A NOTE ON PIEZOMETRIC EFFICIENCY IN AN ORTHODOX GUN

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It has been shown that the muzzle velocity and the piezometric efficiency both may be increased for a suitable composite charge in comparison with those of the single charge. The condition stipulated is that the length of the gun should be sufficiently large in comparison to the initial free space behind the shot. The composite charge considered here consists of two components such that the pressure driving the shot remains absolutely constant throughout the period when the second component burns.

In a paper Ray[1] discussed the possibility of getting constant driving pressure in an orthodox gun all through the period when the second component of the composite charge burns. He showed that in order to satisfy this condition the central ballistic parameter corresponding to the second component will be $2/\gamma$ and the second component has the form factor equal to -1. The author with the above conditions made a comparative study of the composite charge and the single charge regarding their piezometric efficiency (PE) and muzzle velocity (MV). The case of single charge has already been considered in Part I. Under the constant pressure phase during the second stage of burning the piezometric efficiency has been calculated both analytically and numerically. It has been shown that the piezometric efficiency and the muzzle velocity of the composite charge cannot always be made greater than that of the single charge. But if the length of the gun be sufficiently large in comparison to the initial free space behind the shot, then the PE and MV may be increased in comparison with that of the single charge. Also both the quantities increase if M_1 be increased i.e. if the ratio C_1/C be decreased except perhaps for the progressive propellants.

FIRST STAGE OF BURNING

Assuming the values of γ's of the two charges to be equal and neglecting the covolume effect, the basic equations in the first stage of burning as given by Kapur[2] are

$$F_1 C_1 Z_1 + F_2 C_2 Z_2 = p\left[K_0 + A x - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}\right] + \frac{1}{2}\omega(\gamma - 1) v^2 \tag{1}$$

$$\omega v \frac{dv}{dx} = \omega \frac{dv}{dt} = A p \tag{2}$$

$$D_i \frac{df_i}{dt} = -\beta_i p \tag{3}$$

$$Z_i = (1 - f_i)(1 + \theta_i f_i), \qquad (i = 1, 2) \tag{4}$$

Then equations are to be integrated with the initial conditions

$$x = v = p = Z_1 = Z_2 = 0, \qquad f_1 = f_2 = 1$$

From (2) and (3) integrating with the initial conditions one gets

$$v = \frac{A D_1}{\beta_1 \omega}(1 - f_1) = \frac{A D_2}{\beta_2 \omega}(1 - f_2) \tag{5}$$

At all burnt of the first component

$$v_{B1} = \frac{A D_1}{\beta_1 \omega} \text{ by putting } f_1 = 0 \tag{6}$$

Also the value of f_2 at burnt of the first component is

$$f_{2B1} = 1 - \frac{D_1/\beta_1}{D_2/\beta_2} = 1 - \frac{1}{\alpha_0}$$

where $$\alpha_0 = \frac{D_2/\beta_2}{D_1/\beta_1} \tag{7}$$

when $\frac{D_2}{\beta_2} > \frac{D_1}{\beta_1}$ i.e. the charge C_1 burns out first.

SECOND STAGE OF BURNING

Following Kapur[2], the equations at this stage are

$$F_1 C_1 + F_2 C_2 Z_2 = p\left[K_0 + Ax - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}\right] + \frac{1}{2}\,\omega\,(\gamma - 1)\,v^2 \tag{8}$$

$$\omega\, v \frac{dv}{dx} = A\,p \tag{9}$$

$$D_2 \frac{df_2}{dt} = -\beta_2\, p \tag{10}$$

$$Z_2 = (1 - f_2)(1 + \theta_2 f_2) \tag{11}$$

These equations are to be integrated under constant pressure phase i. e. $p = p_{B1}$. The initial conditions are

$$x = x_{B1}\,,\ v = v_{B1}\,,\ Z_2 = Z_{2B1}\,,\ f_2 = f_{2B1} = 1 - \frac{1}{\alpha_0} \tag{12}$$

Also in the second stage of burning

$$v = v_{B1} + \frac{A\,D_2}{\beta_2\,\omega}\left(1 - \frac{1}{\alpha_0} - f_2\right) \tag{13}$$

Now Ray[1] obtained the condition that Second component burns under constant pressure which is the pressure at burnt of the first component. The condition, he obtained, is

$$\theta_2 = -1 \quad \text{and} \quad M_2 = \frac{2}{\gamma} \tag{14}$$

DETERMINATION OF MAXIMUM PRESSURE

From equation (1), (4), (5) and (14) one gets

$$F_1 C_1 \frac{\beta_1\,\omega}{A\,D_1}\,v\left(1 + \theta_1 - \frac{v\,\beta_1\,\omega}{A\,D_1}\,\theta_1\right) + F_2 C_2 \frac{\beta_2^{\,2}\,\omega^2}{A^2 D_2^{\,2}}\,v^2 = A\,p\,(x + l) + \frac{1}{2}\,\omega\,(\gamma - 1)\,v^2$$

