COMBINED LAMINAR AND TURBULENT FLOW OVER ROTATING CONES

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An analysis has been carried out for the combined laminar and turbulent flow over rotating cones in an otherwise undisturbed fluid. The momentum integral equations in meridianal and circumferential directions, which govern the turbulent fluid flow over a rotating cone, have been solved in their dimensionless forms. A continuous eddy diffusivity of momentum, suggested by Van Driest, has been adopted to obtain the velocity components. Taking the central laminar core into account, friction moment coefficients are calculated as functions of cone Reynolds number Re_0. The theoretical findings are compared with other theories and the available experimental data.

NOMENCLATURE

A^+ = damping constant

C_M = friction moment coefficient

K^+ = mixing length constant

L = slant height

$L+$ = dimensionless slant height, Lu^*/ν

l = meridianal distance from the apex

$l+$ = dimensionless meridianal distance from the apex, lu^*/ν

M = friction moment

r = radius of the cone at l, $l \sin \lambda$

$r+$ = dimensionless radius of the cone, ru^*/ν

u^* = friction velocity, $\sqrt{\tau_w/\rho}$

u = resultant relative velocity

$u+$ = dimensionless resultant relative velocity, u/u^*

u_T = total tangential velocity, $\sqrt{u^2_x + u^2_\phi}$

u_T^+ = dimensionless total tangential velocity, u_T/u^*

u_x = meridianal component velocity

u_ϕ = circumferential component velocity

u_x^+ = dimensionless meridianal component velocity, u_x/u^*

u_ϕ^+ = dimensionless circumferential component velocity, u_ϕ/u^*

ωr = surface velocity

$(\omega r)^+$ = dimensionless surface velocity, $(\omega r)/u^*$

z = perpendicular distance from the surface

z^+ = dimensionless perpendicular distance from the surface, zu^*/ν

Greek Symbols

α = ratio of component shear stresses, $\tau_r/-\tau_\phi$

δ = momentum boundary layer thickness

$\delta+$ = dimensionless momentum boundary layer thickness, $\delta u^*/\nu$

ϵ_M = eddy diffusivity of momentum

τ_x = meridianal component of shear stress

τ_ϕ = circumferential component of shear stress

τ_w = total wall shear stress, $\sqrt{\tau^2_x + \tau^2_\phi}$

ρ = density

λ = half the apex angle of the cone

μ = dynamic viscosity

= kinematic viscosity

ω = angular velocity

Subscripts

l = laminar

t = turbulent

w = at the surface

c = critical

0 = outer most

Accurate information of fluid flow from rotating axi-symmetrical surfaces are essential in the designing of rotating parts of many industrial equipments, as diverse as electric motors, gas-turbines and electrogyros. A rotating cone forms an important geometry to be studied in detail since many rotating components can be idealised to this surface.

When a cone rotates about its own axis in an otherwise undisturbed fluid, the tangential friction drag at the surface imparts a circumferential velocity while the centrifugal force induces a meridianal velocity in the fluid. The flow remains laminar near the axis of rotation, the rest of the cone area being under turbulent flow. Wu[1] demonstrated that Karman's[2] and Cochran's[3] solutions for the laminar hydrodynamic field over the rotating disc can be successfully applied for the cone if the Reynolds number for the cone surface is defined as

$$\mathrm{Re}_0 = \frac{\omega L^2_0}{\nu} \sin \lambda \tag{1}$$

Herring & Grosh[4], in a more comprehensive study, obtained friction moment coefficient for the rotating cone, under laminar flow, as

$$C_M = \frac{M}{\frac{1}{2} \rho \omega^2 L^2_0 \sin^2 \lambda \, r^3_0} = 1{\cdot}935 \, \mathrm{Re}_0^{-0{\cdot}5} \tag{2}$$

Experiments with the rotating disc have shown that transition from laminar to turbulent flow occurs at a Reynolds number of 200,000. The problem of turbulent flow over rotating cone, within the knowledge of the author, has not been subjected previously to analytical study. On the other hand, the case of rotating disc, which is a particular case of conical surfaces, has received some attention. Karman[2] and Goldstein[5], adopting the 1/7th power law and the logarithmic profile for the pipe flow respectively obtained the friction moment coefficient for rotating disc as functions of disc Reynolds number. Both these theories give radial velocities which disagree with the experimental values of Gregory et al[6], and further, the friction moment coefficients are found satisfactory only at high Reynolds numbers. In view of this, an attempt to correct the radial velocity profile was made by Richardson & Saunders[7].

