

# RATIONAL TURBIDITY FACTOR IN RELATION TO AIR MASS

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In this Paper the effect of variable air mass has been investigated and a formula has been presented connecting  $T_r$  with  $B$ ,  $W$  and  $m_r$ . Value of  $I$  computed from the proposed formula have been found to agree very well with the values obtained from Schuepp's diagram duly corrected for  $W$ .

The Rational Turbidity Factor  $T_r$  as a new measure of total atmospheric turbidity was introduced by the present authors<sup>1</sup>, in order to overcome the limitations of Linke's Turbidity Factor<sup>2</sup>,  $T$ . In our earlier paper<sup>3</sup>,  $T_r$  was correlated with  $B$  is the Angstrom-Schuepp Turbidity Coefficient and  $W$  is the precipitable water vapour in the atmosphere in cm. It was shown in that paper<sup>3</sup> that the effects of  $B$  and  $W$  are inseparable; because of the interaction of scattering by aerosols and absorption by water vapour within the absorption bands of water vapour.

The object of the present paper is to investigate the effect of varying air masses ( $m_r$ ) on  $T_r$  in relation to  $B$  and  $W$ .

## CORRELATION OF $T_r$ WITH $B$ , $W$ AND $m_r$

From Shuepp's diagram<sup>4</sup>, after applying due corrections as proposed by him, values of direct solar radiation at normal incidence ( $I$ ) and corresponding  $T_r$  were obtained and presented in Table-1, for different values of  $B$  (0 to 1),  $W$  (0.5 cm to 10 cm) and  $m_r$  (3 to 10). Values for  $m_r=1$  have already been presented in our earlier paper<sup>3</sup>.  $T_r$  was computed as usual by the formula<sup>1</sup>.

$$T_r = \frac{1}{m_r} \left( \frac{0.32491 - \log I}{0.072375} \right)^{1/0.57} \quad (1)$$

TABLE 1

INTENSITY OF DIRECT SOLAR RADIATION AT NORMAL INCIDENCE  $I$  IN cal/cm<sup>2</sup>. min AND RATIONAL TURBIDITY FACTOR  $T_r$  FOR DIFFERENT VALUES OF  $B$ ,  $W$  AND  $m_r$ , AT MEAN SOLAR DISTANCE.

( $I$  values from Schuepp<sup>4</sup> with due corrections)

Atmospheric pressure  $\approx 1000$ mb,  $0_s \approx 0.34$  cm NTP (IGY Scale)

		Precipitable water $W$ cm							
$m_r$	$B$	0.5		2.0		5.0		10.0	
		$I$	$T_r$	$I$	$T_r$	$I$	$T_r$	$I$	$T_r$
3		1.23	2.63	1.14	3.31	1.06	4.03	0.975	4.93
	0.1	0.895	6.43	0.770	7.86	0.698	9.25	0.642	10.51
	0.2	0.616	11.16	0.548	13.08	0.490	15.04	0.441	16.99
	0.4	0.335	22.56	0.287	26.00	0.248	29.42	0.223	32.03
	0.6	0.207	33.92	0.170	39.12	0.145	43.55	0.128	47.17
	1.0	0.100	54.70	0.0780	62.75	0.0648	69.07	0.0564	73.97

TABLE 1—contd.

		Precipitable water $W$ cm							
$m_r$	$B$	0.5		2.0		5.0		10.0	
		$I$	$T_r$	$I$	$T_r$	$I$	$T_r$	$I$	$T_r$
6	0	0.969	2.50	0.865	3.17	0.782	3.82	0.720	4.40
	0.1	0.492	7.48	0.427	8.81	0.376	10.07	0.341	11.09
	0.2	0.286	13.04	0.235	15.37	0.204	17.15	0.183	18.57
	0.4	0.120	24.55	0.095	28.16	0.0792	31.12	0.068	33.68
	0.6	0.0627	35.11	0.0475	40.12	0.0383	44.19	0.0325	47.42
	1.0	0.0235	54.07	0.0166	61.61	0.0134	66.46	0.0113	70.44
10	0	0.759	2.42	0.650	3.10	0.589	3.56	0.542	3.98
	0.1	0.292	7.68	0.237	9.16	0.208	10.14	0.188	10.93
	0.2	0.141	13.31	0.108	15.69	0.0920	17.20	0.0800	18.57
	0.4	0.0475	24.07	0.0340	27.91	0.0279	30.30	0.0239	32.23
	0.6	0.0200	34.51	0.0137	39.57	0.0111	42.52	0.0095	44.75
	1.0	0.0055	52.99	0.00355	60.02	0.0029	63.40	0.0024	66.62