where

$$A\,l = K_0 - \frac{C_1}{\delta_1} - \frac{C_2}{\delta_2}$$

By (2) we get the differential equation

$$\frac{dx}{x + l} = \frac{dv}{\dfrac{F_1 C_1 \beta_1 (1 + \theta_1)}{A\,D_1} + v\left(\dfrac{1}{2} - \dfrac{\theta_1}{M_1}\right)} \tag{15}$$

where $$\theta_1 \neq -1$$

when $\theta_1 \neq \frac{M_1}{2}$, integrating the equation (15) with the condition that $x = 0$, $v = 0$ one gets

$$\log\left(1 + \frac{x}{l}\right) = \frac{2\,M_1}{M_1 - 2\theta_1}\,\log\left[1 + v\,\frac{A\,D_1}{F_1\,C_1\,\beta_1\,(1 + \theta_1)}\;\frac{M_1 - 2\,\theta_1}{2\,M_1}\right] \tag{15a}$$

Now $x = x_{B1}$ when $v = v_{B1}$
we have from (6)

$$x_{B1} = l\left[\left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{\left(\frac{2M_1}{M_1-2\theta_1}\right)} - 1\right] \tag{16}$$

Again when $\theta_1 = \frac{M_1}{2}$, equation (15) on integration leads to

$$x_{B1} = l\left[e^{\left(\frac{M_1}{1+\theta_1}\right)} - 1\right] \tag{17}$$

Again for $\theta_1 \neq \frac{M_1}{2}$, differentiating both sides of (15a) with respect to x, we have

$$\frac{dv}{dx} = \frac{F_1\, C_1\, \beta_1}{A\, D_1}\,\frac{1+\theta_1}{l}\left(1+\frac{x}{l}\right)^{-\frac{M_1+2\theta_1}{2M_1}}$$

and from (2) one gets,

$$p = \frac{2\,F_1\,C_1}{A\,l}\,\frac{(1+\theta_1)^2}{M_1-2\theta_1}\left[\left(1+\frac{x}{l}\right)^{-\left(\frac{2\theta_1}{M_1}\right)} - \left(1+\frac{x}{l}\right)^{-\frac{M_1+2\theta_1}{2M_1}}\right] \tag{18}$$

Hence from (16)

$$p_{B1} = \frac{2\,F_1\,C_1}{A\,l}\,\frac{(1+\theta_1)^2}{M_1-2\theta_1}\left[\left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{-\left(\frac{4\theta_1}{M_1-2\theta_1}\right)} - \left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{-\frac{M_1+2\theta_1}{M_1-2\theta_1}}\right] \tag{19}$$

for $\theta_1 = \frac{M_1}{2}$ similarly we have the expression for p_{B1} as

$$p_{B1} = \frac{F_1\,C_1\,(1+\theta_1)}{A\,l}\,e^{-\frac{M_1}{1+\theta_1}} \tag{20}$$

where

$$p = \frac{F_1\,C_1\,(1+\theta_1)^2}{A\,l\,M_1}\left(1+\frac{x}{l}\right)^{-1}\log\left(1+\frac{x}{l}\right) \tag{21}$$

Now to determine the maximum pressure for $\theta_1 \neq \frac{M_1}{2}$, we have noticed that $\frac{d^2 p}{dx^2} < 0$ when $M_1 \neq 2\,\theta_1$.

Hence the pressure will be maximum at x given by

$$1+\frac{x}{l} = \left(\frac{M_1+2\,\theta_1}{4\,\theta_1}\right)^{\frac{2M_1}{M_1-2\theta_1}} \tag{22}$$

But for $\theta_1 = 0$ (22) gives no finite values of x and hence the maximum pressure will occur at burnt of the first component.

Hence

$$p_{max}\Big|_{\theta_1=0} = \frac{2\,F_1\,C_1}{A\,l}\,\frac{1}{M_1+2} \tag{23}$$

From (18) and (22), the expression for the maximum pressure for $\theta_1 \neq 0$ is given by

$$p_{max} = \frac{2\,F_1\,C_1}{A\,l}\,\frac{(1+\theta_1)^2}{M_1-2\theta_1}\left[\left(\frac{M_1+2\,\theta_1}{4\,\theta_1}\right)^{-\frac{4\,\theta_1}{M_1-2\,\theta_1}} - \left(\frac{M_1+2\,\theta_1}{4\,\theta_1}\right)^{-\frac{M_1+2\theta_1}{M_1-2\theta_1}}\right] \tag{24}$$

Similarly for $\theta_1 = \frac{M_1}{2}$ the expression for maximum pressure is found to be

$$p_{max} = \frac{F_1\,C_1}{2\,A\,l}\,\frac{(1+\theta_1)^2}{\theta_1}\,\frac{1}{e} \tag{25}$$