The present work was undertaken with a view to obtain realistic velocity profiles in the turbulent boundary layer over the rotating cones (and discs) and consequently the turbulent friction moments inclusive of the transition region. The analysis is carried out by first deriving the momentum integral equations for the meridianal and tangential directions in their dimensionless forms. The concept of "Continuous Eddy Diffusivity", suggested by Van Driest[8], is adopted to obtain the component velocity profiles. Taking into consideration the central laminar portion near the axis of rotation, the overall friction moments for the complete surface are evaluated by numerically integrating the local turbulent values over the surface. A series of experiments were conducted with a rotating cone to supplement the existing data which are partly reported here, while the complete experimental results will be reported soon.

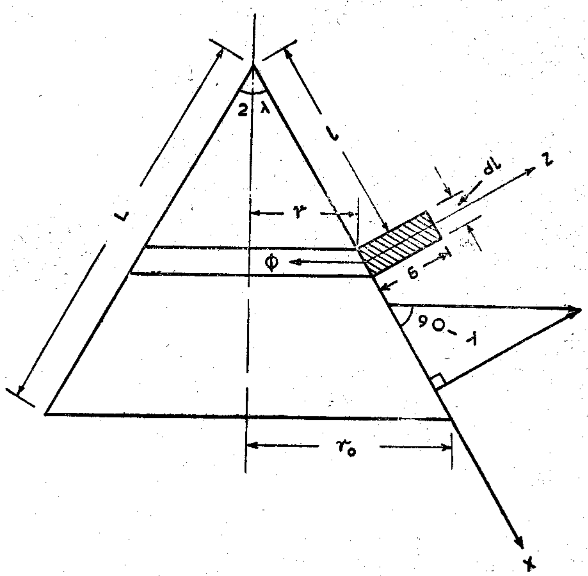

Fig. 1—Hydrodynamic boundary layer over the rotating cone.

MOMENTUM INTEGRAL EQUATIONS

The momentum integral equations in the meridianal and circumferential directions may be written by considering an elemental ring of surface width dl at a distance l from the apex of the cone (Fig. 1). For such an element, the increment of momentum of the fluid in the meridianal of X-direction is [17]

$$\frac{d}{dl}\left[2\,\pi\, r\, \rho \int_0^\delta u^2_x\, dz \right] dl$$

where δ is the hydrodynamic boundary layer thickness at $l = r/\sin\lambda$. The centrifugal force on the element, acting perpendicular to the axis of rotation is

$$-2\,\pi\, r \left[\rho \int_0^\delta \frac{u^2\phi}{r}\, dz \right] dl$$

The component of this force along the meridian together with the increment of momentum must be balanced by the frictional forces acting on the elemental ring which is equal to $2\pi r\, dl\, \tau_x$, where τ_x = meridianal component of the wall shear stress. Thus, the momentum integral equation in the meridianal direction can be written as

$$\frac{d}{dl}\left[l \int_0^\delta u^2_x\, dz \right] - \int_0^\delta u^2\phi\, dz = -\frac{l\,\tau_x}{\rho} \tag{3}$$

The momentum integral equation in the circumferential direction is obtained by balancing the increment of moment of momentum of the fluid in the element of the boundary layer with the moment of frictional forces about the axis of rotation.

The increment of moment of momentum about the axis of rotation is

$$\frac{d}{dl}\left[2\,\pi\, r^2\, \rho \int_0^\delta u_x\, u_\phi\, dz \right] dl$$

while the moment of the frictional forces on the element considered is

$$-2\,\pi\, r^2\, dl\, \tau_\phi$$

where τ_ϕ = circumferential component of the wall shear stress. Hence the momentum integral equation in the circumferential direction becomes

$$\frac{d}{dl}\left[l^2 \int_0^\delta u_x\, u_\phi\, dz \right] = -\frac{l^2\,\tau_\phi}{\rho} \tag{4}$$

The component shear stresses may be replaced by the total shear stress at the wall and the ratio of the component shear stresses.

