ON ATTAINMENT OF CONSTANT PRESSURE IN AN H/L GUN WITH MODERATED CHARGES

R. N. BHATTACHARYYA

Maharaja Monindra Chandra College, Calcutta

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It has been shown that suitable moderated charge with two components can be found such that the pressures in both the first and second chamber in an H/L gun remain absolutely constant during the period the second component burns, the constant pressures being equal to the pressures at burnt of the first component.

The internal ballistics of high-low pressure gun was discussed by Kapur[1] and Aggarwal[2] for the general form function. In the present paper the author has discussed the problem of H/L gun with moderated charges having two components and has demonstrated that if a moderated charge of two components (of which first component is known and the second component is also known except for the size and shape) burns in an H/L gun, pressures in both the chambers can be kept constant during the period of burning of the second component. Generally the solution determines two relations between the four characteristics of the second propellant component of which two may be known from the physical properties of the propellant so that the other two may be calculated. Also the internal ballistics is calculated when the pressures are constant.

The ballistic equations in the non-isothermal model, when the first component of the moderated charge is burning in an H/L gun, are the following :

The equations of state for the gases in the first and second chamber are :

$$P\left[U_1 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2} + \frac{C_1}{\delta_1}\phi_1 - (C_1 + C_2)N\eta\right] = (C_1 + C_2)NRT_1 \quad (1)$$

and

$$P_2\left[U_2 + Ax - C_1\phi_1\eta + (C_1 + C_2)N\eta\right] = \left[C_1\phi_1 - (C_1 + C_2)N\right]RT_2 \quad (2)$$

where N is the fraction of the total charge turned into gas.

The equation of continuity (when $\omega < \omega^*$) is

$$(C_1 + C_2)\frac{dN}{dt} = C_1\frac{d\phi_1}{dt} - \frac{\psi S P_1}{\sqrt{RT_1}} \quad (3)$$

where

$$\psi = \left(\frac{2r}{r-1}\right)^{\frac{1}{2}}$$

we take the law of burning and form function as

$$D_1\frac{df_1}{dt} = -\beta_1 P_1 \quad (4)$$

and

$$\phi_1 = (1 - f_1)(1 + \theta_1 f_1) \quad (5)$$

The equations of Energy[1] for the first and second chamber are :

$$\frac{d}{dt}\left[(C_1 + C_2)NT_1\right] = T_0 C_1\frac{d\phi_1}{dt} - r\,T_1\frac{d}{dt}\left[C_1\phi_1 - (C_1 + C_2)N\right] \quad (6)$$

and

$$\frac{d}{dt}\left[\{ C_1 \phi_1 - (C_1 + C_2) N \} T_2\right] = r\, T_1 \frac{d}{dt}\left[C_1 \phi_1 - (C_1 + C_2) N\right] \qquad (7)$$

The equation of motion of the shot is given by

$$W \frac{dv}{dt} = A\, P_2 \qquad (8)$$

These equations are solved with initial condition. We suppose that this solution gives $P_1 = P_{1B1}$, $P_2 = P_{2B1}$, $T_1 = T_{1B1}$, $T_2 = T_{2B1}$, $N = N_{B1}$, $v = v_{1B}$, and $x = x_{B1}$ when $\phi_1 = 1$ i.e. $f_1 = 0$. For integrations of equations during the second stage of burning, we will require the above quantities at the instant when the first component burns out. Here also we note that for solving the above equations suppose that C_2 and δ_2 (or C_2/δ_2) are known.

Ballistic equations when the second component burns.

The equations of state for the gases in the first and the second chamber are :

$$P_1\left[U_1 - \frac{C_2}{\delta_2} + \frac{C_2}{\delta_2}\phi_2 - (C_1 + C_2) N\eta\right] = (C_1 + C_2) NRT_1 \qquad (9)$$

and

$$P_2\left[U_2 + Ax - (C_1 + C_2\phi_2)\,\eta + (C_1 + C_2) N\eta\right] = \left[C_1 + C_2\phi_2 - (C_1 + C_2) N\right] RT_2 \qquad (10)$$

The equation of continuity (when $\omega < \omega^*$) is

$$(C_1 + C_2)\frac{dN}{dt} = C_2\frac{d\phi_2}{dt} - \frac{\psi\, SP_1}{\sqrt{RT_1}} \qquad (11)$$

