ON COMPUTATION OF PRECIPITABLE WATER FROM UPPER AIR DATA

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An analysis has been undertaken to ascertain the accuracy of the conventional summation method for computing precipitable water in the atmosphere from upper air data. Errors have been found to depend on lapse parameter, α which is known to vary with latitude and season.

Of late, our military planners have been taking growing interest in climatology, particularly, in respect of high altitude and desert regions of this country. Atmospheric humidity is one of the important climatic parameters, and the water vapour content of the local atmosphere is one of the constituents (of atmospheric turbidity that determines the intensity of) incoming solar radiation[1], diffused sky radiation, and their spatial distribution[2], as well as the radiation balance at the earth-air interface[3]. Apart from its obvious application in agricultural meteorology, a quantitative knowledge of the same is useful in the assessment of performance characteristics of solar energy devices, and estimation of solar energy resources.

The most accurate and reliable method of estimating 'precipitable water' in the entire vertical atmosphere, is the sophisticated spectroscopic method[2] which, however, is not suitable for routine practical measurements. A method was developed by the present authors[4] for estimation of precipitable water from the value of surface humidity. This method, however, requires the knowledge of the lapse parameter, characteristic of the vertical moisture profile, which has been found to vary from place to place and in different seasons. At stations provided with radio-sonde equipment, the precipitable water upto the highest level of observation is computed from upper-air data using the conventional summation method[5] which assumes, as a first approximation, a linear relationship between specific humidity and atmospheric pressure between any two levels. We have shown[4] that this assumption is not justified and that the water vapour pressure 'e' varies as p^{α} where 'p' is the total pressure and α the lapse parameter. For six Indian stations studied by us, α has been found to vary from 2·90 to 3·83. It will, therefore, be pertinent to enquire about the accuracy of the conventional method of computation. The present paper makes an attempt to analyse this problem.

CONVENTIONAL SUMMATION METHOD

The mass of water vapour per unit mass of moist air, or the specific humidity, q, varies with total pressure, p, so that the precipitable water, W_p from ground level (p_0) upto any level p, will be given by[5].

$$W_p = \frac{1}{g} \int_{p_0}^{p} q\,dp \qquad (1)$$

where g, the acceleration due to gravity, may be regarded as a constant over the range involved.

For the purpose of computation, the entire range is divided into n layers, separated by pressure levels $p_0, p_1, p_2, \ldots\ldots, p_r, \ldots\ldots p_{n-1}, p_n$, the last being the highest level of observation. Equation (1) is thus replaced by the summation formula,

$$W_{cn} = \frac{1}{g} \sum_{r=1}^{n} \left\{ \frac{1}{2} \left(q_{r-1} + q_r \right) \left(p_{r-1} + p_r \right) \right\} \qquad (2)$$

The expression to be summed up for the n layers is nothing but the product of the mean specific humidity of the rth layer and the pressure drop across it.

Accuracy of the conventional method

Assuming that the actual vertical moisture profile is characterised by a lapse parameter, α, the vapour pressures, e_0 and e, at pressure levels p_0 and p respectively, will be related by the equation[4],

$$e/e_0 = (p/p_0)^{\alpha} \tag{3}$$

The specific humidity, q, on the other hand, is given by[6]

$$q = \frac{0 \cdot 622e}{p - 0 \cdot 378e}$$

Since e/p is maximum of ground level and never exceeds $0 \cdot 05$ even under extreme conditions, the above may be approximated by

$$q \doteq 0 \cdot 622e/p \tag{4}$$

without any significant error[4]. With the help of equation (3), equation (4) becomes

$$q = 0 \cdot 622 (e_0/p_0) (p/p_0)^{\alpha - 1} \tag{5}$$

Substitution of this expression for q in equation (2) yields

$$W_{cn} = \frac{0 \cdot 622e_0/p_0}{2g} \sum_{r=1}^{n} \left\{ (p_{r-1}/p_0)^{\alpha-1} + (p_r/p_0)^{\alpha-1} \right\} (p_{r-1} - p_r) \tag{6}$$

If the layers are taken at equal pressure intervals so that $p_{r-1} - p_r = (p_0 - p_n)/n = \triangle p$, equation (6) becomes

$$W_{cn} = \frac{0 \cdot 622 \, (e_0/p_0) \triangle p}{2g} \sum_{r=1}^{n} \left\{ (p_{r-1}/p_0)^{\alpha-1} + (p_r/p_0)^{\alpha-1} \right\} \tag{7}$$