If

$$\alpha = \tau_x \,/ -\tau_\phi$$

(τ_ϕ is negative since the flow direction is taken as positive) the total shear stress at the wall can be written as

$$\tau_w = -\tau\phi \sqrt{1+\alpha^2}$$

and the component shear stresses will then become

$$\tau_r = \frac{\alpha}{\sqrt{1+\alpha^2}}\,\tau_w, \quad \tau\phi = \frac{-1}{\sqrt{1+\alpha^2}}\,\tau_w$$

Introducing these terms for the component shear stresses in (3) and (4) and when made dimensionless by making use of the definitions given in the nomenclature, the resulting dimensionless momentum integral equations in the meridianal and circumferential directions will be

$$\frac{1}{l^+}\left[\int_0^{\delta^+} u_\phi^{+^2}\, dz^+ - \frac{d}{dl^+}\left\{ l^+ \int_0^{\delta^+} u_x^{+^2}\, dz^+ \right\}\right] = \frac{\alpha}{\sqrt{1+\alpha^2}} \tag{5}$$

and

$$\frac{d}{dl^+}\left[l^{+^2} \int_0^{\delta^+} u^+_x\, u^+_\phi\, dz^+ \right] = \frac{1}{\sqrt{1+\alpha^2}}\, l^{+^2} \tag{6}$$

By integrating, (6) can be rewritten as

$$\int_0^{\{\}} d\left[l^{+^2} \int_0^{\delta^+} u^+_x\, u^+_\phi\, dz^+ \right] = \frac{l^{+^3}}{3\sqrt{1+\alpha^2}} \tag{7}$$

where the upper limit on the outer integral, i.e. the braces, is evaluated numerically at the value of l^+ corresponding to δ^+. This solution procedure is outlined later.

VELOCITY PROFILES

In order that the momentum integral equations may be integrated, it is necessary to select a suitable velocity distribution in the turbulent boundary layer. This begins with the expression relating the time averaged shear stress and the velocity gradient within the boundary layer as

$$\tau = (\mu + \rho\,\epsilon_M)\,\frac{du}{dz}$$

The first term in the parenthesis, the viscosity μ, represents the laminar contribution to the shear; while the second term, the eddy diffusivity of momentum ϵ_M, represents the turbulent contribution. In dimensionless form, the above expression becomes

$$\tau / \tau_w = \left[1 + \epsilon_M/\nu \right] \frac{du^+}{dz^+} \tag{8}$$

If the variation of τ/τ_w across the boundary layer were known and expressions for ϵ_M/ν were available, velocity profiles may be calculated by integrating (8). Because of the enormous difficulties in determining the actual shear stress distribution within the boundary layer, it is common practice to postulate some reasonable distribution for the shear stress. While solving the turbulent boundary layer over the flat plate, Diessler[9] set $\tau/\tau_w = 1$ and obtained very satisfactory velocity profiles and a friction factor—Reynolds number relationship.

Van Driest[8] proposed an expression for the momentum eddy diffusivity as

$$\epsilon_M/\nu = K^{+^2} z^{+^2} \left[1 - \exp\left(-z^+/A^+\right)\right]^2 \frac{du^+}{dz^+}$$

where A^+ is a damping constant of the turbulence field and K^+ is the universal mixing constant. Substituting for ϵ_M/ν in (8) and making $\tau/\tau_w = 1$, the velocity distribution may be shown to be

$$\therefore \quad u^+ = \int_0^{z^+} \frac{2\,dz^+}{1 + \sqrt{1 + 4K^{+2} z^{+2} [\,1 - \exp(-z^+/A^+)\,]^2}} \tag{9}$$