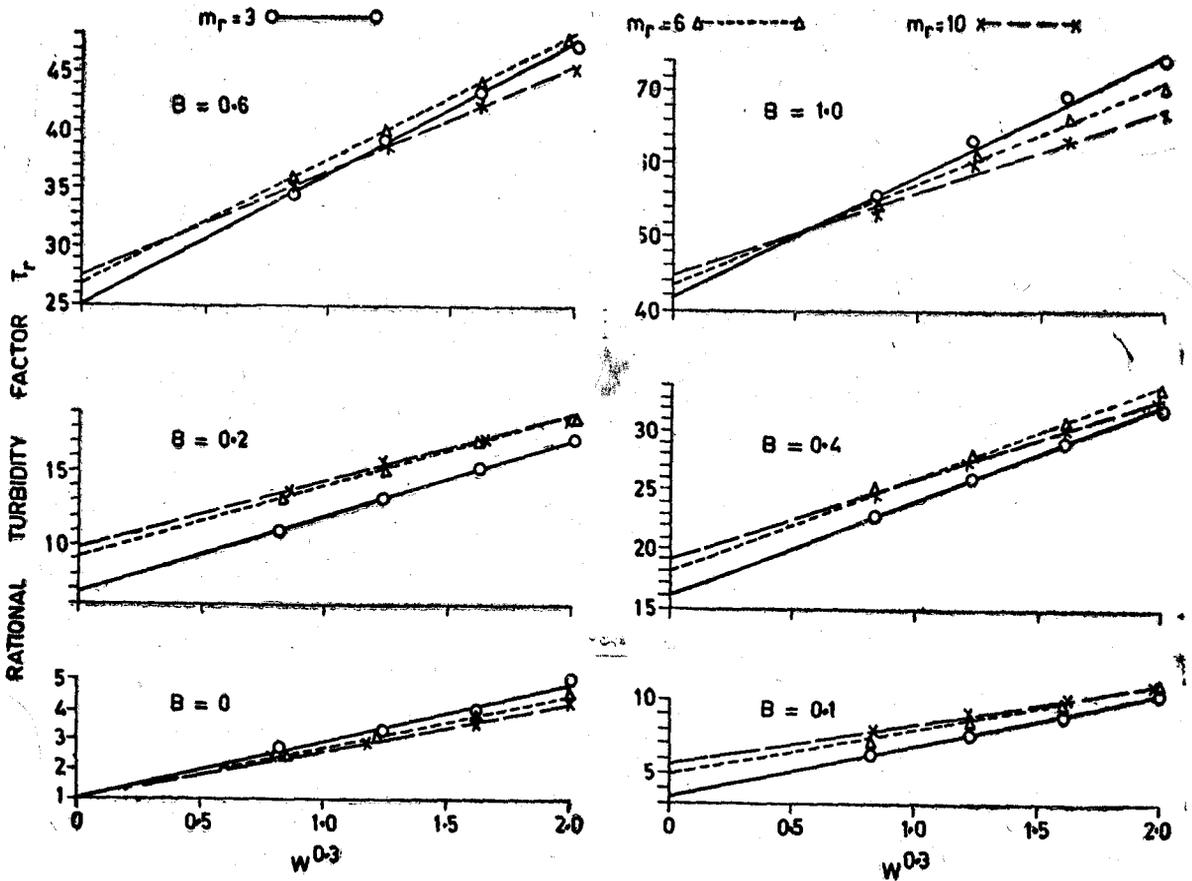


Fig. 1—Linear relationship between  $T_r$  ( $=a+b W^{0.3}$ ) &  $W^{0.3}$  for different values of  $B$  &  $m_r$ .

