ELECTROMAGNETIC WAVE SCATTERING BY PLASMA CONTAINERS

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(*Received* 3 *August* 1971; *revised* 4 *February* 1972)

The scattering cross-section has been derived by an approximate method for a thin plasma plate and for thin spherical and conical shells. The computed results for scattering cross-section for a plate and spherical and conical shells are obtained. The results are presented in the form of scattering cross-section aspect angle plane.

Recent developments in space technology have led to many researches in the electromagnetic theory of a plasma medium. Scattering, reflection and radiation by a plasma is important not only as a part of electromagnetic wave theory, but also in space communication technology. The problem of scattering of electromagnetic waves in plasmas has attracted considerable attention, since this phenomenon is related to long range atmospheric propagation of short waves beyond the limits of the "radio horizon"—the idea for the explanation for the scattering of electromagnetic waves is taken from Booker and Gorden's theory[1]. The theory of scattering of plane electromagnetic waves from an isotropic sphere has been presented by Stratton[2]. It is recognized that scattering is responsible for radar return signals from the wakes of high speed re-entry objects.

This paper presents some theoretical aspects for scattering cross-section for plasma containers with computed results.

THEORETICAL APPROACH TO THE PROBLEM

When an electromagnetic wave is incident on a plasma, a polarization current[3] is induced which produces a scattered radiation pattern. We know that the polarization current density at each point of the plasma body is proportional to the total electric field at that point and the field of the scattered wave from other points of the plasma. The polarization current density is given by

$$i = j\omega \left[\epsilon_p - \epsilon_0 \right] E$$

where ϵ_p is the plasma dielectric constant and ϵ_0 is the free space dielectric constant. For this approximation, each dielectric sheet is replaced by an infinitely thin polarization current[4] sheet located at the centre of the original sheet. The resulting boundary value problem can be rigorously solved by the Wiener Hopf or related function-theoretic technique[5,6]. The solution gives the expressions for the reflection and transmission coefficients in terms of the eigenvalues of propagation constant for the problem.

Here, the solution for the total electric field in a plasma (complex medium) is difficult, but under certain conditions scattering cross-section may be obtained by the use of first order approximation of the total electric field. To find out the electric field within a thin plasma sheet, we consider a case of arbitrary incidence angle on an infinite plasma sheet. In order to solve the problem, we have the following boundary conditions as the thickness of the sheet decreases.

(*a*) the field outside the plasma approaches the incident field,

(*b*) the tangential electric field within the plasma approaches the tangential incident electric field,

(*c*) the normal electric field within the plasma approaches ϵ_0/ϵ_p times the normal incident field.

On this basis, the differential scattering[3] for far field by a small differential area dA of a thin walled plasma is given by

$$d\left[\frac{E_s}{E_i}\right] = \frac{k^2}{4\pi}\left(\frac{\epsilon_p}{\epsilon_0} - 1\right)\left[1 - \left(1 - \frac{\epsilon_0}{\epsilon_p}\right)\sin^2\theta\right]\frac{\exp(-j\,2\,kR)}{R}\,.\,t\,dA \tag{1}$$

where E_i is the amplitude of the incident linear plane wave, E_s is the back scattered field of the incident polarization, R is the distance to the scattering element, θ is the angle between the incident electric field and plane tangent to the surface, t is the wall thickness, and $k = \omega/c = 2\pi/\lambda$.

For a conducting area element, physical optics yields a differential contribution to the back scattered far field given[7] by

$$d\left[\frac{E_s}{E_i}\right] = \frac{-jk}{2\pi}\ \frac{\exp(-j\,2\,kR)}{R}\ .\,dA \tag{2}$$

when $\frac{kt}{2}\,(\epsilon_p/\epsilon_0 - 1) << 1.$

Hence the dielectric-body scatters much less than conductors (say, plasma). For the scattering cross-section (1) is integrated over the surface and the following relation[8] is applied

$$\sigma = 4\,\pi\,R^2\,|\,E_s\,/\,E_i|^2 \tag{3}$$

In order to check this technique analytically we consider a square plate of side length S whose scattering cross-section for the TE-wave (electric field parallel to the surface) is given by

$$\sigma_{\mathrm{TE}} = \frac{1}{4\,\pi}\left|\,k^2\,S^2\,t\,(\epsilon_p/\epsilon_0 - 1)\ \frac{\sin(kS\sin\phi)}{kS\sin\phi}\right|^2 \tag{4}$$

where ϕ is the angle of incidence relative to broadside.

