WOUND BALLISTICS : STUDY OF THE RUPTURE OF HUMAN SKIN MEMBRANE UNDER THE IMPACT OF A PROJECTILE

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The paper attempts to theorize the rupture of human skin membrane under the impact of a projectile. An expression for the threshold velocity for penetration has been derived which is found to give results in fair agreement with experimental values reported in literature.

In a recent communication, Jauhari and Bandyopadhyay[1] have attempted to theorize the elastic breakdown of human skin membrane under the impact of a spherical projectile by having recourse to the theory of elasticity. As a first approximation, they treated the skin as a homogeneous, isotropic, elastic membrane having elastic properties akin to a ductile material. The elastic breakdown of the skin membrane was investigated on the basis of *maximum shear* theory and an expression for the threshold velocity required for the elastic breakdown of the skin membrane was derived. Calculations indicate that for persons in the age group 50—80 years, the threshold velocity is of the order of 4.6 - 5.2 m/sec for a spherical projectile of lead whose radius is ten times the thickness of the skin membrane. The elastic breakdown of the skin membrane is in fact the first stage in the process leading to its ultimate rupture. With the onset of yielding, the inelastic strain increases, eventually leading to the rupture.

When a material is stretched by a tensile force, the proportionality between the tensile force and the corresponding elongation holds only upto a certain limiting value of the tensile stress called the proportional limit, which depends upon the properties of the material. As this limit is exceeded, the relationship between the tensile stress and the elongation assumes a complicated form. To investigate the behaviour of a material beyond the limit of proportionality, the mechanical properties of the material are required to be known beyond the proportional limit. These properties are usually defined by tension and compression test diagrams. Tension test diagrams for skin of human cadavers (without subcutaneous fat) of different age groups are available in literature[2]. These can, therefore, be of assistance in analysing the problem of rupture of human skin membrane under the impact of a projectile. In the present paper, an attempt has been made to study this aspect of the problem of skin penetration by missiles.

RUPTURE OF SKIN MEMBRANE

When a projectile strikes a human body, its impact causes the skin membrane to stretch. As the skin stretches, the kinetic energy of the projectile gets stored in skin as the potential energy of deformation. If the projectile has enough kinetic energy, it may cause the skin membrane to stretch to such an extent that it ruptures. Assuming that (*a*) the projectile does not deform on impact, (*b*) its kinetic energy is spent only in stretching the skin, one can visualize that the velocity of the projectile will continually diminish as the skin stretches and at the point of rupture, it will reduce to zero; the entire kinetic energy of the projectile being stored as the potential energy of deformation of the skin.

It is now stipulated that the skin membrane will just rupture when the kinetic energy of the projectile per unit volume of the strained skin membrane equals the strain energy per unit volume as obtained in a simple tension test of the skin membrane. The latter is obviously represented by the area under the stress-strain curve of the skin membrane right upto the rupture point. If m is the mass of the projectile, V_{th} is the threshold velocity required to just rupture the skin membrane, Δ_0 the volume of the skin strained as a result of the impact of the projectile and A the area under the stress-strain curve of the skin membrane in a simple tension test, the condition of rupture is represented by the following equation :

$$\frac{\frac{1}{2}\, m\, V_{th}^2}{\Delta_0} = A \qquad (1)$$

Eq. (1) gives the general condition under which the skin membrane will rupture under the impact of a projectile, irrespective of its size and shape. From (1)

$$V_{th} = \sqrt{\frac{2\,\Delta_0\,A}{m}}\,, \tag{2}$$

which gives an expression for the threshold velocity for the penetration of skin membrane.

Eq. (2) can be used to calculate the threshold velocity for penetration, provided Δ_0, A and m are known. An attempt will now be made to calculate the threshold velocity for penetration in respect of solid spherical projectiles of lead and steel.