It is apparent that the agreement between (9) and the experimental values will depend on the choice of the damping constant A^+ which characterises the influence of the wall and the universal mixing constant K^+. Van Driest[8] found that by letting $A^+ = 27$ and $K^+ = 0{\cdot}4$, very good agreement is obtained between (9) and the pipe flow experimental velocity profiles of Laufer[10]. Following Goldstein's[5] work on rotating disc, the resultant relative velocity in the boundary layer is assumed to be given by (9). At the edge of the boundary layer, the resultant relative velocity must equal the surface velocity and therefore

$$(\omega r)^+ = u^+ = \int_0^{\delta^+} \frac{2\,dz^+}{1 + \sqrt{1 + 4\,K^{+2} z^{+2} [\,1 - \exp(-z^+/A^+)\,]^2}} \tag{10}$$

where δ^+ is the dimensionless hydrodynamic boundary layer thickness. It now remains to make some suitable assumptions to resolve the resultant relative velocity into meridianal and circumferential components which must be consistent with the boundary conditions at the solid surface and at the edge of the boundary layer. These boundary conditions in dimensionless form are

$$z^+ = 0, \qquad u^+_\phi = (\omega r)^+, \qquad u^+_x = 0$$

$$z^+ = \delta^+, \qquad u^+_\phi = 0, \qquad u^+_x = 0$$

Both Karman[2] and Goldstein[5], in their analyses of turbulent boundary layer over the rotating disc, assumed that the component velocities were related to the resultant relative velocity by some function of α. Karman[2] assumed a linear variation of α to obtain a radial velocity which is zero at the edge of the boundary layer, whereas Goldstein[5] assumed that α was constant and that the velocity components were related to the resultant relative velocity in the same way as the component shear stresses were related to the total value at the wall.

A combination of Karman's[2] and Goldstein's[5] assumptions is made in the sense a new value of α is defined which is not constant so that

$$\alpha_z = \frac{u_x}{(\omega r - u_\phi)}$$

α_z is assumed to be some function of z/δ which need not be linear. Since the resultant relative velocity is given by

$$u = \sqrt{u_x^2 + (\omega r - u_\phi)^2}$$

it can be shown that the relative component velocities, after being made dimensionless, are

$$\left.\begin{aligned} u^+_\phi &= (\omega r)^+ - \frac{u^+}{\sqrt{1 + \alpha_z^2}} \\ u^+_x &= \frac{\alpha_z}{\sqrt{1 + \alpha_z^2}}\; u^+ \end{aligned}\right\} \tag{11}$$

From (11), it can be seen that at $z^+ = 0$, $u^+_\phi = (\omega r)^+$ and $u^+_x = 0$ since u^+ is equal to zero. However, at $z^+ = \delta^+$, for u^+_ϕ and u^+_x to be zeroes, α_z must be zero since $u^+ = (\omega r)^+$. This leads to an assumption that α_z may be represented by an expression

$$\alpha_z = \alpha\,[\,1 - (z/\delta)^n\,] \tag{12}$$

which will meet the boundary conditions specified above. The value of 'n' may be selected to give the best agreement between the measured and theoretical velocity profiles, and friction moments. This is later shown to be $0{\cdot}2$. Having selected the velocity profiles, noting that

$$\mathrm{Re} = \frac{\omega l^2}{\nu} \sin\lambda = (\omega r)^+ l^+ \tag{13}$$

the integral (5) and (6) can now be solved numerically in the following manner: (i) Values of δ^+ and α are assumed and the integrals in (5) and (6) are evaluated for the assumed distribution of u^+_x and u^+_ϕ. (ii) Hence the value of l^+ for the selected value of δ^+ is obtained from (7). (iii) Using this value of l^+, (5) is evaluated to obtain α. For the first attempt, this value of α does not agree with the initial assumed value. This anomaly may be corrected using an iterative procedure until the two values agree within any tolerable allowance. (iv) Then the corresponding value of Reynolds number is obtained from (13). By varying δ^+ and repeating the procedure, the growth of the hydrodynamic boundary layer and the variation of α may be found as functions of Reynolds number.

