta-\alpha+\gamma)}{\beta}\right\}(x/\rho)^{2n-1} - \frac{(n+1)(2n-1)\gamma}{\alpha}(x/\rho)^{2n+1}\right]dx \tag{29}$$

$$2\,(2n+1)\,b_{2n}\,\rho^{n+1} = \int_0^1 h\,(x^2)\left[\frac{2\,(n+1)\,\gamma}{\alpha}\,(x/\rho)^{2n+1} - \frac{(2n+1)\gamma}{\alpha}\,(x/\rho)^{2n-1}\right]dx \quad (30)$$

$$4n\,(2n-1)\,c'_{2n}\,\rho^{-2n-1} = \int_0^1 h\,(x^2)\left[\frac{(2n+1)\,(\alpha-\gamma)\,(\alpha-\beta)}{\alpha\beta}\,(x/\rho)^{2n-1} + \right.$$

$$\left. + \frac{2(n+1)\,(\alpha-\gamma)}{\alpha}\,(x/\rho)^{2n+1}\right]dx \quad (31)$$

$$2\,(4n^2-1)\,d'_{2n}\,\rho^{-2n+1} = -\int_0^1 h\,(x^2)\,\frac{(\alpha-\gamma)\,(2n+1)}{\beta}\,(x/\rho)^{2n-1}\,dx \quad (32)$$

$$c'_{2n+1} = 0 = d'_{n+1}\ ,$$

where

$$\mu = \mu_1/\mu_2\ ;\ k_i = (3-4\eta_i)\ ;\ i = 1, 2\ ;$$

$$\alpha = (k_1+\mu)\ ;\ \beta = (k_2\,\mu+1)\ ;\ \gamma = \mu - 1.$$

Now the expression (19) takes the following form

$$F(r) = p(r) + \int_0^1 x\,h\,(x^2)\,N\,(x, r)\,dx$$

where

$$N\,(x, r) = \frac{A_0}{\rho^2} + A_1\,x^2\,\rho^{-4} - x^{-2}\sum_{n=1}^{\infty}\left(\frac{xr}{\rho^2}\right)^{2n}\left[(2n+1)\,(n+1)\,(\gamma/\alpha) - \right.$$

$$\left. - \left\{(8n^2+16n+7)\,\frac{\gamma}{2\alpha} + \frac{(\beta-\alpha+\gamma)}{2\beta}\right\}(x/\rho)^2 + \frac{(n+2)\,(n+1)\,\gamma}{\alpha}\,(x/\rho)^4\right]. \quad (33)$$

When $\rho > 1$, $N\,(x, r)$ can be approximated to

$$N\,(x, r) = A_0\,\rho^{-2} + (A_1\,x^{-2} + A_2\,r^2)\,\rho^{-4} + A_3\,x^2\,r^2\,\rho^{-6} + O\,(\rho^{-8}) \quad (34)$$

where

$$A_0 = \frac{1}{2}\left\{\frac{3\gamma}{\alpha} + \frac{(\beta-\alpha+\gamma)}{\beta} + \frac{4\gamma}{\alpha+\gamma}\right\}\ ;$$

$$A_1 = -\frac{2\gamma}{\alpha}\ ;\quad A_2 = 3A_1\ ;\ A_3 = \frac{1}{2}\left\{\frac{31\gamma}{\alpha} + (\beta-\alpha+\gamma/\beta)\right\}.$$

The expression (25) now takes the following form

$$g\,(t^2) = -\frac{2}{\pi}\,H\Big[\,S\,(y)\,\Big] - \frac{2}{\pi}\,H\left[\int_k^1 x\,g\,(x^2)\,N\,(x, y)\,dx\right] + \frac{s}{T\,(t^2)} \quad (35)$$

where

$$S(y) = p(y) + \frac{1}{(1-\eta_1)} \left[\int_0^k \frac{x f'(x)}{x^2 - y^2}\, dx - \int_0^k x f'(x)\, N(x, y)\, dx \right] \tag{36}$$

The arbitrary constant s is determined from the condition (23) and it is given by

$$s = \frac{1}{F} \left[\frac{f(k)}{(1-\eta_1)} + \frac{2}{\pi} \int_k^1 H \left[S(y) \right] dt + \frac{2}{\pi} \int_k^1 H \left[\int_k^1 x g(x^2)\, N(x, y)\, dx \right] dt \right] \tag{37}$$