Further we have the equation of burning as

$$D_2\frac{df_2}{dt} = -\beta_2 P_1\ , \qquad (12)$$

the form function as

$$\phi_2 = (1 - f_2)\,(1 + \theta_2 f_2) \qquad (13)$$

and the equations of energy for the first and second chamber as

$$\frac{d}{dt}\left[(C_1 + C_2) NT_1\right] = T_0\, C_2\frac{d\phi_2}{dt} - r\, T_1\frac{d}{dt}\left[C_2\phi_2 - (C_1 + C_2) N\right] \qquad (14)$$

and

$$\frac{d}{dt}\left[\left\{C_1 + C_2\phi_2 - (C_1 + C_2) N\right\} T_2\right] = r\, T_1\frac{d}{dt}\left[C_2\phi_2 - (C_1 + C_2) N\right] \qquad (15)$$

The equation of motion is

$$W\frac{dv}{dt} = A\, P_2 \qquad (16)$$

We are to obtain the solution of these equations with initial conditions $x = x_{B1}$, $v = v_{B1}$, $P_1 = P_{1B1}$, $P_2 = P_{2B1}$, $T_1 = T_{1B1}$, $T_2 = T_{2B1}$ and $N = N_{B1}$ at $f_2 = 1$.

Here x_{B1}, v_{B1}, P_{1B1}, P_{2B1}, T_{1B1}, T_{2B1} and N_{B1} are the values of x, v, P_1, P_2, T_1, T_2 and N when the first component has just burnt out.

Let us assume that the solutions of the above equations are possible when

$$P_1 = P_{1B1} \text{ and } P_2 = P_{2B1} \tag{17}$$

and seek conditions so that this solution may give $x=x_{B1}$, $v=v_{B1}$, $N=N_{B1}$, $T_1=T_{1B1}$ and $T_2=T_{2B1}$ at $f_2=1$ and the system of (9) to (16) may remain consistent for the solutions $P_1=P_{1B1}$ and $P_2=P_{2B1}$ with the (17), (12) and (16), we have

$$\frac{dv}{df_2} = -\frac{A}{W}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}} \tag{18}$$

or

$$\frac{d}{df_2}\left(\frac{df_2}{dt}\cdot\frac{dx}{df_2}\right) = -\frac{A}{W}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}}$$

Which by (12) and (17) reduces to

$$\frac{d^2x}{df_2^2} = \frac{AD_2^2}{\beta_2^2\omega P_{1B1}}\cdot\frac{P_{2B1}}{P_{1B1}} \tag{19}$$

Integrating (18) with the condition $v = v_{B1}$ at $f_2 = 1$, we have

$$v = v_{B1} + \frac{A}{W}\,\frac{D_2}{\beta_2}\,\frac{P_{2B1}}{P_{1B1}}\,(1 - f_2) \tag{20}$$

Now we impose the conditions that (9) and (10) with (17) gives

$$x = x_{B1},\ T_1 = T_{1B1},\ T_2 = T_{2B1} \text{ and } N = N_{B1} \text{ at } f_2 = 1$$

and further that (9) and (10) are consistent with (19).

Now $x = x_{B1}$, $T_2 = T_{2B1}$, $N = N_{B1}$ will satisfy (10), if

$$P_{2B1}\,[\,U_2 + A\,x_{B1} - C_1\eta + (C_1 + C_2)\,N_{B1}\eta\,] = [\,C_1 - (C_1 + C_2)N_{B1}\,]\,RT_{2B1} \tag{21}$$

which is true because (21) is obtained from (2) by considering values when the first component burns out. Also $N = N_{B1}$ and $T_1 = T_{1B1}$ will satisfy (9), if

$$P_{1B1}\left[\,U_1 - \frac{C_2}{\delta_2} - (C_1 + C_2)\,N_{B1}\,\eta\,\right] = (C_1 + C_2)\,N_{B1}\,RT_{1B1} \tag{22}$$

which is true because (22) is obtained from (1) by considering values when the first component burns out.

