HEAT TRANSFER IN RAREFIED MHD LAMINAR CHANNEL FLOW

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The problem of heat transfer for the steady axi-symmetrical laminar source flow of a slightly rarefied electrically conducting gas between two infinite parallel circular disks under transverse magnetic field is analytically investigated where both Joulean and viscous heating are considered. The flow parameters and the temperature are expanded in powers of $1/r$. The quantity of heat transfer per unit time from a finite disk has been calculated. It is found that with the increase of magnetic field, the rate of heat transfer from the lower disk decreases and increases from the upper disk. The maximum temperature increases with the increase of the magnetic field. The rate of heat transfer from both the disks as well as the maximum temperature decreases with the increase of the rarefication of the gas.

Low density gas partially loses its continuum characteristics and becomes rarefied. These rarefication effects are approximated by a slip of the fluid over the solid wall and a temperature jump[1]. When the gas is only slightly rarefied the flow regime is termed as 'Slip-flow' and in this regime the gas density is just slightly less than that characteristic of a completely continuum[2] and it is usually analyzed by applying continuum approach together with modified boundary conditions for velocity slip and temperature jump[3].

The steady axi-symmetric laminar source flow of a slightly rarefied electrically conducting gas between two infinite parallel circular disks in presence of a transverse magnetic field H_0 has been discussed by Khader, Goodling and Vachon[4]. The case of classical flow (MHD with no-rarefication) of the present problem was numerically solved by Khader and others[5] with constant disk temperature (same for both the disks) by neglecting terms of order higher than the first negative power of r. In the present investigation the energy equation is solved analytically, where both Joulean and viscous heating are included, when the upper and lower disks are maintained at two constant temperature T_1^* and T_2^* $(T_2^* > T_1^*)$ respectively. The effect of rarefication has been taken into account by considering velocity slip and temperature jump at the solid boundaries. A source of volumetric flow rate Q has been assumed at the centre of the upper disk (Fig. 1) and the temperature has been expanded in powers of $1/r$ and solution holds good between $r = \frac{r_0^*}{l}$ and $r = \frac{b^*}{l}$. No local heat transfer coefficient but total one is measured and yet this is useful enough to clarify the effect of magnetic field as well as rarefication of the gas. The rate of total heat transfer per unit time from both the disks have been plotted (Fig. 2 & 3) against M for fixed ϵ. The effect of M and ϵ on maximum temperature has been shown in Fig. 4.

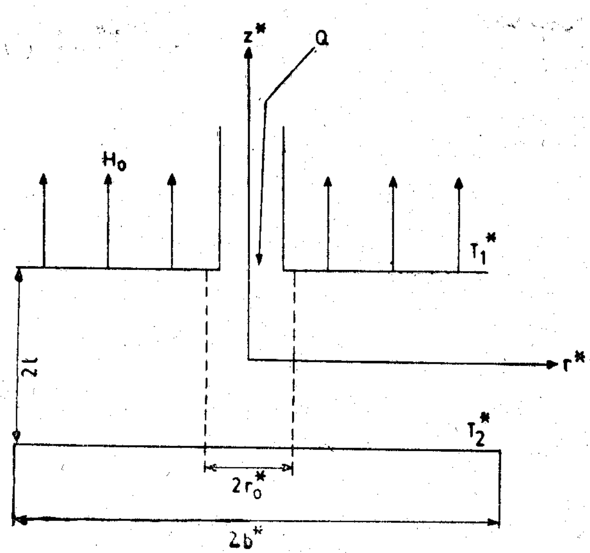

Fig. 1—Configuration and co-ordinate system.

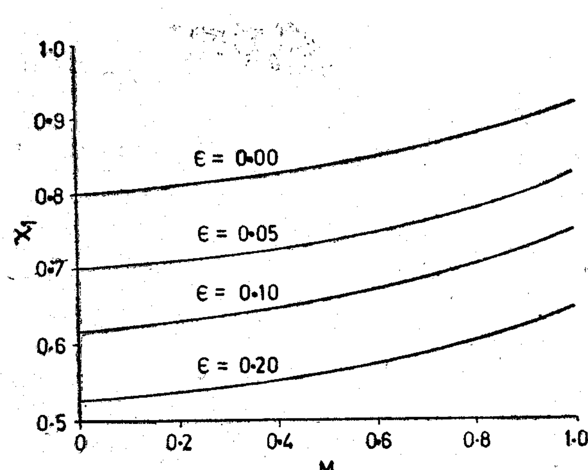

Fig. 2—χ_1 against M (upper disk).