As in the earlier paper<sup>3</sup>,  $T_r$  has been plotted against  $W^{0.3}$  for different values of  $B$  (0 to 1) and different values of  $m_r$  (3, 6, 10) in Fig. 1. As expected,  $T_r$  could be expressed in the form

$$T_r = a + b W^{0.3} \tag{2}$$

where  $a$  and  $b$  are related to  $B$  and  $m_r$ . Values of  $a$  and  $b$  were roughly estimated from the figure. Since for  $W = 0$ , and  $B = 0$ ,  $T_r$  should be 1,  $(a-1)$  has been plotted against  $B$  on log-log scale in Fig. 2, for each

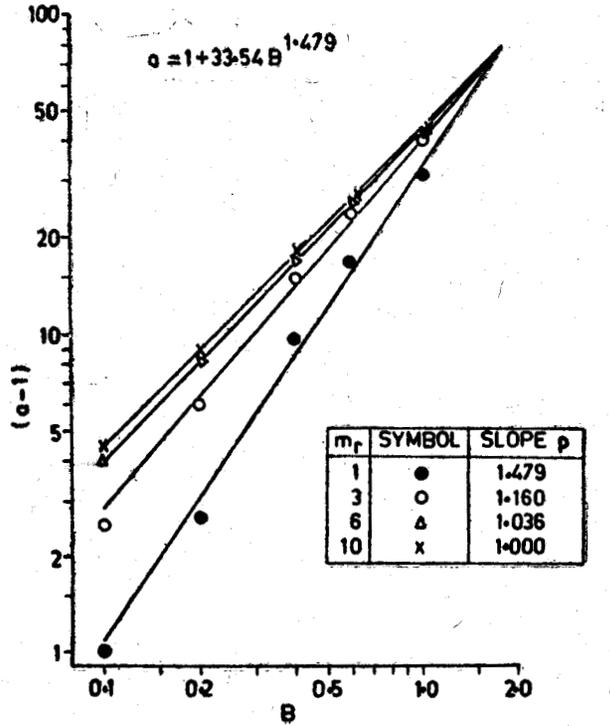


Fig. 2— $(a-1)$  plotted against  $B$  on log-log scale for different values of  $m_r$ .

value of  $m_r$ . The values for unit air mass were taken from our earlier paper<sup>3</sup>. It will be seen that points corresponding to each value of  $m_r$  roughly fall on a straight line, and that all the four straight lines appear to converge at one point given by  $B = 1.8$  and  $(a-1) = 80$ . The slope  $p$  of each line was determined, and the values obtained are shown in Fig. 2. It is observed that with increasing air mass, the slope  $p$  approaches unity almost exponentially, as can be seen from Fig. 3. The best fit was obtained with the equation

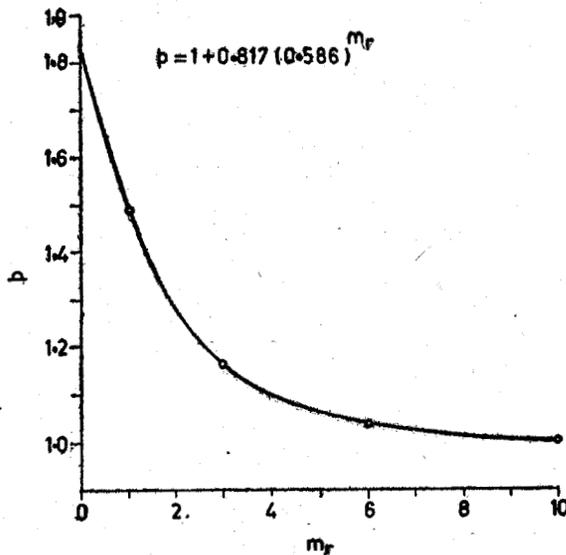


Fig. 3—Exponential fall of  $p$  with  $m_r$ .

$$p = 1 + 0.817 (0.586)^{m_r} \tag{3}$$

The equation relating  $a$  with  $B$  and  $m_r$  is thus given by

$$a = 1 + 80 (B/1.8)^{1 + 0.817 (0.586)^{m_r}} \tag{4}$$

The next problem is to correlate  $b$ , the slopes of the straight lines in Fig., with  $B$  and  $m_r$ . By plotting  $b$  against  $B$  on log-log scale, we obtained curves with concavity upwards. This implies that  $\log b$  should be linearly related not to  $\log B$  but to  $\log (B+k)$  where  $k$  is some suitable positive constant, which was found to be nearly equal to 0.1 after a few trials. In Fig. 4,  $b$  has been plotted against  $(B+0.1)$  on log-log scale, for different values of  $m_r$ . The points for each value of  $m_r$  roughly lie on a straight line, and all the four