Expression for Δ_0

When a spherical projectile strikes the skin membrane, it is easy to visualize that on account of stretching, a portion of the skin, which remains in contact with the projectile will approximately assume the shape of a portion of a spherical shell of radius r, where r is the radius of the spherical projectile. This situation is depicted in Fig. 1, where a plane section of the skin (in a state of tension just at the point of rupture) and the spherical projectile are shown. The volume of skin strained in this manner can be obtained from simple geometrical considerations *i.e.,* by multiplying the area of presentation of the projectile with the thickness of the skin. Therefore,

$$\Delta_0 = \pi r^2\, t_0 \sin^2 \theta, \tag{3}$$

where t_0 is the thickness of the skin in an unstretched condition and 2θ is the angle subtended by the portion of the skin in contact with the projectile at the latter's centre. The maximum area of presentation in case of a spherical projectile can be πr^2 which is attained when $\theta = \frac{\pi}{2}$, *i.e.,* when the skin in contact with the ball envelops the half of the spherical surface of the ball. Therefore, the maximum value of Δ_0 at the point of rupture can be

$$(\Delta_0)_{max} = \pi\, r^2\, t_o. \tag{4}$$

Referring to Fig. 1, a skin of length $2r$ Sinθ is stretched to a length $2r\theta$. Therefore, the percent elongation in this case will be

$$\epsilon = \frac{2r\theta - 2\,r\,\sin\theta}{2r\,\sin\theta} \times 100 = \left(\frac{\theta}{\sin\theta} - 1\right)100 \tag{5}$$

When $\theta = \pi/2$, $\epsilon = 57\%$. Thus it is clear that so long $0 < \epsilon < 57\%$ at the point of rupture, the value of

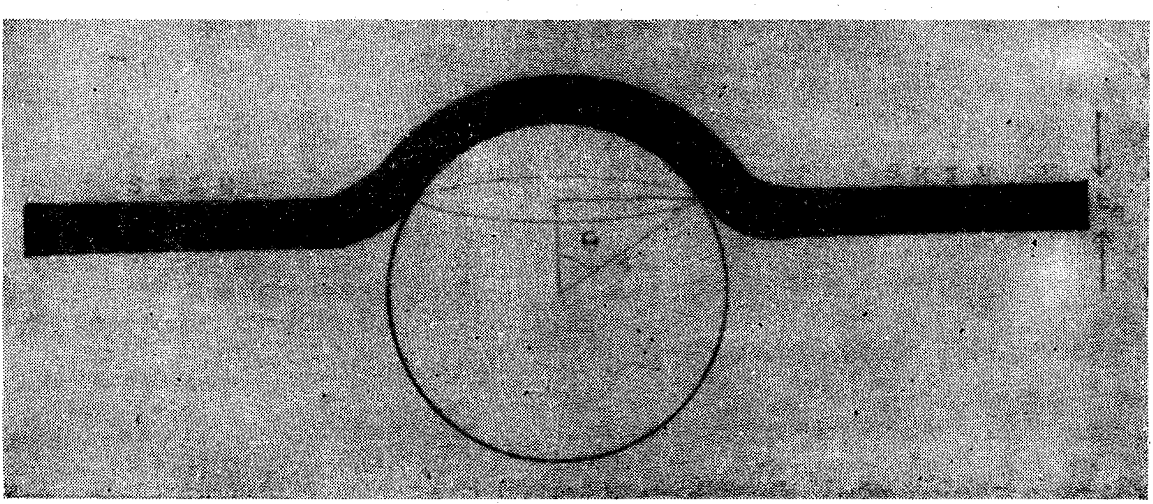

Figure 1. Plate section of the skin and the spherical projectile at the point of rupture.

Δ_0 will depend upon θ as per eq. (3). Once the rupture is recorded at elongations greater or equal to 57% the value of Δ_0 will be given by (4) which is independent of θ. The values of θ and Sinθ for different percentage elongations are given in Table 1 and these will be used for the calculation of threshold velocity.

Expression for A

As stated earlier, A represents the area under the stress-strain curve of skin in a simple tension test. According to Seely[3], this is approximately represented as below in case of ductile materials :

$$A = \frac{S_y + S_u}{2} \epsilon_u, \qquad (6)$$

where S_y and S_u are the yield-point and ultimate strength of the material respectively, and ϵ_u is the strain at the rupture point. In so far as human skin membrane is concerned, stress-strain curves for various age groups are available in literature[2]. The area under these curves was, therefore, directly calculated and is given in Table 2.