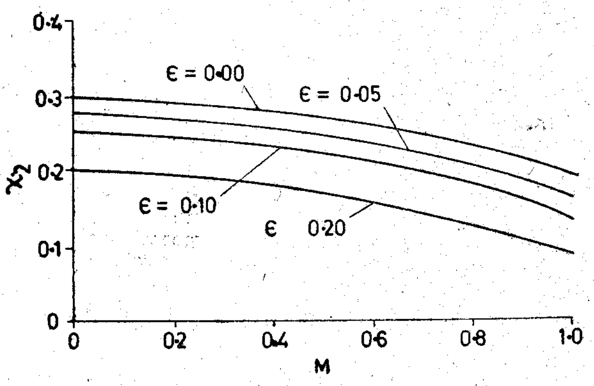

Fig. 3—χ_2 against M (lower disk).

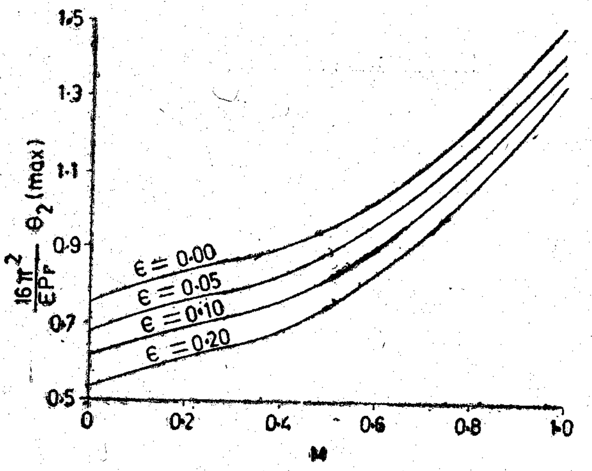

Fig. 4—θ_2 (max) against M.

The Knudsen number and the magnetic Reynolds number are considered to be small for applying continuum approach and to neglect induced magnetic field. The magnetic pressure number is considered of the order of unity. The assumption that the fluid is incompressible is a fairly valid approximation atleast for low Mach number flows.

HEAT TRANSFER ANALYSIS

Consider a steady source flow of volumetric flow rate Q of low density, electrically conducting fluid between two non-conducting disks at $z^* = \pm l$ at two constant temperature T_1^* and T_2^*, ($T_2^* > T_1^*$) respectively (Fig. 1). The radius of the hole is r_0^* and that of the disk is b^*. We considered the velocity and the temperature distribution between the two disks, $r^* = r_0^*$ and $r^* = b^*$. At any point of the fluid, let T^* be the temperature, u^* and w^* be the radial and the axial velocity components. H_z^* be the applied magnetic field in the direction of z^*. We define the following dimensionless quantities.

$$u = \frac{u^* l^2}{Q}, \quad w = \frac{w^* l^2}{Q}, \quad r = \frac{r^*}{l}, \quad z = \frac{z^*}{l}, \quad \theta = \frac{T^* - T_1^*}{T_2^* - T_1^*} \tag{1}$$

First order velocity slip and temperature jump boundary conditions after neglecting thermal creep are given by[3]

$$u = \mp \epsilon_1 \frac{\partial u}{\partial z}, \quad w = 0 \quad \text{at } z = \pm 1 \tag{2}$$

$$\theta = - \epsilon_2 \frac{\partial \theta}{\partial z} \qquad \text{at } z = + 1 \tag{3}$$

and

$$\theta = 1 + \epsilon_2 \frac{\partial \theta}{\partial z} \qquad \text{at } z = -1$$

where

$$\epsilon_1 = \frac{2 - f}{f} \cdot \frac{\lambda}{l}, \quad \epsilon_2 = \frac{2 - g}{g} \cdot \frac{2\gamma}{\gamma + 1} \cdot \frac{\lambda}{l.Pr} \tag{4}$$

f is the Maxwell's reflection coefficient, g is the Maxwell's thermal accommodation coefficient, λ is the mean free path, P_r is the Prandtl number, γ is the ratio of the specific heat coefficient. Parameters ϵ_1 and ϵ_2 are known as velocity-slip and temperature jump coefficients and $\epsilon_1 = \epsilon_2$ ($= \epsilon$) for all practical purpose[6].