With the help of (12), equations (11), (14), and (15) can be written as

$$(C_1 + C_2)\,\frac{dN}{df_2} = C_2\,\frac{d\,\phi_2}{d\,f_2} + \frac{D_2}{\beta_2}\,\frac{\psi\,S}{\sqrt{RT_1}} \tag{23}$$

$$\frac{d}{df_2}\,[\,(C_1 + C_2)\,NT_1\,] = T_0\,C_2\,\frac{d\,\phi_2}{d\,f_2} - r\,T_1\,\frac{d}{d\,f_2}\,[C_2\,\phi_2 - (C_1 + C_2)\,N] \tag{24}$$

and

$$\frac{d}{df_2}\,[\,\{\,C_1 + C_2\,\phi_2 - (C_1 + C_2)\,N\,\}\,T_2\,] = r\,T_1\,\frac{d}{df_2}\,[\,C_2\,\phi_2 - (C_1 + C_2)\,N\,] \tag{25}$$

From (24) and (25) by integration, we get

$$\{\,C_1 + C_2\,\phi_2 - (C_1 + C_2)\,N\,\}\,T_2 + (C_1 + C_2)\,N\,T_1 = T_0\,C_2\,\phi_2 + \\ + \{\,C_1 - (C_1 + C_2)\,N_{B1}\,\}\,T_{2B1} + (C_1 + C_2)\,N_{B1}\,T_{1B1} \tag{26}$$

Again from (6) and (7), we get

$$\{ C_1 - (C_1 + C_2) N_{B1} \} T_{2B1} + (C_1 + C_2) N_{B1} T_{1B1} = T_0 C_1$$

Hence (26) reduces to

$$\{ C_1 + C_2 \phi_2 - (C_1 + C_2) N \} T_2 + (C_1 + C_2) N T_1 = T_0 (C_1 + C_2 \phi_2) \tag{27}$$

Adding (9) and (10) with (17) and (27), we get

$$P_{2B1} [U_2 + A x - (C_1 + C_2 \phi_2) \eta + (C_1 + C_2) N \eta] +$$
$$+ P_{1B1} \left[U_1 - \frac{C_2}{\delta_2} + \frac{C_2}{\delta_2} \phi_2 - (C_1 + C_2) N \eta \right] = R T_0 (C_1 + C_2 \phi_2) \tag{28}$$

Differentiating (28) and (13) with respect to f_2 with the help of (12), (17) and (23), we get

$$\frac{-A P_{2B1} v D_2}{\beta_2 P_{1B1}} + \frac{\eta P_{2B1} \psi S D_2}{\beta_2 \sqrt{R T_1}} + \frac{C_2}{\delta_2} P_{1B1} \{ (1 - f_2) \theta_2 - (1 + \theta_2 f_2) \} -$$
$$- \eta P_{1B1} \left\{ C_2 (\theta_2 - 1 - 2\theta_2 f_2) + \frac{\psi S D_2}{\sqrt{R T_1} \beta_2} \right\} = R T_0 C_2 (\theta_2 - 1 - 2 \theta_2 f_2)$$

Now $v = v_{B1}$, $T_1 = T_{1B1}$ and $f_2 = 1$ will satisfy the above equation, if

$$- A v_{B1} \frac{D_2}{\beta_2} \frac{P_{2B1}}{P_{1B1}} + (1 + \theta_2) \left\{ T_0 R C_2 + C_2 P_{1B1} (\eta - 1/\delta_2) \right\} =$$
$$= \frac{D_2}{\beta_2} \frac{\eta \psi S}{\sqrt{R T_{1B1}}} (P_{1B1} - P_{2B1}) \tag{29}$$

Introducing the following dimensionless constants

$$\frac{C_2}{C_1} = \beta_0, \quad \frac{D_2 / \beta_2}{D_1 / \beta_1} = \alpha_0, \quad \Psi = \frac{\psi S D_1}{\beta_1 C_1 \sqrt{R T_0}}, \quad \eta_{B1} = \frac{v_{B1} A D_1}{C_1 \beta_1 R T_0},$$

$$\frac{P_{2B1}}{P_{1B1}} = \omega_{B1}, \quad \frac{T_{1B1}}{T_0} \quad T_0^1 \quad \frac{\eta P_{1B1}}{R T_0} = r_0 \text{ and } (\eta - 1/\delta_2) \frac{P_{1B1}}{R T_0} = \delta_0 \tag{30}$$

(29) reduces to

$$1 + \theta_2 = \frac{\alpha_0}{\beta_0} \frac{r_0 \Psi (1 - \omega_{B1}) / \sqrt{T_0^1} + \eta_{B1} \omega_{B1}}{1 + \delta_0} \tag{31}$$