FRICTION MOMENT AND ITS COEFFICIENT

The resisting moment from the apex upto L may be written as

$$M = -\int_0^L 2\,\pi\, r^2\, \tau_\phi\, dl$$

But, from (4), it can be seen that

$$-r^2\, \tau_\phi\, dl = \frac{d}{dl}\left[r^2\, \rho \int_0^\delta u_x\, u_\phi\, dz \right] dl$$

Hence the friction moment can be written as

$$M = 2\,\pi \int_0^L d\left\{ l^2 \sin^2 \lambda\, \rho \int_0^\delta u_x\, u_\phi\, dz \right\} \tag{14}$$

This equation, if evaluated between the limits zero and L_0, will yield the friction moment for the whole surface under turbulent flow. However, in practical cases, there is always a portion near the axis of rotation which is under laminar flow.

If L_c is the critical slant height at which transition from laminar to turbulent flow takes place, the friction moment due to turbulent region will be given by (14), when integrated between the limits L_c and L_0. Denoting the friction moment due to turbulent region as M_t,

$$M_t = 2\,\pi\,\rho \int_{L_c}^{L_0} d\left\{ l^2 \sin^2 \lambda \int_0^\delta u_x\, u_\phi\, dz \right\}$$

The total friction moment is obtained by adding to this the moment due to laminar region. Defining the friction moment coefficient as

$$C_M = \frac{M}{\frac{1}{2}\,\rho\, v_0^2\, r_0^3}$$

where $v_0 = \omega\, L_0 \sin \lambda$ and $r_0 = L_0 \sin \lambda$. The friction moment coefficient for the combined laminar and turbulent flow may be written as

$$C_M = \frac{2\,\pi\,\rho \int_{Lc}^{L_0} d\left\{ l^2 \sin^2 \lambda \int_0^\delta u_x\, u_\phi\, dz \right\} + M_l}{\frac{1}{2}\,\rho\, L_0^5\, \omega^2 \sin^5 \lambda}$$

which when made dimensionless, becomes

$$C_M = \frac{4\,\pi}{(\omega r_0)^+\, \mathrm{Re}_0 \sin \lambda}\left[\int_0^{\delta^+_0} u^+_x\, u^+_\phi\, dz^+ - \left(\frac{L^+_c}{L^+_0}\right)^2 \int_0^{\delta^+_c} u^+_x\, u^+_\phi\, dz^+ \right] + \frac{2\, M_l}{\rho\, L_0^5\, \omega^2 \sin^5 \lambda}$$

where δ^+_0 is the dimensionless hydrodynamic boundary layer thickness at L^+_0 at which point the Reynolds number is Re_0, the corresponding values at the critical point being δ^+_c, L^+_c and Re_c. Substituting for the laminar friction moment from (2), the friction moment coefficient for the entire cone under combined laminar and turbulent flows can be written as

$$C_M = \frac{4\pi}{(\omega r_0)^+ \,\mathrm{Re}_0 \sin\lambda}\left[\int_0^{\delta^+_0} u^+_x\, u^+_\phi\, dz^+ - \left(\frac{L^+_c}{L^+_0}\right)^2 \int_0^{\delta^+_c} u^+_x\, u^+_\phi\, dz^+\right] + $$

$$+ \frac{0\cdot 61952\,\pi}{\sqrt{\mathrm{Re}_c}\,\sin\lambda}\left[\frac{L^+_c}{L^+_0}\right]^5 \qquad (15)$$

It is to be noted that $0\cdot61952\,\pi = 1\cdot935$ and when $\lambda = 90$, (15) yields, the overall friction moment coefficient for one side of the disc.

All the numerical integrations were performed by using either the fourteen point Gaussian quadrature or Simpson's one-third rule. In performing the integrations, finer meshes were used over areas where the curves have higher gradients, as for example, while integrating the quantity $\int_0^{\delta^+} u^{+2}_x\, dz^+$, finer meshes were used both near the surface and towards the outer edge of the boundary layer. It was also ensured that further reduction in integration steps did not yield any substantial improvement in accuracy.