The expressions for the velocity components have been calculated by Khader and others by expanding them in powers of $\frac{1}{r}$, as

$$u = \sum_{1}^{n} \frac{F_n}{r^n}, \quad w = \sum_{1}^{n} \frac{G_n}{r^n} \tag{5}$$

The induced magnetic field components of $H_z{}^*$ is taken to be zero under the condition of small magnetic Reynolds number ($R_m << 1$) and $H_z{}^*$ is approximated by H_0.

The energy equation governing the distribution of temperature in the cylindrical coordinate system for an axi-symmetric steady motion in dimensionless variable is

$$u\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}=\frac{1}{Re\,Pr}\left[\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}\right]+$$
$$+\frac{2E}{Re}\left[\left(\frac{\partial u}{\partial r}\right)^2+\frac{u^2}{r^2}+\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\frac{M^2E}{Re}u^2 \qquad (6)$$

where $M=\dfrac{\mu_m H_0 l}{\sqrt{(\rho_v/\sigma)}}$, is the Hartmann number, $Re = Q/\nu l$, is the source Reynolds number, $Pr = \nu C_p/\rho k$, is the Prandtl number, $E = Q^2/C_p(T_2{}^* - T_1{}^*)\,l^4$, is the Eckert number, μ_m is the magnetic permeability, σ is the electrical conductivity of the fluid, ρ = density, ν = coefficient of viscosity, k = thermometric conductivity and C_p = specific heat at constant pressure.

We confine ourselves for the series upto 4th power of $\frac{1}{r}$ and with this approximation the equation (6) has been solved by expanding u, w and θ in powers of $\frac{1}{r}$ and taking

$$\theta=\sum_{0}^{n}\frac{\theta_n}{r^n} \qquad (7)$$

the boundary conditions (3) can be written as

$$\begin{aligned}\theta_n &= -\epsilon\,\theta_n{}', \quad \text{at } z=+1, \text{ for } n=0,1,2,\ldots\ldots\\ \theta_0 &= 1+\epsilon\,\theta_0{}', \quad \text{at } z=-1\\ \theta_n &= \epsilon\,\theta_n{}', \quad \text{at } z=-1 \text{ for } n=1,2,3,\ldots\ldots\end{aligned} \qquad (8)$$

where a prime denotes the differentiation w.r. to z.

We start the series of θ from n=0 to take into account the effect of conduction. Substituting (5) and (7) in (6) and equating like powers of $\frac{1}{r}$ we get a set of ordinary differential equations.

$$\theta_0{}''=0, \qquad \theta_1{}''=0$$
$$\theta_4{}''+E\,Pr\left[F_1{}'^2+M^2F_1{}^2\right]=0, \qquad \theta_3{}''+\left[1+Re\,Pr\,F_1\right]\theta_1=0 \qquad (9)$$
$$\theta_4{}''+2\left[2+Re\,Pr\,F_1\right]\theta_2+2\,E\,Pr\left[2F_1{}^2+F_1{}'F_3{}'+M^2F_1F_3\right]-Re\,Pr\,G_4\theta_0{}'=0$$

taking F_n and G_n as given by[4] and solving (9) under boundary conditions (8), we get

$$\theta_0=\frac{1}{2}-\frac{z}{2(1+\epsilon)} \qquad (10)$$

$$\frac{16\pi^2\beta^2}{E\,Pr\,M^4}\cdot\theta_2=-\left[\frac{ch\,2Mz}{4M^2}-2\alpha\frac{ch\,Mz}{M^2}+\frac{\alpha^2z^2}{2}\right]+$$
$$+\left[\left\{\frac{ch\,2M}{4M^2}-2\alpha\frac{ch\,M}{M^2}+\frac{\alpha^2}{2}\right\}+\epsilon\left\{\frac{sh\,2M}{2M}-2\alpha\frac{sh\,M}{M}+\alpha^2\right\}\right] \qquad (11)$$

where

$$\alpha=\epsilon\,M\,sh\,M+ch\,M\,, \qquad \beta=\left[1-\epsilon M^2\right]sh\,M-M\,ch\,M \qquad (12)$$

$$\theta_1=\theta_3=0 \qquad (13)$$

The expression for θ_4 has been calculated and found that the inclusion of θ_4 in temperature distribution has negligible effect and so we have dropped θ_4 from the expression of θ_4. Also for large values of r i.e. $r >> 1$, one can neglect terms of order higher than the first negative power of r.

