STRUCTURE OF RADIATIVE BOUNDARY SHOCK WAVE

B. PRASAD

Department of Mathematics

D. A. V. P. G. College, Azamgarh—276001

&

R. P. RAI

T. D. College, Ballia

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In this paper the structure of a radiative shock wave in viscous compressive thin layer adjacent to the surface of a body has been studied and the expressions for the variations of the quantities through this wave have been derived.

Assuming the occurrence of boundary shock wave as postulated by Martin[1], Prasad and Rai[2] extended his results to radiative gases and discussed the properties of radiative boundary shock waves. Here our aim is to obtain the expressions for the variation of quantities through these waves.

STRUCTURE AND BOUNDARY CONDITIONS

Integrating the equations for conservation of mass, momentum and energy in one dimensional steady flow in a non-accelerating co-ordinate system[2] with the boundary conditions at

$$\begin{aligned} &x = x_b = 0^+, \\ &u = u_b,\ T = T_b,\ \rho = \rho_b,\ \tau = \tau_b,\ q = q_b, \\ &p = p_b,\ q_r = q_{r_b} \\ &\text{and at } x \longrightarrow \infty, \end{aligned} \tag{1}$$

and at $x = \infty$,

$$\frac{du}{dx} \longrightarrow 0,\ \frac{dT}{dx} \longrightarrow 0 \tag{2}$$

we get

$$\rho u = \text{constant} \tag{3}$$

$$\rho u^2 + p + \frac{a\,T^4}{3} - \tau = \text{constant} \tag{4}$$

$$\rho u \left(e + \frac{1}{2} u^2 \right) + q - uf = \text{constant} \tag{5}$$

To make the equations dimensionless we define and substitute the dependent variables.

$$\bar{\mu} = \frac{u}{u_b},\ \bar{T} = \frac{T}{T_b} \tag{6}$$

and a new independent variable

$$\xi = \rho_b\, u_b \int_0^x \frac{dx}{\bar{\mu}}, \tag{7}$$

With the assumption $\widetilde{P}_r$ = constant, $\widetilde{\mu}$ is proportional to k since C_p is constant. Then from equations (4) and (5)

$$\frac{d\bar{u}}{d\xi} - \bar{u} - \frac{1}{\gamma M_b^2}\frac{\bar{T}}{\bar{u}} - \frac{\bar{T}^4}{\gamma M^2_{rb}} = \frac{\tau_b}{\rho_b u^2_b} - 1 - \frac{1}{\gamma M_b^2} - \frac{1}{\gamma M^2_{rb}} \tag{8}$$

$$\frac{d}{d\xi}\left[\frac{1}{\widetilde{P}_r}\left(\frac{\bar{T}}{(\gamma-1)M_b^2} + \frac{4\bar{T}}{\gamma M^2_{rb}}\right) + \frac{1}{2}\bar{\mu}^r\right] - \left[\frac{\bar{T}}{(\gamma-1)M^2_b} + \frac{\bar{u}\bar{T}^4}{\gamma M^2_{rb}} + \frac{1}{2}\bar{\mu}^r\right] =$$

$$= -\left[\frac{1}{(\gamma-1)M^2_b} + \frac{1}{\gamma M^2_{r_b}} + \frac{1}{2}\right] + \left[-\frac{q_b}{\rho_b u^3_b} + \frac{\tau_b}{\rho_b u^2_b}\right] \tag{9}$$

and the boundary conditions

$$\xi \longrightarrow \infty : \frac{d\bar{u}}{d\xi} \longrightarrow 0, \frac{d\bar{T}}{d\xi} \longrightarrow 0. \tag{10}$$

In the special case where we have consider $\widetilde{P}_r = 1$, it is convenient to define

$$\bar{L} = \frac{C_p T + \frac{1}{2}u^2}{u^2_b} = \frac{\bar{T}}{(\gamma+1)M^2_b} + \frac{4\bar{T}}{\gamma M^2_{rb}} + \frac{1}{2}\bar{u}^2 \tag{11}$$

so that

$$\frac{d\bar{L}}{d\xi} = \frac{k\frac{dT}{dx}}{\rho_b u^3_b} + \frac{\widetilde{\mu} u \frac{du}{dx}}{\rho_b u_b^3} = \frac{-q + u_\tau}{\rho_b u^3_b} \tag{12}$$

where $k = k_c + k_r$. Further defining

$$\theta = \bar{L} - \bar{L}_b \tag{13}$$

the equation (9) can be written as

$$\frac{d\theta}{d\xi} - \theta = \left(\frac{d\theta}{d\xi}\right)_b \tag{14}$$

where by definition $\theta = 0$ at $\xi = 0$ and from the boundary conditions (10) $\frac{d\theta}{d\xi} \to 0$ as $\xi \to \infty$. The only possible solution is obviously

$$\theta \equiv 0 \equiv \frac{d\theta}{d\xi} = \frac{d\bar{L}}{d\xi}, \tag{15}$$