Now to satisfy that (19) and (28) should be consistent, we differentiate (28) twice with respect to f_2 and substitute (19), and with the help of (13) and (23), we get

$$\frac{A^2 D_2^2 P_{2B1}^2}{\beta_2^2 \omega P_{1B1}^2} - \frac{\eta P_{2B1} D_2 \psi S}{2 \beta_2 \sqrt{R} T_1^{3/2}} \frac{d T_1}{d f_2} - 2 \theta_2 \frac{C_2}{\delta_2} P_{1B1} -$$
$$- \eta P_{1B1} \left\{ - 2 \theta_2 C_2 - \frac{D_2 \psi S}{2 \beta_2 \sqrt{R} T_1^{3/2}} \frac{d T_1}{d f_2} \right\} = - 2\theta_2 R T_0 C_2$$

or

$$- 2 \theta_2 C_2 [R T_0 + (\eta - 1/\delta_2) P_{1B1}] =$$
$$= \frac{A^2 D_2^2 P_{2B1}^2}{\beta_2^2 \omega P_{1B1}^2} + \frac{\eta D_2 \psi S}{2 \beta_2 \sqrt{R} T_1^{3/2}} (P_{1B1} - P_{2B1}) \frac{d T_1}{d f_2} \tag{32}$$

Again from (23) and (24), we get

$$T_1 \frac{d}{d f_2} (C_1 + C_2) N + (C_1 + C_2) N \frac{d T_1}{d f_2} = T_0 C_2 \frac{d \phi_2}{d f_2} + r T_1 \frac{D_2}{\beta_2} \frac{\psi S}{\sqrt{R T_1}} .$$

Hence

$$\frac{dT_1}{df_2} = \frac{1}{(C_1+C_2)\,N}\left[C_2\,(T_0-T_1)\,(\theta_2-1-2\,\theta_2 f_2) + \frac{D_2}{\beta_2}\;\frac{\psi S\,(r-1)}{\sqrt{R}}\;\sqrt{T_1}\right] \quad (33)$$

Since $T_1 = T_{1B1}$, $N = N_{B1}$ and $f_2 = 1$ will satisfy (32) and (33), we get

$$-2\theta_2\,C_2\left[R\,T_0 + \left(\eta - \frac{1}{\delta_2}\right)P_{1B1}\right] = \frac{A^2 D_2^2 P_{2B1}^2}{\beta_2^2\,\omega\,P_{1B1}^2} +$$

$$+\frac{\eta\,D_2\,\psi S\,(P_{1B1}-P_{2B1})}{2\beta_2\sqrt{R}\,T_{1B1}^{3/2}\,(C_1+C_2)\,N_{B1}}\left[-(1+\theta_2)\,C_2\,(T_0-T_{1B1}) + \frac{D_2}{\beta_2}\;\frac{\psi S\,(r-1)}{\sqrt{R}}\;\sqrt{T_{1B1}}\right]$$

Now, we introduce the central ballistic parameter

$$M_1 = \frac{A^2\,D_1^2}{\beta_1^2\,\omega\,C_1\,R\,T_0}$$

Then from (30) the above equation can be written in the non-dimensional form as

$$-2\theta_2\,(1+\delta_0) = M_1\,\frac{\alpha_0^2}{\beta_0}\;\omega_{B1}^2 + \frac{r_0\,(1-\omega_{B1})\,\Psi\,\alpha_0}{2\,\beta_0\sqrt{T_0^1}\,N_{B1}\,(1+1/\beta_0}\;\cdot$$

$$\cdot\left[(1+\theta_2)\left(1-\frac{1}{T_0^1}\right) + \frac{\Psi\,\alpha_0\,(r-1)}{\beta_0\sqrt{T_0^1}}\right]$$

or

$$-2\theta_2\,(1+\delta_0) = M_1\,\frac{\alpha_0^2\omega_{B1}^2}{\beta_0} + \frac{r_0\,\Psi\,\alpha_0\,(1-\omega_{B1})}{2\,(1+\beta_0)\sqrt{T_0^1}\,N_{B1}}\cdot\left[(1+\theta_2)\left(1-\frac{1}{T_0^1}\right) + \frac{\Psi\alpha_0\,(r-1)}{\beta_0\sqrt{T_0^1}}\right] \quad (34)$$

The simultaneous satisfaction of (31) and (34) gives the condition that $P_1 = P_{1B1}$ and $P_2 = P_{2B1}$ may be the solutions of (9) to (16). Equations (31) and (34) actually give two equations connecting fou parameters α_0, θ_2, β_0 and δ_0 defining the second propellant component, the properties and mass of the first propellant component being assumed to be known. δ_0 involves η and $1/\delta_2$ and β_0 involves C_2 which are supposed to be known, as noted earlier, in the integration of equations for the first stage of burning.

Hence we may look upon (31) and (34) as two equations for θ_2 and α_0.