For a TM-wave (magnetic field parallel to the surface) scattering cross-section is

$$\sigma_{\mathrm{TM}} = \sigma_{\mathrm{TE}}\left[1 - \left(1 - \frac{\epsilon_p}{\epsilon_0}\right)\sin^2\theta\right]^2 \tag{5}$$

Also, for a spherical shell of radius a, the scattering cross-section is given by

$$\sigma = \pi\left|\,kat\,(\epsilon_p/\epsilon_0 - 1)\left(\sin(2\,ka) - [1 - \epsilon_0/\epsilon_p]\left[\frac{\sin(2\,ka)}{(2\,ka)^2} - \frac{\cos(2\,ka)}{2\,ka}\right]\right)\right|^2 \tag{6}$$

For a conical shell (without a base) in which the dielectric constant is so low that $(1-\epsilon_0/\epsilon_p)\sin^2\theta$ may be set equal to zero in (1), the back scattered field for nose-on incidence is given by

$$\frac{E_s}{E_i} = \frac{t}{8}\,(\epsilon_p/\epsilon_0 - 1)\left[\frac{\tan\psi/2}{\cos\psi/2}\right][(1 + j\,2\,kL)\exp(-j\,2\,kL) - 1]\cdot\frac{\exp(-j\,2\,k\,R_0)}{R_0} \tag{7}$$

where L is the cone length, ψ is the cone angle, and R_0 is the distance from the base.

The scattering cross-section for a TE-wave (electric field parallel to the surface) for the plasma conical shell simplifies to

$$\left.\begin{aligned} \sigma &= \frac{\pi}{4}\left|\,ktL\,(\epsilon_p/\epsilon_0 - 1)\ \frac{\tan\psi/2}{\cos\psi/2}\right|^2 \\ &\text{and} \\ \sigma_{\mathrm{TE}} &= \frac{\pi}{4}\left|\left(ktL\,(\epsilon_p/\epsilon_0 - 1)\,\frac{\tan\psi/2}{\cos\psi/2}\right)\frac{\sin(kL\sin\phi)}{kL\sin\phi}\right|^2 \end{aligned}\right\} \tag{8}$$

For a TM-wave (magnetic field parallel to the surface) the scattering cross-section from (5) is given by

$$\sigma_{TM} = \sigma_{TE}\,[\,1 - (\,1 - \epsilon_0/\epsilon_p)\,\sin^2\theta\,] \tag{9}$$

RESULTS AND DISCUSSION

Fig. 1 gives calculated results for two types of polarizations for a plasma square plate 5″×5″×1/5″ (side length S = 5 inch and thickness t = 1/5 inch) with $\epsilon_p/\epsilon_0 = 0\cdot5$ at 2000 MH$_z$. The results for a spherical shell for low frequency limit are given in Tables 1 & 2 for a fixed value of $\epsilon_p/\epsilon_0 = 0\cdot5$ and relative dielectric

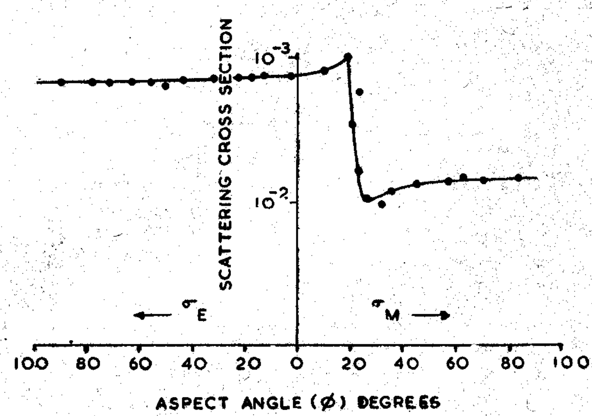

Fig. 1—Computed scattering cross-section for a plasma square plate (5″ × 5″ × 1/5″).

constant near unity. Table 3 represents the scattering cross-section for the limiting cases of a very small dielectric constant.