Hence equations (12) and (14) give

$$1 + \frac{\gamma-1}{2} M^2_b\left[1 - \bar{u}^2 + \frac{8}{\gamma M^2_{rb}}\right] = \bar{T}\left[1 + \frac{4(\gamma-1)}{\gamma}\frac{M^2_b}{M^2_{rb}}\right] \tag{16}$$

and

$$q_c = u_\tau \tag{17}$$

Putting $\gamma = 5/3$, $C_{h_c} = \frac{1}{2}$, $M_b = 1$, $M_{r_b} = 2$ in equation (8) we get after simple algebraic manipulation.

$$\bar{u}\frac{d\bar{u}}{d\xi} = 0{\cdot}001\,\bar{u}^9 - 0{\cdot}011\,\bar{u}^7 + 0{\cdot}077\,\bar{u}^5 - 0{\cdot}272\,\bar{u}^3 + 0{\cdot}857\,\bar{u}^2 - 1{\cdot}646\,\bar{u} + {\cdot}742 \tag{18}$$

where by definition

$$\bar{u} = 1 \quad \text{as} \quad \xi \to 0 \tag{19}$$

and

$$\frac{d\bar{u}}{d\xi} \to 0 \quad \text{as} \quad \xi \to \infty \tag{20}$$

Since the roots of

$$0{\cdot}001\,\bar{u}^9 - 0{\cdot}011\,\bar{u}^7 + 0{\cdot}77\,\bar{u}^5 - 0{\cdot}272\,\bar{u}^3 + 0{\cdot}857\,\bar{u}^2 - 1{\cdot}646\,\bar{u} + 0{\cdot}742 = 0 \tag{21}$$

are all real, integrating equation (18) in the usual manner and applying the condition (19) we get

$$\left(\frac{\bar{u}-e_1}{1-e_1}\right)^{\frac{e_1}{(e_1-e_2))\,(e_1-e_3)\ldots\ldots(e_1-e_9)}} \cdot \left(\frac{\bar{u}-e_2}{1-e_2}\right)^{\frac{e_2}{(e_2-e_1)\,(e_2-e_3)\ldots\ldots(e_2-e_9)}} \cdot$$

$$\left(\frac{\bar{u}-e_8}{1-e_8}\right)^{\frac{e_8}{(e_8-e_1)\,(e_8-e_2)\ldots\ldots(e_8-e_9)}} \cdot \left(\frac{\bar{u}-e_9}{1-e_9}\right)^{\frac{e_9}{(e_9-e_1)\,(e_9-e_2)\ldots\ldots(e_9-e_8)}} = e^{\xi}\,. \tag{22}$$

Solving the equation (21) numerically the most physically significant root comes out to be $0{\cdot}60$. Evidently when $\bar{u}=1$, $\xi \to 0$ and when $\bar{u}=0{\cdot}60$, $\xi \to \infty$. For values of $\bar{u}$ between 1 and $0{\cdot}60$, ξ takes steadily large values. There is no need of finding the values of ξ correspondng to $\bar{u}$ greater than 1 and negative values of $\bar{u}$ since they are inconsistant with the physical problem under consideration. Keeping in mind that

$$\bar{\rho} = \frac{\rho}{\rho_b} = \frac{1}{\bar{u}} \tag{23}$$

$$\bar{p} = \frac{p}{p_b} = \bar{\rho}\,\bar{T} \tag{24}$$

the corresponding values of $\bar{\rho}$, $\bar{T}$ and $\bar{p}$ are given in the table 1 and the variations of these quantities with ξ are plotted in figure 1.

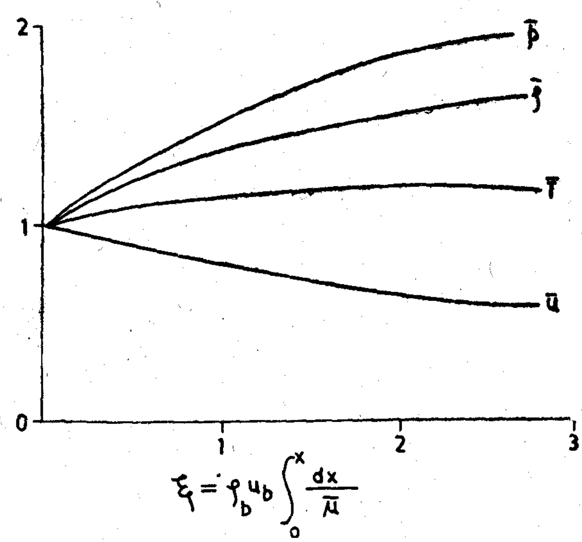

Fig. 1—Variation of pressure, density, temperature & velocity within radiative boundary shock for
$\gamma = \frac{5}{3}$, $M_b = 1$, $C_{b_c} = \frac{1}{2}$, $M_{r_b} = 2$, $\bar{P}_r = 1$.

TABLE 1

VALUES OF $\bar{\rho}$, $\bar{T}$ AND $\bar{p}$ CORRESPONDING TO REPRESENTATIVE VALUES OF ū

$\bar{\mu}$	$\bar{\rho}$	$\bar{T}$	$\bar{p}$
0·60	1·66	1·15	1·91
0·70	1·43	1·12	1·60
0·80	1·25	1·08	1·35
0·90	1·11	1·04	1·15
1·00	1·00	1·00	1·00

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2. PRASAD, B. & RAI, R. P., *Def. Sci. Jour.*, **28** (1978), 67.