Eliminating θ_2 from (31) and (34), we get a quadratic equation for α_0. To complete our solution we should show that as given by (31) and (34), α_0 is positive and θ_2 satisfies $-1 < \theta_2 \leqslant 1$ for practical values of constants involved in (31) and (34). As the solution of the Ballistic equation in the non-isothermal model is not known, we illustrate it considering the isothermal model to have some idea about the results. From Kapur[1] and Tables given by Corner[3], we take the values of the constants.

With the constants, thus determined, we tabulate the value of α_0 and θ_2. The shape and size of the second component charge as follows. Since the range of T_0^1 is not known, we take its value considering that $T_0^1 < 1$.

Considering isothermal model, we get from Corner[3]

$$b = \frac{C_1\,[\,1/\delta_1 - \eta\,(1-\Psi)\,]}{U_1 - C_1/\delta_1}\;,\quad N_{B1} = (1-\Psi)$$

Let η = 25 cu. in./lb., $1/\delta_1 = 1/\delta_2 = 17{\cdot}5$ cu. in./lb.,

then
$$\delta_0 = \left(\eta - \frac{1}{\delta_2}\right) \frac{b\,(1-\Psi)}{[1/\delta_1 - \eta\,(1-\Psi)](1+b)}$$

$$r_0 = \eta \frac{b\,(1-\Psi)}{[1/\delta_1 - \eta\,(1-\Psi)](1+b)}$$

and
$$\omega_{B1} = \frac{(U_1 - C_1/\delta_1)\,(1+b)}{U_2 + Ax_{B1} - C_1/\delta_1\,\Psi} \cdot \frac{\Psi}{1-\Psi}$$

with
$$X_B = \mu \left(\frac{W}{\lambda C_1 \psi}\right)^{\frac{1}{2}} \frac{U_2 + Ax_{B1}}{A}$$

and
$$X_0 = \mu \left(\frac{W}{\lambda C_1 \psi}\right)^{\frac{1}{2}} \frac{U_2}{A}$$

For given X_0, ν and b, x_B is determined from the Table[3] and then,
$$\omega_{B1} = \frac{\delta_1\,[\,1/\delta_1 - \eta\,(1-\Psi)\,]\,(1+b)}{b\,(1-\Psi)\,(X_B/\nu - 1)}$$

with
$$\nu = \frac{\mu\eta}{A} \left(\frac{\omega C_1 \psi}{\lambda}\right)^{\frac{1}{2}}$$

It is known that $\nu < 0{\cdot}3$ and $-0{\cdot}5 < b < 0{\cdot}5$ and in H/L gun it is expected that ν is less than b. Given b, ν and X_0, ω_{B1} is determined.

$$\eta_{B1} = \left(\frac{1}{1+b} \frac{dX}{d\phi}\right)_B \frac{\Psi\,(1-\Psi)\,b}{\nu\,\delta_1\,[1/\delta_1 - \eta\,(1-\Psi)]}$$

For given b, ν and X_0, $\left(\frac{1}{1+b} \frac{dX}{d\phi}\right)_B$ is determined from the Table[1] and then η_{B1} is determined.

TABLE 1

VALUES OF T_0^1, a_0 AND θ_2 FOR $r = 1{\cdot}25$, $\psi = 0{\cdot}5$, $\beta_0 = 1$ AND $b = 0.2$, $\nu = 0{\cdot}1$, $X_0 = 0.1$

T_0^1	a_0	θ_2	T_0^1	a_0	θ_2
0·3	1·287	—0·2497	0·7	1·343	—0·3312
0·4	1·310	—0·2795	0·8	1·354	—0·3400
0·5	1·317	—0·3012	0·9	1·370	—0·3698
0·6	1·330	—0·3283			

TABLE 2

VALUES OF T_0^1, a_0 AND θ_2 FOR $r = 1.25$, $\psi = 0.5$, $\beta_0 = 1$ AND $b = 0.4$, $\nu = 0.2$, $X_0 = 1.0$

T_0^1	a_0	θ_2	T_0^1	a_0	θ_2
0·4	1·465	—0·1232	0·7	1·612	—0·1823
0·5	1·523	—0·1578	0·8	1·620	—0·2012
0·6	1·578	—0·1636	0·9	1·630	—0·2339

Also M_1 can be determined from the formula:

$$M_1 = \frac{\Psi (1 - \Psi)^2 b^2 \eta^2}{\nu^2 [1/\delta_1 - \eta (1 - \Psi)]^2}$$