Expression for m

If the projectile is in the form of a solid spherical ball of radius r and density ρ,

$$m = \frac{4}{3} \pi r^3 \rho. \qquad (7)$$

Therefore, the final expression for V_{th} is obtained by substituting for Δ_0 and m from eqs. (3) and (7) respectively in equation (2) *i.e.*,

$$V_{th} = \sqrt{\frac{3}{2} \frac{A\, t_o}{r\rho} \sin\theta} \qquad (8)$$

Calculation of V_{th} for different values of A, $\frac{t_o}{r}$ and Sinθ is given in Tables 3 — 6 for solid spherical projectiles of steel ($\rho = 7 \cdot 8$ gms/c.c.) and lead [ρ=11gms/c.c].

TABLE 1

PERCENT ELONGATION (ϵ) VERSUS θ AND SIN θ

Elongation (%)	θ degrees	Sin θ
10	44	.6947
20	59	.8571
30	70	.9396
40	79	.9816
57	90	1

TABLE 2

A, PERCENT ELONGATION AT RUPTURE AND THRESHOLD VELOCITY (for r=1/16″ and t_o 5=mm) FOR DIFFERENT AGE GROUPS

Age group	A (ft-poundal/cu. inch)	Elongation at rupture (%)	Threshold velocity (ft/sec.)
From 2 months prematurely born infants to 3 years old children	632.15	47	102
15—30 years	822.55	35	113
30—50 years	837.79	33	113
50—80 years	891.10	31	115

TABLE 3

V_{th} IN FT/SEC FOR THE AGE GROUP 50—80 YEARS

t_0/r	Elongation at rupture	10%	20%	30%	40%	57%
0.1	lead	13	16	17	18	18
	steel	15	19	20	21	22
0.5	lead	29	35	39	40	41
	steel	34	42	46	48	49
1	lead	40	50	55	57	58
	steel	48	59	65	68	69
2	lead	57	70	77	81	82
	steel	68	83	91	96	97
3	lead	70	86	94	98	100
	steel	83	102	112	117	119
4	lead	81	99	109	114	116
	steel	96	118	129	135	138
5	lead	90	111	122	127	130
	steel	107	132	145	151	154

TABLE 4

V_{th} IN FT/SEC FOR THE AGE GROUP 30—50 YEARS

t_0/r	Elongation at rupture	10%	20%	30%	40%	57%
0.1	lead	13	15	17	18	18
	steel	15	18	20	21	21
0.5	lead	28	34	37	39	39
	steel	33	40	44	46	47
1	lead	39	48	53	55	56
	steel	46	57	63	66	67
2	lead	55	68	75	78	79
	steel	66	81	89	93	94
3	lead	67	83	92	96	98
	steel	80	99	109	114	116
4	lead	78	96	105	110	113
	steel	93	114	125	131	134
5	lead	87	108	118	124	125
	steel	104	128	140	147	149

TABLE 5

V_{th} IN FT/SEC FOR THE AGE GROUP 15—30 YEARS

t_0/r	Elongation at rupture	10%	20%	30%	40%	57%
0.1	lead	13	15	17	18	18
	steel	15	18	20	21	21
0.5	lead	27	34	37	39	39
	steel	32	40	44	46	47
1	lead	39	48	52	55	55
	steel	46	57	62	65	66
2	lead	55	67	74	77	78
	steel	65	80	88	92	93
3	lead	67	82	91	95	97
	steel	80	98	108	113	115
4	lead	77	95	104	109	111
	steel	92	113	124	130	132
5	lead	87	107	117	122	125
	steel	103	127	139	145	148

