SLOWING DOWN DENSITY OF NEUTRONS IN A CYLINDRICAL PILE

V. L. DESHPANDE

Pratap College, Amalner, Maharastra

(*Received* 19 *July* 1972)

In this paper, an equation governing the slowing down of neutrons in matter for a pile in the form of a cylinder has been established and the solution for various types of sources is obtained.

In a nuclear pile, uranium nuclei undergo fission and liberate neutrons, which when passing through moderator, diffuse through the material and are absorbed producing neutrons of the next generation. Thus the chain reaction takes place. The equation governing the slowing down of neutrons in matter is obtained with the help of transport equation for neutrons which under certain assumptions, is reduced to the basic equation of age theory.

Sneddon[1] has discussed the solution for the equation governing the slowing down of neutrons in matter for a pile in the form of a rectangular parallelopiped. The aim of this paper is to consider a pile in the form of a cylinder of infinite height and obtain the solution for various types of sources. We[2] have already considered the case of a cylinder of finite height.

BASIC EQUATION

By taking the following formulae from Sneddon[1], we have

If
$$\bar{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(z) . e^{i\xi z} dz \tag{1}$$

then
$$f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{f}(\xi) . e^{-i\xi z} d\xi \tag{2}$$

Following Sneddon[1], if $f(x)$ satisfies Dirichlet's conditions in the interval $(0, a)$ and if its finite Hankel transform in that range is defined to be

$$f_J(\xi_i) = \int_0^a r . f(r) . J_0(\xi_i r) . dr \tag{3}$$

where ξ_i is a root of the equation

$$J_0(\xi_i a) = 0 \tag{4}$$

then at any point of $(0, a)$ at which the function of $f(x)$ is continuous

$$f(x) = \frac{2}{a^2} \sum_i \frac{f_J(\xi_i) . J_0(r\xi_i)}{[J_1(a\xi_i)]^2} \tag{5}$$

where the sum is taken over all the positive roots of equation (4)

The equation, governing the slowing down density $\psi(r, \phi, z, \theta)$ of neutrons in cylindrical polar coordinates[1] is

$$\frac{\partial \psi}{\partial \theta} = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \phi^2} + T(r, \phi, z, \theta) \tag{6}$$

where $T(r, \phi, z, \theta)$ is the known sources of neutrons in the material and θ is the symbolic age defined by Sneddon[1].

Assuming the symmetry about the axis of the cylinder, we have

$$\frac{\partial \psi}{\partial \theta} = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} + T(r, z, \theta), \tag{7}$$

where $$\psi \equiv \psi(r, z, \theta).$$

Consider the case of an infinite cylinder of radius 'a' and bounded by the planes $z = \infty$ and $z = -\infty$. By assuming that the slowing down density vanishes on the boundary of the cylinder i. e.

$$\psi(a, z, \theta) = \psi(r, \infty, \theta) = \psi(r, -\infty, \theta) = 0, \tag{8}$$

we further assume that the source function $T(r, z, \theta)$ is such that

$$T(r, z, \theta) = S(r, z).\, U(\theta).$$

Now, multiplying (7) throughout by $1/(2\pi)^{\frac{1}{2}} \exp(i\xi z)$, integrating with respect to z from $z = -\infty$ to $z = \infty$, making use of boundary conditions (8) and employing the notation of (1), we have

$$\frac{\partial \bar{\psi}}{\partial \theta} + \xi^2 \bar{\psi} = \frac{\partial^2 \bar{\psi}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\psi}}{\partial r} + U(\theta) \cdot \bar{S}(r, \xi) \tag{9}$$

Again multiplying (9) throughout by $r J_0(\xi_i r)$, integrating with respect to r from $r = 0$ to $r = a$ employing the formulae (8), (1, p. 87, (62)) and (3), we have

$$\frac{d \bar{\psi}_J}{d \theta} + (\xi^2 + \xi_i^2)\, \bar{\psi}_J = U(\theta) \cdot \bar{S}_J(\xi_i, \xi) \tag{10}$$

The solution of this ordinary differential equation (10) is

$$\bar{\psi}_J = \exp[-(\xi^2 + \xi_i^2)\theta] \cdot \bar{S}_J(\xi_i, \xi) \cdot \int_0^{\theta} U(\theta') \exp[(\xi^2 + \xi_i^2)\theta']\, d\theta' \tag{11}$$

which with the help of the inversion formulae (2) and (4) reduces to

$$\psi(r, z, \theta) = \frac{1}{a^2} \cdot \sqrt{\frac{2}{\pi}} \cdot \sum_i \frac{J_0(r\xi_i)}{(J_1(a\xi_i))^2} \int_{-\infty}^{\infty} e^{-i\xi z} \int_0^{\theta} U(\theta') \cdot \exp(-(\xi^2 + \xi_i^2)(\theta - \theta'))\, d\theta' \cdot d\xi \tag{12}$$

Special Cases

(*i*) If in (12), $S(r, z) = S(r).\, g(z)$ where $S(r)$ and $g(z)$ are functions of r and z respectively, then,

$$\bar{S}_J(\xi_i, \xi) = S_J(\xi_i).\, \bar{g}(\xi);$$

so (12) becomes

$$\psi(r, z, \theta) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{a^2} \sum_i \frac{J_0(r\,\xi_i) \cdot S_J(\xi_i)}{(J_1(a\,\xi_i))^2} \int_{-\infty}^{\infty} \exp(-i\,\xi\,z)\,\bar{g}(\xi) \cdot$$

$$\cdot \int_0^{\infty} U(\theta') \cdot \exp[-(\xi^2 + \xi_i^2)(\theta - \theta')]\, d\theta'\, d\xi.$$

(*ii*) Again of $S(r, z) = S_0 \cdot \delta(r - r') \cdot \delta(z - z')$ at (r', z') then making use of (1, p. 33, (77f)),

$$\bar{S}_J(\xi_i, \xi) = S_0 \cdot r' \cdot J_0(\xi_i\, r') \exp(i\,\xi\,z'),$$

so (12) becomes

$$\psi(r, z, \theta) = (2/\pi)^{\frac{1}{2}} \cdot \frac{1}{a^2} S_0 \cdot r' \cdot \sum_i \frac{J_0(r\,\xi_i) \cdot J_0(r'\,\xi_i)}{(J_1(a\,\xi_i))^2} \cdot \int_{-\infty}^{\infty} \exp[-i\,\xi(z - z')] \cdot$$

$$\cdot \int_0^{\theta} U(\theta') \cdot \exp[-(\xi^2 + \xi_i^2)(\theta - \theta')]\, d\theta'\, d\xi.$$

ACKNOWLEDGEMENTS

I am thankful to Dr. V. M. Bhise of G. S. Institute of Technology and Science, Indore for his valuable guidance in the preparation of this paper. My thanks are also due to Dr. S. M. Das Gupta, Director, G.S.I.T.S., Indore for providing facilities to carry out the work.

REFERENCES

1. SNEDDON, I. N., "Fourier Transforms" (McGraw Hill Book Company Inc., New York), 1951, pp. 206–226, p. 19 (38) p. 83 & p. 87 (62).
2. DESHPANDE, V. L. & BHISE, V. M., Slowing down of Neutrons in a cylindrical pile—Koninkl., *Nederl. Akade, Van Wetens chappen* —Amsterdam, series A, 74. No. 4, p. 297–300 (1971).