Travel and Velocity during the Burning of the Second Component

We can show from (20), that

$$\frac{v_{B2}}{v_{B2}} = 1 + \frac{M_1 \alpha_0}{\eta_{B1}} \omega_{B1} \tag{35}$$

Which is the velocity ratio. From the above examples with $T_0^1 = 0{\cdot}8$, we get

$$\frac{v_{B2}}{v_{B1}} = 2{\cdot}67 \text{ and } 2{\cdot}32$$

Also from (19), (12) and (17), we get

$$-\frac{dx}{df_2} \cdot \frac{\beta_2 P_{1B1}}{D_2} = v_{B1} + \frac{A}{W} \frac{D_2}{\beta_2} \frac{P_{2B1}}{P_{1B1}} (1 - f_2)$$

Integrating this taking $x = x_{B1}$ at $f_2 = 1$

$$\frac{\beta_2 P_{1B1}}{D_2} (x - x_{B1}) = v_{B1} (1 - f_2) + \frac{A D_2 P_{2B1}}{2 \omega \beta_2 P_{1B1}} (1 - f_2)^2$$

Let $x = x_{B2}$ when the second component burns out i.e. $f_2 = 0$.

Therefore

$$x_{B2} - x_{B1} = \frac{D_2 v_{B1}}{\beta_2 P_{1B1}} + \frac{A D_2^2 P_{2B1}}{2 \omega \beta_2^2 P_{1B1}^2} = \frac{D_2}{\beta_2 P_{1B1}} \left[v_{B1} + \frac{A D_2}{2 \beta_2 \omega} \frac{P_{2B1}}{P_{1B1}} \right]$$

Then from (30), we get

$$\frac{x_{B2} - x_{B1}}{x_{B1}} = \frac{\eta C_1}{r_0 A x_{B1}} \left[\alpha_0 \eta_{B1} + \frac{M_1 \alpha_0^2}{2} \omega_{B1} \right]$$

Let us introduce another dimensionless quantity

$$\xi_{B1} = \frac{\eta C_1}{A x_{B1}}$$

Hence the travel ratio

$$= \frac{\xi_{B1}}{r_0} \left[\alpha_0 \eta_{B1} + \frac{M_1 \alpha_0^2}{2} \omega_{B1} \right] \tag{36}$$

Now differentiating (9) with respect to f_2 and using (17), (23) and (24)

$$P_{1B1} \frac{C_2}{\delta_2} \frac{d\phi_2}{df_2} - \eta P_{1B1} \left[C_2 \frac{d\phi_2}{df_2} + \frac{D_2}{\beta_2} \frac{\psi S}{\sqrt{RT_1}} \right] = R \left[T_0 C_2 \frac{d\phi_2}{df_2} + \frac{r \psi S D_2}{\beta_2 \sqrt{R}} \sqrt{T_1} \right]$$

or $$C_2 (\theta_2 - 1 - 2\theta_2 f_2) \left[R T_0 + \left(\eta - \frac{1}{\delta_2} \right) P_{1B1} \right] = - \frac{\psi S D_2}{\beta_2} \left[\frac{\eta P_{1B1}}{\sqrt{RT_1}} + r \sqrt{RT_1} \right]$$

From (30) this can be reduced to

$$(\theta_2 - 1 - 2\,\theta_2 f_2)(1 + \delta_0) = -\,\Psi \frac{\alpha_0}{\beta_0}\left[\frac{r_0}{\sqrt{T^1}} + r\sqrt{T^1}\right] \tag{37}$$

where

$$T^1 = \frac{T_1}{T_0}$$

This determines T^1 as a fraction of f_2.

Then (23) can be written as

$$\left(1 + \frac{1}{\beta_0}\right)\frac{dN}{df_2} = \frac{d\phi_2}{df_2} + \Psi\,\frac{\alpha_0}{\beta_0}\;\frac{1}{\sqrt{T_0^1}}$$

Putting the value of T^1 from (37) and then integrating the above equation, we get N as a function of f_2. Hence

$$N = \frac{\beta_0}{1 + \beta_0}(1 - f_2)(1 + \theta_2 f_2) + \frac{\Psi\alpha_0}{1 + \beta_0}\int \frac{1}{\sqrt{T^1}}\,df_2 + B \tag{38}$$

where B is determined by the initial condition $N = N_{B1}$

when $f_2 = 1$

As T_1 and N are determined from (37) and (38), T_2 is determined as a function of f_2 from the relation (27).

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