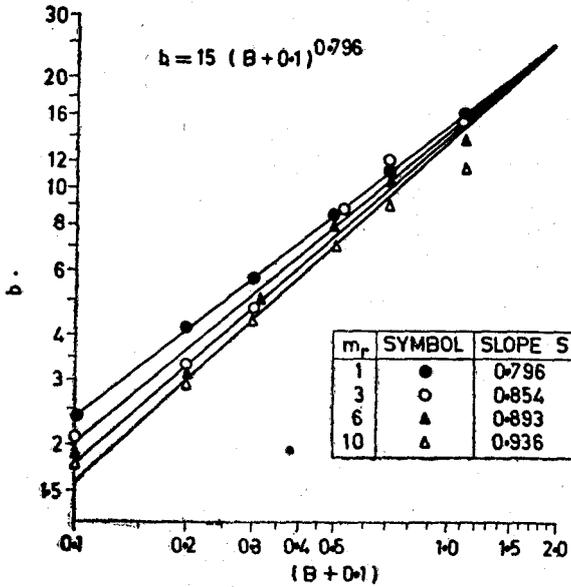


Fig. 4  $b$  — Plotted against  $(B+0.1)$  on log-log scale for different values of  $m_r$ .

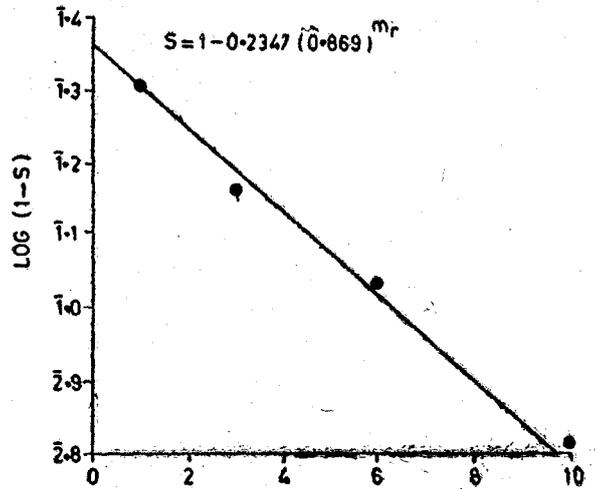


Fig. 5—Exponential rise in  $S$  with  $m_r$ .

lines appear to converge at one point corresponding to  $B = 1.8$  and  $b = 25$ . It may be noted that the point of convergence in Fig. 2 also corresponded with  $B = 1.8$ . Only one point corresponding to  $B = 1.0$ ,  $m_r = 10$ , fell very much out of the picture, which cannot be considered to be sufficiently reliable since the four values of  $I$  related to this point are all below  $0.01 \text{ cal/cm}^2 \cdot \text{min.}$  (Table 1). The values of the slope  $s$  of the four lines in Fig. 4, where estimated and indicated in the same figure. It is observed in this case as well, that with increasing air mass, the slope  $s$  approaches unity almost exponentially as will be evident from Fig. 5, in which  $\log (1-s)$  plotted against  $m_r$  reveals a linear relationship.

The best fit was obtained with the equation.

$$S = 1 - 0.2347 (0.869)^{m_r} \tag{5}$$

The equation relating  $b$  with  $B$  and  $m_r$  will, therefore, be given by

$$b = 25 \left( \frac{B + 0.1}{1.9} \right)^{1 - 0.896 (0.896)^{m_r}} \tag{6}$$

By substituting the expressions for  $a$  and  $b$  from equations (4) and (6) in equation (2), we finally obtain the equation relating  $T_r$  with  $B$ ,  $W$  and  $m_r$ , namely,

$$T_r = 1 + 80 (B/1.8)^{1 + 0.817 (0.586)^{m_r}} + \left[ 25 \left( \frac{B + 0.1}{1.9} \right)^{1 - 0.2347 (0.869)^{m_r}} \right] W^{0.3} \tag{7}$$