For a conical shell plasma, we have $t = 0{\cdot}00595\,L$, $L/\lambda = 6{\cdot}28$, $\psi = 45°$ with $\epsilon_p/\epsilon_0 = 0{\cdot}5$ at 2000 MHz. The calculated results are shown in Fig. 2. Tables 4-6 represent the scattering cross-section for the limiting cases of a very small dielectric constant. When we increase the radius of the shell (or the cone length), the electrical size of the shell is increased for $ka >> 1$ and using (6) the scattering cross-section becomes

$$\sigma \approx \pi \mid kat\,(\epsilon_p/\epsilon_0 - 1)\sin(2\,ka)\mid^2, \text{ for } ka >> 1 \quad (10)$$

From this statement we conclude that only dielectric constant and the shell thickness affect the scattering cross-section by a factor $\mid t\,(\epsilon_p/\epsilon_0 - 1)\mid^2$. Hence it is possible to replace an actual shell with an equivalent thinner one of appropriately high dielectric constant[6] to determine the back scattering cross-section of a thin dielectric shell[8]. The reverse case in case of plasmas is also true. This type of problem was studied by Aden[9] for a shell of arbitrary thickness but these results were, however, complicated for numerical treatment. Table 3 also illustrates that such treatment is suitable for large spheres when the dielectric constant is small. The scattering in case of large sphere is produced by the front and rear surfaces where the electric field is tangential, but for a small sphere the entire surface is considered. For the first case angle θ is equal to zero and for the second case angle θ has a complete range. Hence, the geometrical optics approach discussed by Peters & Thomas[7] for solution for scattering from a sphere with concentric spherical shell yields results that are most similar to the large sphere case of (10). In the case of frustum the net scattered field may be computed from the contribution of the full cone minus the contribution of the removed conical tip. In the limit as the cone angle goes to zero, a frustum becomes a cylindrical ring[10].

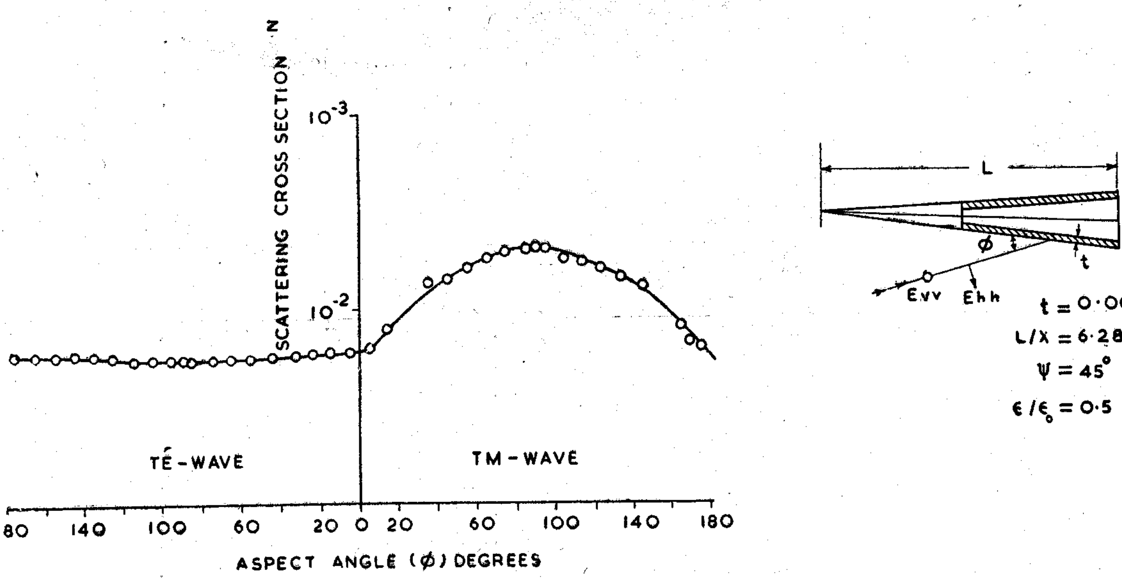

Fig. 2—Computed scattering cross-section for a conical shell of plasma ($t = 0{\cdot}00595$L, $L/\lambda = 6{\cdot}28$, ψ—cone angle = 45°, $\epsilon_p/\epsilon_0 = 0{\cdot}5$).

CONCLUSION

The present paper presents the computed results for the scattering cross-section for a thin walled plasma square plate and spherical and conical shells. The scattering from cylindrical plasma containers will be considered in a subsequent communication. It may also be possible to reconsider this study for three dimensional plasma bodies for a very small dielectric constant and a relative dielectric constant very near unity by using a volume integral rather than surface integral. The application of this problem has direct bearing to certain problems such as radar meteorology. The scattering phenomenon is useful to facilitate radar scattering cross-section control by changes in the geometrical configurations and dielectric properties of the plasma bodies (medium) which can be varied at will.