Eliminating 's' between (35) and (36) we get,

$$g(t^2) = \phi(t^2) + \int_k^1 x\, g(x^2)\, M(x^2, t^2)\, dx \tag{38}$$

where

$$\phi(t^2) = -\frac{2}{\pi} H \left[S(y) \right] + \frac{2}{\pi F T(t^2)} \int_k^1 H \left[S(y) \right] dt$$

and

$$M(x^2, t^2) = \frac{2}{\pi T(t^2)} \left[\left(t^2 - \frac{E}{F} \right) \left(A_0\, \rho^{-2} + A_1\, x^2 \rho^{-4} \right) + \right.$$
$$\left. + \frac{1}{2} \left\{ 2t^4 - (k^2+1)\, t^2 + \delta \right\} (A_2\, \rho^{-4} + A_3\, \rho^{-6}) + O(\rho^{-8}) \right],$$

with

$$\delta = \frac{1}{3} \left[2k^2 - (k^2 + 1)\, E/F \right].$$

The equation (38) is a Fredholm integral equation of the second kind and can be solved iteratively by making a representation of the kind,

$$g(t^2) = \sum_{n=0}^{\infty} g_n(t^2)\, \rho^{-2n} \tag{39}$$

PHYSICAL CASE OF A RECTANGULAR WEDGE

We consider the case where crack is opened by a rectangular wedge of length $2l$ and height 2ϵ. The part of the crack which is not in contact with the wedge is opened by a constant pressure p_0.

So that

$$f(r) = \epsilon p_0,\ p(r) = p_0 \ \text{and} \ k = l.$$

The equations (38) and (39) will now yield

$$g_0(t^2) = \phi(t^2) = \frac{1}{T(t^2)} \left[\frac{2p_0}{\pi} \left(t^2 - \frac{E}{F} \right) + \frac{\epsilon\, p_0}{F(1-\eta_1)} \right] \tag{40}$$

$$g_1(t^2) = \frac{2(t^2 - E/F)}{\pi T(t^2)} \left[p_0 c_0 + \frac{\epsilon d_0 p_0}{F(1-\eta_1)} \right] \tag{41}$$

$$g_2(t^2) = \frac{2}{\pi T(t^2)} \left[\left\{ p_0 c_1 + \frac{\epsilon d_1 p_0}{F(1-\eta_1)} \right\} (t^2 - E/F) + \right.$$

$$\left. + \left\{ p_0 c_2 + \frac{\epsilon d_2 p_0}{F(1-\eta_1)} \right\} \left\{ 2t^4 - (k^2+1) t^2 + \delta \right\} \right] \tag{42}$$

$$g_3(t^2) = \frac{2}{\pi T(t^2)} \left[\left\{ p_0 c_3 + \frac{\epsilon d_3 p_0}{F(1-\eta_1)} \right\} (t^2 - E/F) \right.$$

$$\left. + \left\{ p_0 c_4 + \frac{\epsilon d_4 p_0}{F(1-\eta_1)} \right\} \left\{ 2t^4 - (k^2+1) t^2 + \delta \right\} \right] \tag{43}$$

and

$$g(t^2) = \frac{2p_0}{T(t^2)\pi} \left[(1 + c_0 \rho^{-2} + c_1 \rho^{-4} + c_3 \rho^{-6}) (t^2 - E/F) + \right.$$

$$\left. + (c_2 \rho^{-4} + c_4 \rho^{-6}) \left\{ 2t^4 - (k^2+1) t^2 + \delta \right\} + O(\rho^{-8}) \right] +$$

$$+ \frac{2\epsilon p_0}{\pi(1-\eta_1) F T(t^2)} \left[\frac{\pi}{2} + (d_0 \rho^{-2} + d_1 \rho^{-4} + d_3 \rho^{-6}) (t^2 - E/F) + \right.$$

$$\left. + (d_2 \rho^{-4} + d_4 \rho^{-6}) \left\{ 2t^4 - (k^2+1) t^2 + \delta \right\} + O(\rho^{-8}) \right] \tag{44}$$

where

$$c_0 = A_0 \left\{ \frac{k^2+1}{2} - \frac{E}{F} \right\} ;$$

$$c_1 = c_0^2 + \frac{A_1}{2} \left\{ \frac{c_0 (k^2+1)}{A_0} + \left(\frac{k^2-1}{1} \right)^2 \right\} ;$$

$$c_2 = \frac{A_2 c_0}{2A_0} ; \; c_3 = c_0 \left\{ c_1 + (c_1 - c_0)^2 \left(1 + \frac{A_2}{3A_1} \right) \right\} ;$$

$$c_4 = 2c_0 c_2 + \frac{A_3 (c_1 - c_0^2)}{A_1} ; \; d_0 = \frac{\pi}{2} A_0 ; \; d_1 = c_0 d_0 \frac{\pi}{4} A_1 (k^2+1) ;$$

$$d_2 = A_2 \pi ;$$

$$d_3 = c_0 d_1 + \frac{A_0 d_2}{4} \left[4\delta + (1-k)(2k^3 - k^2 + k^{+1}) \right] +$$

$$+ \frac{d_0 A_1}{8} \left[3k^3 + 2k^2 + 3 - 4(k^2+1) E/F \right] ;$$

$$d_4 = \frac{c_0 d_0 A_2}{2A_0} + \frac{A_3 \pi}{2} (k^2+1)$$

From (6) and (20) we see that the normal component of stress is given by

$$\sigma_\theta^{(1)}\ (r, 0) = -\int_0^1 \frac{xh\,(x^2)}{x^2 - r^2}\, dx + \int_x^1 xh\,(x^2)\, N\,(x, r)\, dx$$