RESULTS AND DISCUSSION

Fig. 2 shows the tangential velocity i.e. $u^+_T = \sqrt{u^{+2}_x + u^{+2}_\phi}$ plotted against z^+ / δ^+ for varying Reynolds number while Fig. 3 shows measured values over a rotating disc by Cobb & Saunders[11] along with the theoretical curves of Karman[2], Goldstein[5] and of the present analysis for a disc Reynolds number of 440,000. It can be seen that the gradient of total tangential velocity near the wall increases with Reynolds number, this being a characteristic of turbulent velocity profiles. The experimental values of Cobb & Saunders[11] show very good agreement with the present theory. As was discussed earlier, the meridianal (radial for the disc) velocity distribution over the rotating cone is of great importance in calculating the mass flow in the meridianal (radial) direction. Fig. 4 shows the distribution of meridianal velocity for different Reynolds number while Fig. 5 shows the measured and theoretical values. Agreement of the theory with the present experimental values seems to be very good. But the present theoretical curve is some 15% higher than the measured values by Gregory et al[6]. The reason for this may be due to the difficulty of measuring velocities close to the wall as suggested by the authors themselves[6]. However, while Fig. 5 shows that the proposed profile lies within the theoretical curves of Karman[2], Goldstein[5] and Dorfman[12], it does give a substantially lower values than those of Richardson[7]. The theoretical flow direction i.e. the angle the total tangential velocity vector makes with the circumferential direction is obtained

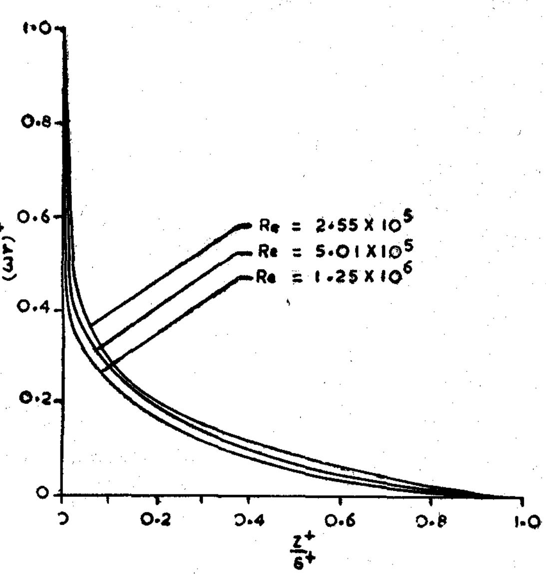

Fig. 2—Total tangential velocity distribution.

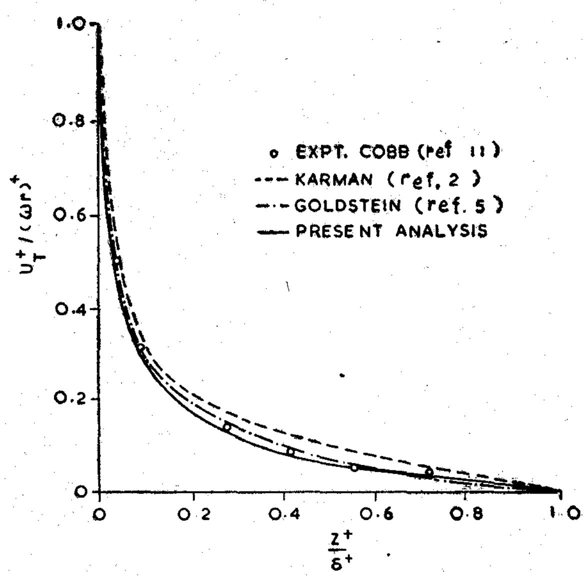

Fig. 3—Total tangential velocity over the rotating disc.