The accurate value, W_n, of the precipitable water from p_0 to p_n can be obtained by performing the integration in equation (1) after substituting the expression for q from equation (5). The result is,

$$W_n = \frac{0 \cdot 622e_0}{g\alpha} \left[1 - (p_n/p_0)^{\alpha} \right] \tag{8}$$

From equation (7) and (8) we have

$$W_{cn}/W_n = \frac{\alpha \triangle p/p_0}{2 \, [1 - (p_n/p_0)^{\alpha}]} \sum_{r=1}^{n} \left\{ (p_{r-1}/p_0)^{\alpha-1} + (p_r/p_0)^{\alpha-1} \right\} \tag{9}$$

Introducing dimensionless pressure $\phi = p/p_0$ we have

$$\phi_r = p_r/p_0, \; \phi_0 = 1, \; \triangle\phi = \triangle p/p_0 = (1 - \phi_n)/n \tag{10}$$

with these values, equation (9) takes the form

$$W_{cn}/W_n = \frac{\alpha \, (1 - \phi_n)}{2n \, (1 - \phi_n^{\alpha})} \sum (\phi_{r-1}^{\alpha-1} + \phi_r^{\alpha-1}),$$

which after some simplification, leads to

$$W_{cn}/W_n = \frac{\alpha \, (1 - \phi_n)}{n \, (1 - \phi_n^{\alpha})} \left[\frac{1}{2} (1 - \phi_n^{\alpha-1}) + \sum_{r=1}^{n} \phi_r^{\alpha-1} \right] \tag{11}$$

The percentage error, ξ, due to the conventional method will be given by

$$\xi = (W_{cn}/W_n - 1) \times 100\% \tag{12}$$

Since $\phi_r = 1 + (1 - \phi_n)\, r/n$, it may be seen from equations (11) and (12) that ξ is a function of α, ϕ_n and n (and, therefore, $\triangle \phi$). Equation (11) is not suitable for routine computational work, since it involves the sum of a complicated series. It was, therefore, considered necessary to evolve a suitable empirical formula giving percentage error ξ as a function of α, ϕ_n and $\triangle \phi$. For this purpose the series in equation (11) was worked out for four values of α, viz., 2, 3, 4, and 5. This range was chosen because, although most of the stations have α lying between 3 and 4, some stations have been found with α below 3 or above 4. The results of the calculations are given below :

I. For $\alpha = 2$, $\xi = 0$ for all values of ϕ_n and $\triangle\phi$.

This is obvious because with $\alpha = 2$, e/p and hence q, should be proportional to p [vide equations (3) and (4)].

II. For $\alpha = 3$,

$$\xi/(\triangle \phi)^2 = \frac{1-\phi_n}{2\,(1-\phi_n^3)} \times 100 \tag{13}$$

III. For $\alpha = 4$,

$$\xi/(\triangle \phi)^2 = \frac{1}{1+\phi_n^2} \times 100 \tag{14}$$

IV. For $\alpha = 5$,

$$\xi/(\triangle \phi)^2 = \left\{ \frac{5}{3} \cdot \frac{(1-\phi_n^3)}{(1-\phi_n^5)} - \frac{(1-\phi_n)}{6\,(1-\phi_n^5)} \cdot (\triangle \phi)^2 \right\} \times 100 \tag{15}$$

It will be seen from the foregoing, that the ratio $\xi/(\triangle \phi)^2$ is independent of $\triangle \phi$ for all the cases upto $\alpha = 4$. For $\alpha = 5$, the ratio shows a slight dependence on $\triangle\phi$, but within the practical range, with ground pressure 1000 mb, highest level of observation from 200 to 800 mb, and pressure interval for each layer from 50 to 300 mb (ϕ_n from 0·2 to 0·8, $\triangle \phi$, from 0·05 to 0·3), the effect has been found to be negligible.

Values of $\xi/(\triangle \phi)^2$ computed from equation (13), (14) and (15) for $\phi_n = 0{\cdot}2, 0{\cdot}4, 0{\cdot}6$ and $0{\cdot}8$ are given in Table 1.

TABLE 1

$\xi/(\triangle\phi)^2$ AS A FUNCTION OF α AND ϕ_n

Highest level of observation (ϕ_n)	Lapse parameter (α)			
	2	3	4	5
0·2	0	40·32	96·15	165·4
0·4	0	32·05	86·21	157·6
0·6	0	25·51	73·53	141·7
0·8	0	20·49	60·98	121·0

Values of log $[\xi/(\triangle\phi)^2]$ obtained from Table 1 were plotted against log $(\alpha-2)$ as shown in Fig. 1. This was done because $\xi = 0$ when $\alpha = 2$. It will be seen that the points for each value of ϕ_n practically lie on a straight line, and that all the straight lines appear to converge to one point, corresponding to log $(\alpha - 2) = 0{\cdot}805$ and log $[\xi/(\triangle\phi)^2] = 2{\cdot}635$.