TABLE 6

V_{th} IN FT/SEC FOR THE AGE GROUP FROM 2 MONTHS PREMATURELY BORN INFANTS TO CHILDREN OF 3 YEARS

t_0/r	Elongation at rupture	10%	20%	30%	40%	57%
0.1	lead	11	13	14	15	15
	steel	13	16	17	18	18
0.5	lead	23	29	32	34	34
	steel	28	35	38	40	41
1	lead	34	42	45	48	49
	steel	40	50	54	57	58
2	lead	48	59	65	67	69
	steel	57	70	77	80	82
3	lead	59	71	79	83	84
	steel	70	85	94	99	100
4	lead	67	83	92	96	98
	steel	80	99	109	114	116
5	lead	76	93	103	107	109
	steel	90	111	122	127	130

DISCUSSION

In the derivation of the expression for the threshold velocity, it was assumed that the skin while resisting the impact of a moving projectile behaved in the same manner as while resisting a static load, *i.e.*, the stress-strain curves in the two cases were identical. The validity of this assumption may be questionable especially if the load is applied with extreme suddenness. It is, however, experimentally observed that the threshold velocity for the penetration of human skin is of the order of 125—170 ft/sec [4]. The impulse transmitted at such low velocities may not be of such a high magnitude as to invalidate our assumption for approximate calculations.

Another assumption made was that the entire kinetic energy of the projectile was consumed in causing deformation of the skin membrane. In actual practice, a part of this energy is used up in deforming the ball or setting up stresses in its body and a part is consumed in displacing the subcutaneous material. Therefore, the threshold velocity as calculated by the formula derived in this paper is expected to be on the lower side of the velocity observed experimentally.

A reference to Table 2 brings out an interesting fact that the value of A is more or less constant for persons belonging to the different age groups, although their elastic constants are known to differ considerably. This suggests that for the rupture of human skin membrane, there is a threshold strain energy/unit volume. Once the strain energy/unit volume reaches this threshold figure, the skin ruptures. However, according to Beyer[4], there is a threshold velocity (125—170 ft/sec) for the rupture of skin. Missiles with velocities lower than this figure cause contusions without the rupture of skin membrane. The contention of Beyer can be justified provided the factor Δ_0/m in (2) has a more or less constant value irrespective of the size and shape of the missile. This is so because in the general expression for V_{th} given in (2), this factor is involved in addition to A which, as stated above, is more or less constant for the different age-groups. The expression for V_{th} for a spherical projectile given by (8) involves t_o/r, ρ and $\sin\theta$ as variable factors. Prima facie, therefore, V_{th} cannot be assumed to have the same threshold value irrespective of the size and shape of the missile. This is quite evident from Tables 3—6 where V_{th} has been calculated for varying values of t_o/r, ρ and $\sin\theta$. It will be noted from these Tables that the factor t_o/r (ratio of thickenss of skin to the radius of the ball) has considerable influence on the value of V_{th}.

We may now calculate the value of V_{th} for a steel ball of (4/32)″ diameter which was used by Beyer for his experiments. To have an idea about the longitudinal strain at the point of rupture, guidance may be obtained from the stress-strain curves of human skin reproduced by Rothman[2]. From these curves, it is observed that the rupture in various age groups occurs at elongations shown in Table 2. Taking these elongations as observed in a simple tension test to be approximately valid at the rupture point when a projectile strikes the skin membrane, the threshold velocity for persons of various age groups for skin measuring 5 mm in thickness is given in Table 2. These figures are quite close to the value of the threshold velocity reported by Beyer (125—170 ft/sec). However, the fact remains that for smaller skin thickness the threshold velocity is lower for the same elongations. When $t_o/r \approx 0.1$, it may not be objectionable to assume without much error that the stresses across the thickness of the skin membrane are uniformly distributed. However, when t_o/r has a higher value, the stress distribution is far from uniform. In such a case, the layer in contact with the ball is stressed to a maximum extent and the outermost layer to the minimum extent. The strain energy density may, therefore, not be constant throughout. As the ball loses energy and thus more and more of its energy is stored in skin, successive layers may gradually be thrown into a stage of breakdown. Eventually, the entire thickness of the skin membrane is brought into a stage of breakdown. The inelastic strain in this manner continually increases leading to the rupture of the membrane.

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