TABLE 1

SCATTERING CROSS-SECTION FOR A SPHERICAL SHELL OF PLASMA : WHEN $\epsilon p/\epsilon_0 = 0 \cdot 5$ (FIXED VALUE)

Electrical size of the shell ka	Scattering cross-section σ	Electrical size of the shell ka	Scattering cross-section σ
0·10000	0·13751		
0·15000	$0\cdot81556\times10^{-1}$	$0\cdot13499\times10^{1}$	$0\cdot15442\times10^{1}$
0·20000	$0\cdot35320\times10^{-1}$	$0\cdot13999\times10^{1}$	$0\cdot12267\times10^{1}$
0·24999	$0\cdot62761\times10^{-2}$	$0\cdot14499\times10^{1}$	0·90633
0·29999	$0\cdot15246\times10^{-2}$	$0\cdot14999\times10^{1}$	0·60409
0·34999	$0\cdot27279\times10^{-1}$	$0\cdot15000\times10^{1}$	0·34200
0·39999	$0\cdot88367\times10^{-1}$	$0\cdot15999\times10^{1}$	0·14179
0·44999	$0\cdot18779\times10^{-1}$	$0\cdot16499\times10^{1}$	$0\cdot23897\times10^{-1}$
0·49999	0·32640	$0\cdot16999\times10^{1}$	$0\cdot62080\times10^{-2}$
0·55000	0·50263	$0\cdot17499\times10^{1}$	0·10306
0·60000	0·71244	$0\cdot17999\times10^{1}$	0·32423
0·64999	0·94934	$0\cdot18499\times10^{1}$	0·67414
0·69999	$0\cdot12046\times10^{1}$	$0\cdot18999\times10^{1}$	$0\cdot11512\times10^{1}$
0·74999	$0\cdot14677\times10^{1}$	$0\cdot19499\times10^{1}$	$0\cdot17478\times10^{1}$
0·79999	$0\cdot17267\times10^{1}$	$0\cdot19999\times10^{1}$	$0\cdot24498\times10^{1}$
0·84999	$0\cdot19688\times10^{1}$	$0\cdot20499\times10^{1}$	$0\cdot32371\times10^{1}$
0·89999	$0\cdot21814\times10^{1}$	$0\cdot20999\times10^{1}$	$0\cdot40843\times10^{1}$
0·94999	$0\cdot23524\times10^{1}$	$0\cdot21499\times10^{1}$	$0\cdot49614\times10^{1}$
0·99999	$0\cdot24712\times10^{1}$	$0\cdot21999\times10^{1}$	$0\cdot58354\times10^{1}$
$0\cdot10499\times10^{1}$	$0\cdot25293\times10^{1}$	$0\cdot22499\times10^{1}$	$0\cdot66709\times10^{1}$
$0\cdot10999\times10^{1}$	$0\cdot25212\times10^{1}$	$0\cdot22999\times10^{1}$	$0\cdot74324\times10^{1}$
$0\cdot11499\times10^{1}$	$0\cdot24446\times10^{1}$	$0\cdot23499\times10^{1}$	$0\cdot80859\times10^{1}$
$0\cdot11999\times10^{1}$	$0\cdot23012\times10^{1}$	$0\cdot23999\times10^{1}$	$0\cdot86003\times10^{1}$
$0\cdot12499\times10^{1}$	$0\cdot20944\times10^{1}$	$0\cdot24499\times10^{1}$	$0\cdot8949\times10^{1}$
$0\cdot12999\times10^{1}$	$0\cdot18396\times10^{1}$	$0\cdot24999\times10^{1}$	$0\cdot91119\times10^{1}$

TABLE 2

SCATTERING CROSS-SECTION FOR A SPHERICAL PLASMA SHELL FOR RELATIVE DIELECTRIC CONSTANT NEAR UNITY $\epsilon p \approx \epsilon_0$