## Validation of proposed formula

It now remains to be seen how well the formula for  $T_r$  as given by equation (7) can predict  $I$  for values of  $m_r$ ,  $B$  and  $W$  as listed in Table 1 and in our earlier paper<sup>3</sup>.  $T_r$  estimated from equation (7), was used to compute  $I$ , according to the usual equation<sup>1</sup>, viz.,

$$\log I = 0.32491 - 0.072375 (m_r T_r)^{0.57} \quad (8)$$

These values of  $I$  computed from the proposed formula were compared against the values obtained from Schuepp's diagram<sup>4</sup> with due corrections for  $W$  as shown in Table 1. The result is shown in Fig. 6. Values of  $I$  below  $0.01 \text{ cal/cm}^2 \cdot \text{min}$ . have been omitted for reasons explained in the foregoing. This means that out of a total number of 96 points (72 from Table 1 and 24 from earlier paper<sup>3</sup>), only 5 points have been excluded from Fig. 6, which reveals a remarkably close agreement between the two sets of values, considering the wide range of values of  $m_r$ ,  $B$  and  $W$  employed.

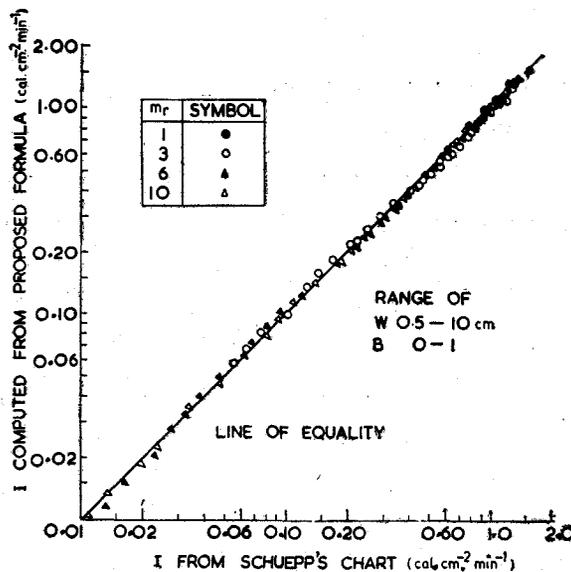


Fig. 6— $I$  computed from proposed formula compared against  $I$  obtained from Schuepp's chart for different values of  $C$ ,  $B$  &  $m_r$ ,

## DISCUSSION

The Rational Turbidity Factor,  $T_r$ , has been defined as the number of pure and dry standard atmospheres producing the same attenuation of direct solar radiation as the given turbid atmosphere. Since the entire solar spectrum (measurement without filters) is involved,  $T_r$  represents total atmospheric turbidity including absorption by precipitable water.

In this paper, an expression for  $T_r$  has been obtained (vide equation 7) in terms of Angstrom-Schuepp Turbidity Coefficient,  $B$ , precipitable water,  $W$ , and relative air mass,  $m_r$ . The first term (equal to 1) on the right hand side of equation (7) represents the effect of pure and dry air; the second term represents the contribution due to aerosols (dust, smoke, haze) while the third term reflects the contribution due to atmospheric water vapour. Our earlier finding<sup>3</sup> that the effect of  $W$  cannot be isolated from that of  $B$  is confirmed by equation (7). This has been explained as due to strong interaction between scattering by aerosols and absorption by water vapour within the absorption bands of water vapour in the infra-red region of solar radiation.

The chief merits of the Rational Turbidity Factor,  $T_r$ , are

- (i) It can be easily and quickly estimated with the help of the nomogram<sup>1</sup>, from the knowledge of intensity of direct solar radiation at normal incidence ( $I$ ) and relative air mass ( $m_r$ ).

- (ii) Unlike Angstrom Turbidity Coefficient,  $\beta$ , and Angstrom-Schuepp Turbidity Coefficient,  $B$ ,  $T_r$  does not assume any fixed size-distribution of aerosols.
- (iii) Measurement with filters is not required.
- (iv) With known value of precipitable water,  $W$ , the turbidity component due to aerosols can be worked out from equation (7). Estimation of  $W$  does not present any serious problem as shown in earlier papers<sup>5,6</sup>.

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