The slope, m, of each line, being a function of ϕ_n, a relationship between them was established as shown in Fig. 2, in which log $(m-1{\cdot}23)$ was plotted against log ϕ_n, yielding a straight line with a slope of $1{\cdot}570$.

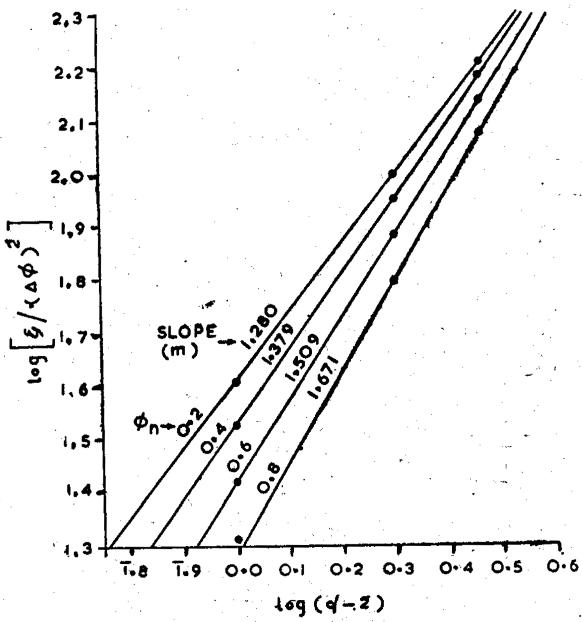

Fig. 1—Relationship between Log $[\xi/(\triangle\phi)^2]$ and Log $(\alpha - 2)$.

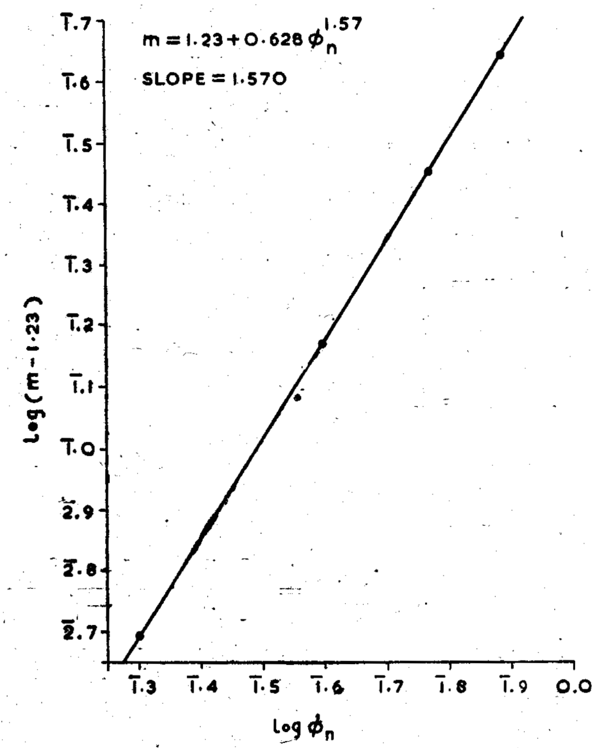

Fig. 2—Relationship between m and ϕ_n.

The final formula arrived at on the basis of the foregoing is

$$\xi/(\triangle\phi)^2 = \frac{44{\cdot}14\,(\alpha-2)^{1\cdot23+0\cdot628\phi_n^{1\cdot57}}}{3{\cdot}203^{\phi_n^{1\cdot57}}} \tag{16}$$

Replacing the original variables, equation (16) becomes

$$\xi = \frac{44{\cdot}14\,(\alpha-2)^{1\cdot23+0\cdot628\,(p_n/p_0)^{1\cdot57}}}{3{\cdot}203^{(p_n/p_0)^{1\cdot57}}}\cdot(\triangle p/p_0)^2\% \tag{17}$$

With the help of equation (17), it should be possible to estimate the percentage error due to the conventional summation method of computing precipitable water for given values of α, p_0, p_n and $\triangle p$. However, Table 2 has been prepared for ready use, wherefrom values can be read for ground pressure 1000 mb or near about, with the highest level of observation at 200, 400, 600 and 800 mb, α ranging from 2·5 to 5·0, and pressure interval of observations varying from 50 mb to 400, 300 or 200 mb, as the case demands.