Electrical size of the shell ka	Scattering cross-section σ	Electrical size of the shell ka	Scattering cross-section σ
0·55000	0·24026	$0\cdot10499\times10^{1}$	0·82150
0·60000	0·31273	$0\cdot10999\times10^{1}$	0·79093
0·64999	0·39226	$0\cdot11499\times10^{1}$	0·73541
0·69999	0·47584	$0\cdot11999\times10^{1}$	0·65700
0·74999	0·55968	$0\cdot12499\times10^{1}$	0·55963
0·79999	0·63945	$0\cdot12999\times10^{1}$	0·44910
0·84999	0·71050	$0\cdot13499\times10^{1}$	0·33288
0·89999	0·76818	$0\cdot13999\times10^{1}$	0·21994
0·94999	0·80817	$0\cdot14499\times10^{1}$	0·12034
0·99999	0·82682	$0\cdot14999\times10^{1}$	$0\cdot44808\times10^{-1}$

TABLE 3

SCATTERING CROSS-SECTION FOR SPHERICAL SHELL OF PLASMA : CASE I : $\epsilon p << \epsilon_0$

$\epsilon p/\epsilon_0$	Electrical size of the shell ka	Scattering cross-section σ
	$0\cdot10000\times10^{-1}$	$0\cdot82704\times10^{3}$
	$0\cdot20000\times10^{-1}$	$0\cdot21072\times10^{3}$
	$0\cdot30000\times10^{-1}$	$0\cdot95434\times10^{2}$
	$0\cdot40000\times10^{-1}$	$0\cdot54691\times10^{2}$
0·01	$0\cdot50000\times10^{-1}$	$0\cdot35655\times10^{2}$
	$0\cdot59999\times10^{-1}$	$0\cdot25217\times10^{2}$
	$0\cdot69999\times10^{-1}$	$0\cdot18866\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot14706\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot11828\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot97518\times10^{1}$
	$0\cdot10000\times10^{-1}$	$0\cdot16856\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot42721\times10^{3}$
	$0\cdot30000\times10^{-1}$	$0\cdot19247\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot10973\times10^{3}$
0·02	$0\cdot50000\times10^{-1}$	$0\cdot71181\times10^{2}$
	$0\cdot59999\times10^{-1}$	$0\cdot50094\times10^{2}$
	$0\cdot69999\times10^{-1}$	$0\cdot37294\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot28931\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot23160\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot19005\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot26753\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot67623\times10^{3}$
	$0\cdot30000\times10^{-1}$	$0\cdot30385\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot17278\times10^{3}$
0·03	$0\cdot50000\times10^{-1}$	$0\cdot11178\times10^{3}$
	$0\cdot59999\times10^{-1}$	$0\cdot78470\times10^{2}$
	$0\cdot69999\times10^{-1}$	$0\cdot58272\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot45092\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot36008\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot29475\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot36395\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot91822\times10^{3}$
	$0\cdot30000\times10^{-1}$	$0\cdot41182\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot23375\times10^{3}$
	$0\cdot50000\times10^{-1}$	$0\cdot15075\times10^{3}$
0·04	$0\cdot59999\times10^{-1}$	$0\cdot10577\times10^{3}$
	$0\cdot69999\times10^{-1}$	$0\cdot78408\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot60567\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot48280\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot39453\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot44247\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot11144\times10^{4}$
	$0\cdot30000\times10^{-1}$	$0\cdot49895\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot28273\times10^{3}$
	$0\cdot50000\times10^{-1}$	$0\cdot18227\times10^{3}$
0·05	$0\cdot59999\times10^{-1}$	$0\cdot12750\times10^{2}$
	$0\cdot69999\times10^{-1}$	$0\cdot94362\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot72771\times10^{2}$
	$0\cdot89999\times10^{-1}$	0·57914 × 10[illegible]
	$0\cdot99999\times10^{-1}$	$0\cdot47249\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot49057\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot12331\times10^{4}$
	$0\cdot30000\times10^{-1}$	$0\cdot55103\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot31163\times10^{3}$
	$0\cdot50000\times10^{-1}$	$0\cdot20052\times10^{3}$
0·06	$0\cdot59999\times10^{-1}$	$0\cdot14000\times10^{3}$
	$0\cdot69999\times10^{-1}$	$0\cdot10341\times10^{3}$
	$0\cdot79999\times10^{-1}$	$0\cdot79603\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot63233\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot51492\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot50051\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot12549\times10^{4}$
	$0\cdot30000\times10^{-1}$	$0\cdot55940\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot31558\times10^{3}$
	$0\cdot50000\times10^{-1}$	$0\cdot20256\times10^{3}$
0·07	$0\cdot59999\times10^{-1}$	$0\cdot14108\times10^{3}$
	$0\cdot69999\times10^{-1}$	$0\cdot10395\times10^{3}$
	$0\cdot79999\times10^{-1}$	$0\cdot79824\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot63255\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot51386\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot47064\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot11760\times10^{4}$
	$0\cdot30000\times10^{-1}$	$0\cdot52244\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot29373\times10^{3}$
	$0\cdot50000\times10^{-1}$	0·18790 × 10[illegible]
0·08	$0\cdot59999\times10^{-1}$	$0\cdot13042\times10^{3}$
	$0\cdot69999\times10^{-1}$	$0\cdot95780\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot73297\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot57886\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot46866\times10^{2}$
	$0\cdot10000\times10^{-1}$	$0\cdot40565\times10^{4}$
	$0\cdot20000\times10^{-1}$	$0\cdot10086\times10^{4}$
	$0\cdot30000\times10^{-1}$	$0\cdot44593\times10^{3}$
	$0\cdot40000\times10^{-1}$	$0\cdot24949\times10^{3}$
	$0\cdot50000\times10^{-1}$	$0\cdot15882\times10^{3}$
0·09	$0\cdot59999\times10^{-1}$	$0\cdot10972\times10^{3}$
	$0\cdot69999\times10^{-1}$	$0\cdot80161\times10^{2}$
	$0\cdot79999\times10^{-1}$	$0\cdot61042\times10^{2}$
	$0\cdot89999\times10^{-1}$	$0\cdot47969\times10^{2}$
	$0\cdot99999\times10^{-1}$	$0\cdot38644\times10^{2}$