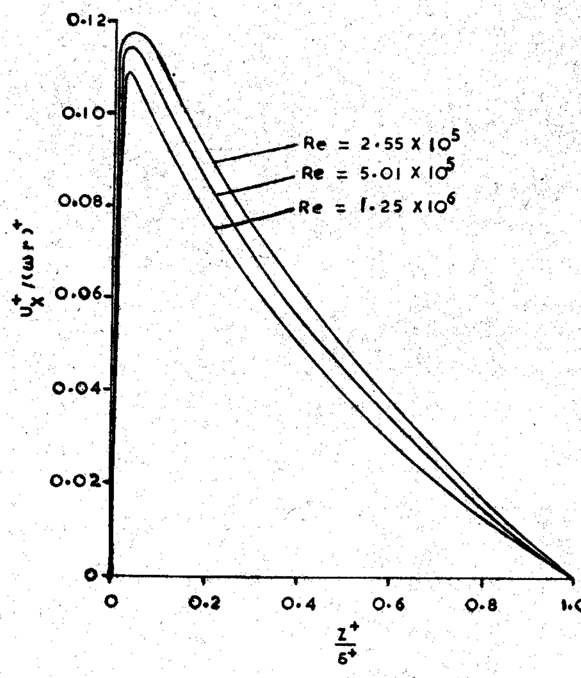

Fig. 4—Meridianal (radial) velocity distribution.

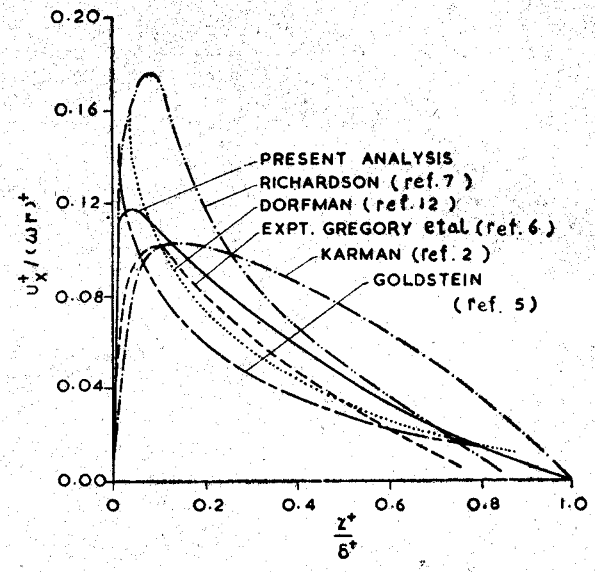

Fig. 5—Radial velocity distribution over the rotating disc.

from the ratio of meridianal and circumferential velocity components and a comparison of the same with the present experimental values and those of Gregory et al.[6] is done in Fig. 6. It was explained earlier that the values of the velocity components and therefore the flow direction depended on the assumption of the variation of α_z and to obtain the agreements shown in Fig. 2 and 3, the exponent 'n' in (12) was made equal to 0·2. Existing theories do not predict the flow direction throughout the boundary layer but give only the ratio of the radial and circumferential components of shear stress at the surface. This is most important since the circumferential component of wall shear stress is the direct contribution to the friction moment of the rotating surface. The values of α obtained from all the theories are therefore presented in Table 1.

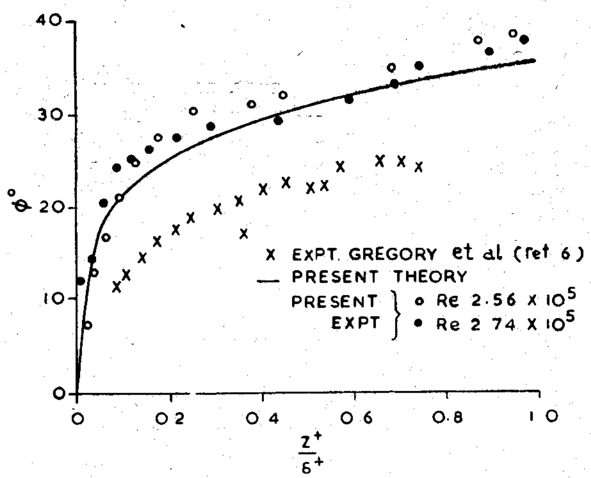

Fig. 6—Measured and theoretical turbulent flow direction.