Using (21) and (22) we get

$$\sigma_\theta^{(1)}\ (r, 0) = \int_k^1 xg\,(x^2)\ \left[N\,(x, r) - \frac{1}{x^2 - r^2} \right] dx \tag{45}$$

Using (44) we get

$$\sigma_\theta^{(1)}\ (r, 0) = \begin{cases} Q_1\,(r^2) + Q\,(r^2) & , \quad 0 < r < k \\ Q_2\,(r^2) + Q\,(r^2) & , \quad r > 1 \end{cases} \tag{46}$$

where

$$\begin{aligned} Q_1\,(r^2) = -\int_k^1 \frac{xg\,(x^2)}{x^2 - r^2}\, dx = p_0 \Bigg[& \left\{ \frac{E/F - r^2}{R\,(r^2)} - 1 \right\} (1 + c_0\,\rho^{-2} + \\ & + c_1\,\rho^{-4} + c_3\,\rho^{-6}) - \left\{ 2r^2 + \frac{2r^4 - (k^2 + 1)\, r^2 + \delta}{R\,(r^2)} \right\} \cdot \\ & \cdot (c_2\,\rho^{-4} + c_4\,\rho^{-6}) + O\,(\rho^{-8}) \Bigg] + \frac{\epsilon p_0}{(1 - \eta_1)\, F} \Bigg[\frac{-\pi}{2R\,(r^2)} + \\ & + \left\{ \frac{E/F - r^2}{R\,(r^2)} - 1 \right\} (d_0\,\rho^{-2} + d_1\,\rho^{-4} + d_3\,\rho^{-6}) - \\ & - \left\{ 2r^2 + \frac{2r^4 - (k^2 + 1)\, r^2 + \delta}{R\,(r^2)} \right\} (d_2\,\rho^{-4} + d_4\,\rho^{-6}) + \\ & + O\,(\rho^{-8}) \Bigg] \ ; \quad 0 < r < k \end{aligned} \tag{47}$$

$$\begin{aligned} Q_2\,(r^2) = -\int_k^1 \frac{xg\,(x^2)}{x^2 - r^2}\, dx = p_0 \Bigg[& \left\{ \frac{r^2 - E/F}{R_1(r^2)} - 1 \right\} (1 + c_0\,\rho^{-2} + \\ & + c_1\,\rho^{-4} + c_3\,\rho^{-6}) + \left\{ \frac{2r^4 - (k^2 + 1)\, r^2 + \delta}{E_1\,(r^2)} - 2r^2 \right\} (c_2\,\rho^{-4} + \\ & + c_4\,\rho^{-6}) + O\,(\rho^{-8}) \Bigg] + \frac{\epsilon p_0}{(1 - \eta_1)\, F} \Bigg[\frac{\pi}{2R_1\,(r^2)} + \left\{ \frac{r^2 - E/F}{R_1\,(r^2)} - \right. \\ & \left. - 1 \right\} (d_0\,\rho^{-2} + d_1\,\rho^{-4} + d_3\,\rho^{-6}) + \left\{ \frac{2r^4 - (k^2 + 1)\, r^2 + \delta}{R_1\,(r^2)} - \right. \\ & \left. - 2r^2 \right\} (d_4\,\rho^{-4} + d_6\,\rho^{-6}) + O\,(\rho^{-8}) \Bigg] , \ r > 1 \end{aligned} \tag{48}$$

$$Q(r^2) = \int_k^1 x g(x^2)\, N(x, r)\, dx$$

and

$$R(r^2) = \left[(k^2 - r^2)(1 - r^2) \right]^{\frac{1}{2}},$$

$$R_1(r^2) = \left[(r^2 - k^2)(r^2 - 1) \right]^{\frac{1}{2}}.$$

The expression for stress intensity factor at the tip of the crack is given by

$$N = \underset{r \to 1}{Lt} + (r-1)^{\frac{1}{2}} \left[\sigma_\theta^{(1)}(r, 0) \right], \quad r > 1 \tag{49}$$

which reduces to the following using (46)

$$N = \frac{p_0}{\{2(1-k^2)\}^{\frac{1}{2}}} \Big[(1 - E/F)(1 + c_0 \rho^{-2} + c_1 \rho^{-4} + c_3 \rho^{-6}) +$$
$$+ \left\{ (1-k^2) + \delta \right\} (c_2 \rho^{-4} + c_4 \rho^{-6} (+ O(\rho^{-8})] +$$
$$+ \frac{\epsilon p_0}{\{2(1-k^2)\}^{\frac{1}{2}} (1-\eta_1) F} \left[\frac{\pi}{2} + (1 - E/F)(d_0 \rho^{-2} + d_1 \rho^{-4} + d_3 \rho^{-6}) + \right.$$
$$\left. + \left\{ (1-k^2) + \delta \right\} (d_2 \rho^{-4} + d_4 \rho^{-6}) + O(\rho^{-8}) \right]. \tag{50}$$