TABLE 3—*contd.*

$\epsilon p/\epsilon_0$	Electrical size of the shell ka	Scattering cross-section σ	$\epsilon p/\epsilon_0$	Electrical size of the shell ka	Scattering cross-section σ
	$0{\cdot}10000\times10^{-1}$	$0{\cdot}31586\times10^{4}$	$0{\cdot}10$	$0{\cdot}59999\times10^{-1}$	$0{\cdot}82302\times10^{2}$
	$0{\cdot}20000\times10^{-1}$	$0{\cdot}77974\times10^{3}$		$0{\cdot}69999\times10^{-1}$	$0{\cdot}59683\times10^{2}$
	$0{\cdot}30000\times10^{-1}$	$0{\cdot}34217\times10^{3}$		$0{\cdot}79999\times10^{-1}$	$0{\cdot}45698\times10^{2}$
	$0{\cdot}40000\times10^{-1}$	$0{\cdot}19002\times10^{3}$		$0{\cdot}89999\times10^{-1}$	$0{\cdot}35165\times10^{2}$
	$0{\cdot}50000\times10^{-1}$	$0{\cdot}12006\times10^{3}$		$0{\cdot}99999\times10^{-1}$	$0{\cdot}28107\times10^{2}$

TABLE 4

SCATTERING CROSS-SECTION FOR A CONICAL SHELL OF PLASMA : WHEN $\epsilon p << \epsilon_0$

