ELASTIC WAVES AND THEIR CONVERSION DUE TO HEATING

S. B. SRIVASTAVA

University of Gorakhpur (U.P.)

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The purpose of this paper is to consider the effect of thermal waves on solids producing thereby elastic waves and the secondary thermal waves, conversion of elastic waves into electromagnetic waves and their application to Defence problems, for example, projection of rockets and missiles. The investigation is restricted to the case when neither melting nor evaporation takes place during heat supply.

Boit[1], Lesson[2] and Chadwick & Sneddon[3], considered coupling of thermal and elastic waves, separately, in different conditions. When the incident thermal waves impact on some portion of the solid surface, secondary thermal waves with much less velocity emerge from other part of the surface. Also, some part of thermal waves get converted in other forms of energy in different circumstances. Formation of both elastic and thermal waves depends on temperature difference, conductivity, specific heat as well as on the particular behaviour of the solid. As the change in stress and strain is due to the thermal waves, isothermal Hooke's relation as given by Love[4], is

$$\sigma_{ij} = 2\,\mu\,(\gamma_{ij} - a\,\theta\,\delta_{ij}) + \lambda\,(\triangle - 3\,a\theta)\,\delta_{ij} \tag{1}$$

where σ_{ij} is the stress tensor, $\gamma_{ij} = \frac{1}{2}(\xi_{ij} + \xi_{ji})$, the strain tensor, $\triangle = \gamma_{i,i}$ the dilatation, μ and λ are the isothermal Lame's constants; a, the coefficient of linear expansion and $\theta = T - T_0$ is the deviation from standard temperature. Chadwick[5] considered the equation of motion of thermoions, the thermal energy equation, the second law of thermodynamics and the conductivity tensor as

$$\rho\,\frac{{}^2u_i}{\partial t^2} = \frac{\partial\sigma_{ij}}{\partial x_j} \tag{2}$$

$$\rho\,\frac{T\,\partial s}{\partial t} = -\frac{\partial J_k}{\partial x_k} \tag{3}$$

$$du = Tds + \frac{1}{\rho}\,\sigma_{ij}\,d\epsilon_{ij} \tag{4}$$

$$\sigma_{ij} = (C_{ijkl})_T\,\epsilon_{kl} - B_{ij}\,\theta \tag{5}$$

respectively, where isothermal constants are defined by

$$(C_{ijkl})_T = \left(\frac{\partial\sigma_{ij}}{\partial\epsilon_{kl}}\right)_T$$

and the thermal expansion coefficients by

$$\alpha_{ij} = \left(\frac{\partial\epsilon_{ij}}{\partial T}\right)$$

and

$$B_{ij} = -\left(\frac{\partial\sigma_{ij}}{\partial T}\right) = \alpha_{kl}\,(C_{ijkl})_T$$

From (3), (4) and (5) we get

$$ds = \left(\frac{\partial s}{\partial\epsilon_{ij}}\right)_T d\epsilon_{ij} + \left(\frac{\partial s}{\partial T}\right)_\epsilon T \tag{6}$$

$$\left(\frac{\partial s}{\partial\epsilon_{ij}}\right)_T = -\frac{1}{\rho}\left(\frac{\partial\sigma_{ij}}{\partial T}\right)_\epsilon = \frac{1}{\rho}\,B_{ij} \tag{7}$$

and

$$ds = \frac{1}{\rho}\,B_{ij}\,d\epsilon_{ij} + \frac{C_\epsilon}{T}\,dT \tag{8}$$

where

$$C_\epsilon = T\left(\frac{\partial s}{\partial T}\right)_\epsilon$$

From (2), (4), (5) and (8), we get

$$\rho\,\frac{\partial^2 u_i}{\partial t^2} = (C_{ijkl})_T\,\frac{\partial \epsilon_{kl}}{\partial x_j} - B_{ij}\,\frac{\partial \theta}{\partial x_j} \qquad (9)$$

$$\rho\, C_\epsilon\,\frac{\partial \theta}{\partial t} + B_{ij}\,T_0\,\frac{\partial \epsilon_{ij}}{\partial t} = K_{ij}\,\frac{\partial^2 \theta}{\partial x_i\,\partial x_j} \qquad (10)$$

Helmat D Weymann[6] also obtained two waves i.e. thermal and elastic waves due to induced heating effect. An interesting phenomenon due to thermoions or thermal waves can be seen in the case of Seeback and Peltier effect when thermoelastic waves are converted into electric current and vice versa. For example, in case of Seeback effect, if one end of thermocouple be heated keeping the other end cool, the more the temperature difference, the greater is the creation of thermoions. As both metals possess different atomic number, electrons emitted due to heating effect, differ and electrons flow due to potential difference and thermal pressure. Upto a certain extent, increment in electromotive force occurs which stops at neutral temperature, while at temperature higher than the neutral temperature i.e. temperature of inversion, flow stops and reverse effect begins. It is quite clear that thermoions get converted into electric current. Trait investigated the e.m.f. of thermoelectric current in different metals having their junctions at different temperatures and expressed e.m.f. of the circuit by the relation

$$E = a\,(t_1 - t_2)\left[t_0 - \frac{1}{2}\,(t_1 + t_2)\right] \qquad (11)$$

where t_1 is absolute temperature of hot junction; t_2, the absolute temperature of cold junction; t_0, the neutral temperature and a is the coefficient depending on the nature of the two metals. He explained that the specific heat responsible for production of electric current is proportional to absolute temperature in each pair of metals. Its magnitude and sign vary in different metals.

The limit of $T - T_0$ i.e. θ plays an important role in heating effect as well as in conversion of elastic waves. When θ is less than the neutral temperature, first the intermolecular space increases, changes in stress and strain occur for the time being and thereafter thermal waves form, when the increment in intermolecular space stops from other surface. In the extreme case, however, when θ becomes large i.e. near the temperature of inversion, σ_{ij}, as a consequence of (5), becomes zero. When $B_{ij}\,\theta > (C_{ijkl})_T\,\epsilon_{kl}$ the stress tensor becomes negative and current flows in opposite direction. This phenomenon is continued till melting or evaporation takes place. Thus, due to heating effect in suitable circumstances thermal waves are converted into thermo-elastic waves and electric current. The direction of flow of current depends on θ and the change of sign of σ_{ij}.

This phenomenon also explains the mechanism, projection and the graze of missiles and rockets. On account of strong mechanical stroke, elastic waves form. Since the frequency of the wave is high, the conversion of elastic waves into heat energy occurs. Due to this heat, sudden explosion of the propellant takes place, which causes projection of rockets and missiles. When a missile or rocket strikes and penetrates a particular place, heat produced on account of friction causes the formation of elastic waves. Consequently, the secondary thermal waves are formed, which accelerate explosion.

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REFERENCES

1. BOIT, M.A., *J. Appl. Phys.*, **27** (1956), 240.
2. LESSON, M., *J. Mech. Phys. & Solids.*, **5** (1956), 57.
3. CHADWICK, P. & SNEDDON, I.N., *J. Mech. Phys. & Solids.*, **6** (1958), 223.
4. LOVE, A.E.H., "Treatise on Mathematical Theory of Elasticity" Dover Publications (CUP), (1944), 108.
5. CHADWICK, P., *J. Mech. Phys. & Solids*, **10** (1962), 99.
6. HELMAT, D., WEYMANN., *Phys. Rev.*, **176** (1968), 838.