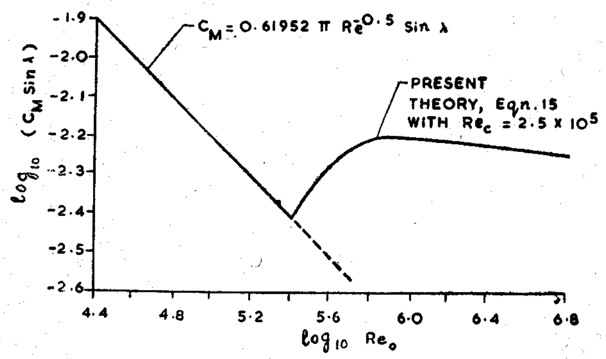

Fig. 7—Friction moment coefficient for cones and one side of the disc.

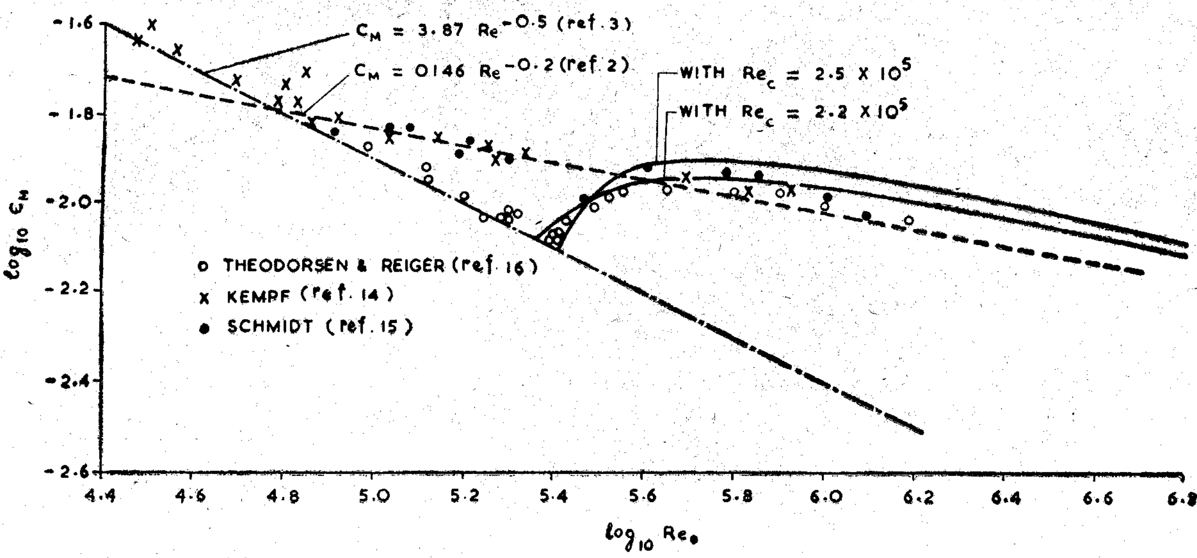

Fig. 8—Friction moment coefficient for both sides of the disc.

TABLE 1

COMPARATIVE VALUES OF α

Re	Karman	Goldstein	Dorfman	Present analysis
3×10^5	0·162	0·333	0·373	0·3989
5×10^5	0·162	0·333	0·370	0·3646
7×10^5	0·162	0·333	0·359	0·3521
1×10^6	0·162	0·333	0·350	0·3465
5×10^6	0·162	0·333	0·341	0·3318

It is seen that α from two of them decreases while those of Karman[2] and Goldstein[5] remain constant with the increase of Reynolds number. This decrease of α indicates that the flow becomes more circumferential with the increase of speed, a fact which was verified experimentally on rotating disc by Bussman[13].

Fig. 7 shows the friction moment coefficient for the cone and for one side of the disc ($\lambda = 90°$) plotted against Reynolds number. Present and other theoretical friction moment coefficients for both sides of the disc along with the experimental measurements of Kempf[14], Schmidt[15] and Theodorsen & Reiger[16] are presented in Fig. 8. The present theory takes into account the laminar portion of the boundary layer around the axis of rotation and is shown as full lines based on two different critical Reynolds numbers. It is seen that the agreement between the theory and experiment depends on the selection of the critical Reynolds number, a precise measurement of which is difficult in practice. Karman[2] formula does not allow for the central laminar portion and is therefore applicable at very high Reynolds numbers only.

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