The stress intensity factor is calculated numerically for various values of ρ (the radius of inner plate). Fig. 1 shows the variation of stress intensity factor with ρ for the case where the inner plate is made of steel while weaker materials form the outer region. Fig. 2 shows the variation of stress intensity factor for the case where outer plate is of steel while weaker materials are taken to form inner plate

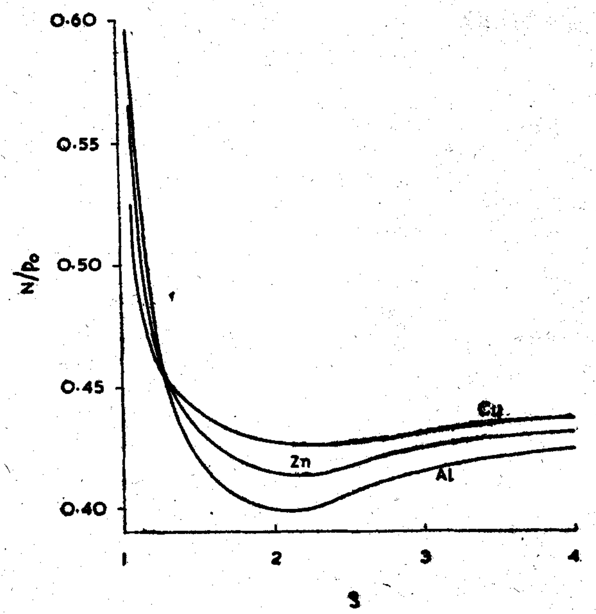

Fig. 1—Stress intensity factor with ρ.

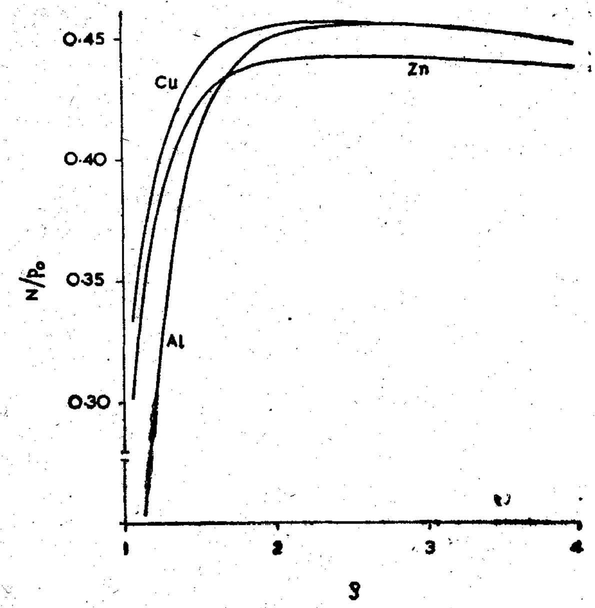

Fig. 2—Stress intensity factor with ρ.

It is interesting to note that if we let $\rho \to \infty$, $\eta_1 = \eta_2 = \eta$ and $\mu_1 = \mu_2 = \mu$, the stress intensity factor N reduces to

$$N_\infty = \frac{p_0 (F - E)}{F \{ 2(1-k^2) \}^{\frac{1}{2}}} + \frac{\epsilon \pi p_0}{2 \{ 2(1-k^2) \}^{\frac{1}{2}} (1-\eta) F}$$

which completely agrees with the expression obtained by Tweed[3] (except the change in notation).

REFERENCES

1. MARKUZON, I.A., *Prikl. Math. Mech.*, **25** (1961), 359.
2. SMITH, E., *Int. J. Engg. Sci.*, **4** (1966), 671.
3. TWEED, J., J. *Elasticity*, **1** (1971), 29.
4. LOWENGRUB M., & SRIVASTAVA, K.N., *Int. Eng. Sci.*, **6** (1968), 359.
5. LOWENGRUB M., & SRIVASTAVA, K.N., *Roc. R. Soci. Edinb.*, **68** (1969), 309.
6. SRIVASTAVA, K.N., KUMAR, M., & JHA, A.C., *J. Japan Soci. for Strength and Fract. of Materials*, **10** (1975), 1.
7. SRIVASTAVA, K.N., & KUMAR, M., *Ind. J. Pure & Appl. Mathematics*, **3** (1972), 830.