TABLE 2

PERCENTAGE ERROR (ξ) OF CONVENTIONAL SUMMATION METHOD OF COMPUTATION OF PRECIPITABLE WATER IN RELATION TO HIGHEST LEVEL OF OBSERVATION (p_n), LAPSE PARAMETER (a), AND PRESSURE INTERVAL OF OBSERVATIONS (Δp).

(GROUND PRESSURE p_o=1000 mb)

Highest level of observation p_n (mb)	Pressure internal Δp (mb)	Lapse parameter (a)					
		2·5	3·0	3·5	4·0	4·5	5·0
	50	0·04	0·10	0·17	0·24	0·32	0·41
	100	0·17	0·40	0·68	0·98	1·30	1·64
	150	0·37	0·90	1·52	2·20	2·92	3·69
200	200	0·66	1·61	2·70	3·91	5·00	6·56
	250	1·04	2·51	4·22	6·10	8·12	10·26
	300	1·49	3·62	6·08	8·79	11·70	14·77
	400	2·65	6·44	10·81	15·63	20·79	26·26
	50	0·03	0·08	0·15	0·22	0·30	0·38
	100	0·13	0·33	0·59	0·87	1·18	1·52
	150	0·29	0·75	1·32	1·96	2·67	3·43
400	200	0·52	1·34	2·34	3·48	4·74	6·09
	250	0·80	2·09	3·66	5·44	7·41	9·52
	300	1·16	3·01	5·27	7·84	10·66	13·71
	50	0·02	0·07	0·12	0·19	0·26	0·34
600	100	0·09	0·26	0·48	0·75	1·05	1·38
	150	0·21	0·59	1·09	1·68	2·36	3·10
	200	0·37	1·05	1·93	2·99	4·19	5·52
	50	0·02	0·05	0·10	0·15	0·22	0·31
800	100	0·06	0·19	0·38	0·62	0·90	1·22
	150	0·14	0·44	0·86	1·39	2·02	2·75
	200	0·24	0·78	1·53	2·48	3·60	4·88

DISCUSSION

It appears from available literature that no serious attempt has been made so far to ascertain the accuracy of the conventional summation method for computing precipitable water in the atmosphere from upper air data which include new point temperatures at various specified pressure levels. The formula used for the purpose implies a tacit assumption of a linear relationship between specific humidity and total pressure. If this were true, the value of; α, the lapse parameter, should have been equal to 2 in all cases, while most of the stations studied by us have yielded values of 'α' between 3 and 4.

It can, however, be seen from Table 2 that for pressure interval upto 100 mb, errors lie within reasonable limits for all values of α upto 5, error increasing with increasing α. Pressure interval of 150 mb seems to be alright for α upto 3·5, while that of 200 mb for α upto 3.

An improved method of computation

The question then naturally arises as to what should be a better method of computation, since 'α' is known to vary from station to station and also shows a seasonal variation. We have attempted to answer this question in the following manner.

Let α_r be the lapse parameter for the rth layer between pressure levels p_{r-1} and p_r. Then the precipitable water, W^r_{r-1}, in this layer, according to equation (8) will be given by[4]

$$W^r_{r-1} = \frac{0 \cdot 635\, e_{r-1}}{\alpha_r} \left[1 - (p_r/p_{r-1})^{\alpha_r}\right] \text{ cm} \tag{18}$$

where the vapour pressure is expressed in millibars.

From eqation (3), we have

$$e_r/e_{r-1} = (p_r/p_{r-1})^{\alpha_r} \tag{19}$$

so that equation (18) reduces to

$$W^r_{r-1} = \frac{0 \cdot 635}{\alpha_r} (e_{r-1} - e_r) \text{ cm} \tag{20}$$

Now from equation (19) we can write

$$\alpha_r = \frac{\log (e_r/e_{r-1})}{\log (p_r/p_{r-1})} \tag{21}$$

With this expression for α_r, equation (20) becomes

$$W^r_{r-1} = 0 \cdot 635\, (e_{r-1} - e_r) \Big/ \frac{\log (e_r/e_{r-1})}{\log (p_r/p_{r-1})} \text{ cm} \tag{22}$$

Hence the total precipitable water in all the n layers from p_0 to p_n, will be given by

$$W_n = 0 \cdot 635 \sum_{r=1}^{n} \left[(e_{r-1} - e_r) \Big/ \frac{\log (e_r/e_{r-1})}{\log (p_r/p_{r-1})} \right] \text{cm} \tag{23}$$

It may be noted that this method is quite general in so far as it does not require the knowledge of 'α', nor does it assume a constant value of 'α' for all the layers. Equation (23) may, therefore, be regarded as an improved method for computation of precipitable water from upper air data.

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