DISCUSSION

The heat flux from the disks $z = +1$ and $z = -1$ in slip flow due to boundary conditions are

$$q_1^* = \left\{ k (T_2^* - T_1^*) / l \right\} (\theta/\epsilon) \Big|_{z = +1}$$

and

$$q_2^* = \left\{ k (T_2^* - T_1^*) / l \right\} \left\{ (1 - \theta) / \epsilon \right\} \Big|_{z = -1}$$

Neglecting edge effects, the rate of total heat transfer per unit time from the circular disks from $r^* = r_0^*$ and $r^* = b^*$ are given by

$$\chi_1^* = \frac{1}{\pi (b^{*2} - r_0^{*2})} \int_{r_0^*}^{b^*} 2 \pi r^* q_1^* dr^*$$

or

$$\chi_1 = \frac{l . \chi_1^*}{k (T_2^* - T_1^*)} = \frac{1}{2 (1 + \epsilon)} + \widetilde{E} \frac{M^4}{\beta^2} \left[\frac{sh\, 2M}{2M} - 2 \alpha \frac{sh\, M}{M} + \alpha^2 \right] \tag{14}$$

and

$$\chi_2^* = \frac{1}{\pi (b^{*2} - r_0^{*2})} \int_{r_0^*}^{b^*} 2 \pi r^* q_2^* dr^*$$

or

$$\chi_2 = \frac{l . \chi_2^*}{k (T_2^* - T_1^*)} = \frac{1}{2 (1 + \epsilon)} - \widetilde{E} \frac{M^4}{\beta^2} \left[\frac{sh\, 2M}{2M} - 2 \alpha \frac{sh\, M}{M} + \alpha^2 \right] \tag{15}$$

where $\widetilde{E} = l^2 EPr \log (b^*/r_0^*)/8\pi^2 (b^{*2} - r_0^{*2})$, a dimensionless number.

The maximum temperature occurs at $z=0$ and θ_2 (max) is given by

$$\theta_2 (\text{max}) = \frac{EPr}{16\pi^2} \cdot \frac{M^4}{\beta^2} \left[\left\{ \frac{ch\, 2M}{4M^2} - 2\alpha \frac{ch\, M}{M^2} + \frac{\alpha^2}{2} \right\} + \right.$$
$$\left. + \epsilon \left\{ \frac{sh\, 2M}{2M} - 2\alpha \frac{sh\, M}{M} + \alpha^2 \right\} - \left\{ \frac{1}{4M^2} - \frac{2\alpha}{M^2} \right\} \right] \tag{16}$$

The rate of total heat transfer χ_1 and χ_2 have been plotted against M (Fig. 2 & 3) for $E = 0.1$ and for fixed ϵ. The graphs reveal that χ_2 decreases whereas χ_1 increases with the increase of M for fixed value of ϵ. Both χ_1 and χ_2 decreases with the increase of ϵ.

The function θ_0 represents the usual radiation temperature which increases with the increase of slip coefficient (ϵ), but is independent of Hartmann number (M). The function θ_2 represents the temperature due to convection and θ_2 (max) has been plotted against M(Fig. 4). The maximum temperature occurs at the middle of the channel and increases with the increase of M, but decreases with the increase of ϵ.

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REFERENCES

1. LIPMANN, H. W. & ROSHKE, A., 'Elements of Gas Dynamics' (John Wiley and Sons, Inc.), 1956, p. 376.
2. SCHAFF, S. A. & CHAMBRE, P. L., 'Flow of Rarefied Gases' (Princeton University Press), 1961, p. 25.
3. KENNARD, E. H., 'Kinetic Theory of Gases' (Allied Pacific Pvt. Ltd.), 1962, p. 291.
4. KHADER, M. S., GOODLING, J. S. & VACHON, R. I., *Appl. Sci. Res.*, **30** (1974), 127.
5. KHADER, M. S. & VACHON, R. I., *Appl. Sci. Res.*, **29** (1974), 321.
6. KAWAMATA, S., *Trans. Japan Soc. Aero and Space Sci.*, **4** (1961), 12.