$\epsilon p/\epsilon_0$	Cone Angle ψ	Scattering cross-section σ	$\epsilon p/\epsilon_0$	Cone Angle ψ	Scattering cross-section σ
	0	$0{\cdot}000$		0	$0{\cdot}000$
	15	$0{\cdot}3331\times10^{-5}$		15	$0{\cdot}299\times10^{-5}$
$0{\cdot}01$	30	$0{\cdot}144\times10^{-4}$		30	$0{\cdot}130\times10^{-4}$
	45	$0{\cdot}377\times10^{-4}$	$0{\cdot}06$	45	$0{\cdot}340\times10^{-4}$
	60	$0{\cdot}835\times10^{-4}$		60	$0{\cdot}753\times10^{-4}$
	75	$0{\cdot}175\times10^{-3}$		75	$0{\cdot}158\times10^{-3}$
	90	$0{\cdot}375\times10^{-3}$		90	$0{\cdot}338\times10^{-3}$
	0	$0{\cdot}000$		0	$0{\cdot}000$
	15	$0{\cdot}324\times10^{-5}$		15	$0{\cdot}292\times10^{-5}$
$0{\cdot}02$	30	$0{\cdot}141\times10^{-4}$		30	$0{\cdot}127\times10^{-4}$
	45	$0{\cdot}370\times10^{-4}$	$0{\cdot}07$	45	$0{\cdot}333\times10^{-4}$
	60	$0{\cdot}818\times10^{-4}$		60	$0{\cdot}737\times10^{-4}$
	75	$0{\cdot}172\times10^{-3}$		75	$0{\cdot}155\times10^{-3}$
	90	$0{\cdot}368\times10^{-3}$		90	$0{\cdot}331\times10^{-3}$
	0	$0{\cdot}000$		0	$0{\cdot}000$
	15	$0{\cdot}318\times10^{-5}$		15	$0{\cdot}286\times10^{-5}$
$0{\cdot}03$	30	$0{\cdot}138\times10^{-4}$		30	$0{\cdot}124\times10^{-4}$
	45	$0{\cdot}362\times10^{-4}$	$0{\cdot}08$	45	$0{\cdot}326\times10^{-4}$
	60	$0{\cdot}802\times10^{-4}$		60	$0{\cdot}721\times10^{-4}$
	75	$0{\cdot}168\times10^{-3}$		75	$0{\cdot}151\times10^{-3}$
	90	$0{\cdot}360\times10^{-3}$		90	$0{\cdot}324\times10^{-3}$
	0	$0{\cdot}000$		0	$0{\cdot}000$
	15	$0{\cdot}311\times10^{-5}$		15	$0{\cdot}280\times10^{-5}$
$0{\cdot}04$	30	$0{\cdot}136\times10^{-4}$		30	$0{\cdot}122\times10^{-4}$
	45	$0{\cdot}355\times10^{-4}$	$0{\cdot}09$	45	$0{\cdot}319\times10^{-4}$
	60	$0{\cdot}785\times10^{-4}$		60	$0{\cdot}705\times10^{-4}$
	75	$0{\cdot}165\times10^{-3}$		75	$0{\cdot}148\times10^{-3}$
	90	$0{\cdot}353\times10^{-3}$		90	$0{\cdot}317\times10^{-3}$
	0	$0{\cdot}000$		0	$0{\cdot}000$
	15	$0{\cdot}305\times10^{-5}$		15	$0{\cdot}274\times10^{-5}$
	30	$0{\cdot}133\times10^{-4}$		30	$0{\cdot}119\times10^{-4}$
$0{\cdot}05$	45	$0{\cdot}348\times10^{-4}$	$0{\cdot}10$	45	$0{\cdot}312\times10^{-4}$
	60	$0{\cdot}769\times10^{-4}$		60	$0{\cdot}690\times10^{-4}$
	75	$0{\cdot}161\times10^{-3}$		75	$0{\cdot}145\times10^{-3}$
	90	$0{\cdot}345\times10^{-3}$		90	$0{\cdot}310\times10^{-3}$

TABLE 5

SCATTERING CROSS-SECTION FOR A CONICAL SHELL OF PLASMA

WHEN : $\epsilon p/\epsilon_0 = 0{\cdot}5$ (fixed value), $\psi = 45°$

Electrical size of the shell L/λ	Scattering cross-section σ	Electrical size of the shell L/λ	Scattering cross-section σ
5	—0·01390	15	—0·12580
6	—0·02012	16	—0·14314
7	—0·02739	17	—0·16159
8	—0·03578	18	—0 18116
9	—0·04529	19	—0·20185
10	—0·05391	20	—0·22366
11	—0·06765	21	—0·24658
12	—0·08051	22	—0·27062
13	—0·09449	23	—0·32207
14	—0·10959	24	—0·32207
		25	0·34947

TABLE 6

SCATTERING CROSS-SECTION FOR A CONICAL SHELL OF PLASMA

CASE $\epsilon p << \epsilon_0$

$\epsilon p/\epsilon_0$	Electrical size of the shell L/λ	Scattering cross-section σ	$\epsilon p/\epsilon_0$	Electrical size of the shell L/λ	Scattering cross-section σ
0·01	5	$0{\cdot}1509\times10^{-3}$	0·06	5	$0{\cdot}1360\times10^{-3}$
	10	$0{\cdot}2415\times10^{-2}$		10	$0{\cdot}2177\times10^{-2}$
	15	$0{\cdot}1222\times10^{-1}$		15	$0{\cdot}1102\times10^{-1}$
	20	$0{\cdot}2864\times10^{-1}$		20	$0{\cdot}3486\times10^{-1}$
	25	$0{\cdot}9635\times10^{-1}$		25	$0{\cdot}8506\times10^{-1}$
0·02	5	$0{\cdot}1479\times10^{-3}$	0·07	5	$0{\cdot}1332\times10^{-3}$
	10	$0{\cdot}2366\times10^{-2}$		10	$0{\cdot}2131\times10^{-2}$
	15	$0{\cdot}1198\times10^{-1}$		15	$0{\cdot}1079\times10^{-1}$
	20	$0{\cdot}3788\times10^{-1}$		20	$0{\cdot}3410\times10^{-1}$
	25	$0{\cdot}9245\times10^{-1}$		25	$0{\cdot}8326\times10^{-1}$
0·03	5	$0{\cdot}1449\times10^{-3}$	0·08	5	$0{\cdot}1303\times10^{-3}$
	10	$0{\cdot}2318\times10^{-2}$		10	$0{\cdot}2085\times10^{-2}$
	15	$0{\cdot}1173\times10^{-1}$		15	$0{\cdot}1055\times10^{-1}$
	20	$0{\cdot}3710\times10^{-1}$		20	$0{\cdot}3337\times10^{-1}$
	25	$0{\cdot}9057\times10^{-1}$		25	$0{\cdot}8147\times10^{-1}$
0·04	5	$0{\cdot}1419\times10^{-3}$	0·09	5	$0{\cdot}1275\times10^{-3}$
	10	$0{\cdot}2271\times10^{-2}$		10	$0{\cdot}2040\times10^{-2}$
	15	$0{\cdot}1149\times10^{-1}$		15	$0{\cdot}1033\times10^{-1}$
	20	$0{\cdot}3633\times10^{-1}$		20	$0{\cdot}3265\times10^{-1}$
	25	$0{\cdot}8871\times10^{-1}$		25	$0{\cdot}7971\times10^{-1}$
0·05	5	$0{\cdot}1390\times10^{-3}$	0·10	5	$0{\cdot}1247\times10^{-3}$
	10	$0{\cdot}2224\times10^{-2}$		10	$0{\cdot}1996\times10^{-2}$
	15	$0{\cdot}1125\times10^{-1}$		15	$0{\cdot}1010\times10^{-1}$
	20	$0{\cdot}3558\times10^{-1}$		20	$0{\cdot}3193\times10^{-1}$
	25	$0{\cdot}8688\times10^{-1}$		25	$0{\cdot}7797\times10^{-1}$

ACKNOWLEDGEMENTS

The author would like to express his heartfelt thanks to Prof. A. K. Duttagupta, Dean, Faculty of Science, Birla Institute of Technology & Science, Pilani, for his stimulating help during the course of this work and also of the members of plasma research group with whom I continuously confer.

REFERENCES

1. Booker, H. G. & Gorden, W. F., Theory of radio scattering in the troposphere, *Proc. Inst. Radio Engrs.*, **38** (1950) 401.
2. Stratton, J. A., "Electromagnetic Theory", (McGraw Hill Book Co., Inc, New York) 1941, pp. 568-573.
3. Collins, R. E., Scattering by an infinite array of thin-dielectric sheets, *Inst. Fadio Engrs, Trans. Ante na Propagat.*, **Ap-8** (1960) Jan., pp. 62 67.
4. Carlson, J. F. & Heins, A.E., "The reflection of an electromagnetic plane wave by an infinite set of plates". *Quart. J. Appl. Math.*, **IV** (1947) Jan., 313-329.
5. Karp, S. N., An application of Sturm—Liouville theory to a class of two part boundary value problems, *Proc. Camb. Phil. Soc.*, **53** (1957), April, 368-381.
6. Berz, F., Reflection and transmission of microwaves at a set of parallel metallic plates, *Proc Inst. Radio Engrs.*, **98** Pt. III 1951 Jan, 47-55.
7. Peters, (Jr.) L. & Thomas, D. T., "A geometrical optics approach for the radar cross-section of thin shells". *J. Geo. Phys. Res.* **67** (1962) May, 2073-75.
8. Andreasen, M. G., Back scattering cross-section of a thin dielectric sphere shell., *Inst. Radio Engrs, Trans. Antenna Propagat.*, Vol **Ap-5** 1957 July, 267-270.
9. Aden, A. L. & Kerker, M., "Scattering of electromagnetic waves from two concentric spheres". *J. Appl. Phys.* **23** (1951) Oct., p. 1242-1246.
10. Philipson, L. L., "An analytical study of scattering by thin dielectric rings", *Inst. Radio Engrs, Trans. Antenna Propagat.*, AP-6 (1958) Jan., 3-8.