Now we consider the second stage of burning to deduce v_{B2} and x_{B2}, the pressure remaining constant during this stage. From (13) by putting $f_2 = 0$ and the value of v_{B1}, we have

$$v_{B2} = \frac{A\,D_1}{\beta_1\,\omega}\,\alpha_0 \tag{26}$$

From (9) and (10) we have
$$\frac{dv}{df_2} = -\frac{A\,D_2}{\beta_2\,\omega},$$

which by (10) and $p = p_{B1}$ gives

$$\frac{d^2 x}{df_2^2} = \frac{A\,D_2^2}{\beta_2^2 \omega\, p_{B1}}$$

and on integrating we have
$$\frac{dx}{df_2} = -\frac{A\,D_2^2}{\beta_2^2 \omega\, p_{B1}}\,(1-f_2)$$

Again integrating with the condition that

$$x = x_{B1},\quad f_2 = f_{2B1} = 1 - \frac{1}{\alpha_0},$$

we have

$$x = x_{B1} + \frac{A\,D_2^2}{2\beta_2^2\, p_{B1}\,\omega}\left[(1-f_2)^2 - \frac{1}{\alpha_0^2}\right]$$

Putting the value of x_{B1} and p_{B1} when $\theta_1 \neq \dfrac{M_1}{2}$

$$\frac{x_{B2}}{l} = \left[\left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{\frac{2M_1}{M_1-2\theta_1}} - 1\right] + \frac{A^2 D_2^2}{4\,\beta_2^2 F_1 C_1\,\omega}\;\frac{M_1-2\theta_1}{(1+\theta_1)^2}\left(1-\frac{1}{\alpha_0^2}\right)\cdot$$
$$\cdot\left[\left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{-\frac{4\theta_1}{M_1-2\theta_1}} - \left\{\frac{M_1+2}{2(1+\theta_1)}\right\}^{-\frac{M_1+2\theta_1}{M_1-2\theta_1}}\right]^{-1} \tag{27}$$

and when

$$\theta_1 = \frac{M_1}{2};$$

$$\frac{x_{B2}}{l} = e^{\frac{M_1}{1+\theta_1}} + \frac{M_1}{2(1+\theta_1)}\left(\alpha_0^2 - 1\right) e^{-\frac{M_1}{1+\theta_1}} - 1 \tag{28}$$

DETERMINATION OF MUZZLE VELOCITY

After all burnt the gases expand adiabatically and the corresponding equations are

$$\omega\, v\,\frac{dv}{dx} = A\,p$$

$$p = p_{B1}\left(\frac{x_{B2}+l}{x+l}\right)^{\gamma} \tag{29}$$

Integrating equations (29) we have

$$v^2 = v_{B2}^2 + \frac{2\,A}{\omega\,(1-\gamma)}\, p_{B1}\,(x_{B2}+l)^{\gamma}\left[(x+l)^{1-\gamma} - (x_{B2}+l)^{1-\gamma}\right] \tag{30}$$

Now for $\theta_1 \neq \dfrac{M_1}{2}$ if V_1 be the muzzle velocity and η_1 is the corresponding quantity in non-dimensional form then by putting $x = d$ (length of the gun) in (30)

$$V_1^2 = \frac{A^2 D_1^2}{\beta_1^2 \omega^2} \left[\alpha_0^2 - \frac{2Al}{\omega(1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D_1^2} p_{B1} M_0' \left\{ 1 - \left(\frac{1+d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \tag{31}$$

and

$$\eta_1^2 = M_1^2 \left[\alpha_0^2 - \frac{2Al}{\omega(1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D_1^2} p_{B1} M_0' \left\{ 1 - \left(\frac{1+d/l}{M_0'} \right)^{1-\gamma} \right\} \right] \tag{32}$$

where

$$M'_0 = \left\{ \frac{M_1+2}{2(1+\theta_1)} \right\}^{\frac{2M_1}{M_1-2\theta_1}} + \frac{A D_2^2}{2\beta_2^2 \omega p_{B1} l} \left(1 - \frac{1}{\alpha_0^2} \right)$$

For $\theta_1 = \frac{M_1}{2}$, we have,

$$V_2^2 = \frac{A^2 D_1^2}{\beta_1^2 \omega^2} \alpha_0^2 - \frac{2Al}{\omega(1-\gamma)} p_{B1} \left\{ e^{\frac{M_1}{1+\theta_1}} + \frac{M_1(\alpha_0^2-1)}{2(1+\theta_1)} e^{-\frac{M_1}{1+\theta_1}} \right\} \cdot$$

$$\cdot \left[1 - \left(\frac{1+d/l}{e^{\frac{M_1}{1+\theta_1}} + \frac{M_1(\alpha_0^2-1)}{2(1+\theta_1)} e^{-\frac{M_1}{1+\theta_1}}} \right)^{1-\gamma} \right] \tag{33}$$

and

$$\eta_2^2 = M_1^2 \left[\alpha_0^2 - \frac{2Al}{\omega(1-\gamma)} \frac{\beta_1^2 \omega^2}{A^2 D_1^2} p_{B1} M_0 \left\{ 1 - \left(\frac{1+d/l}{M_0} \right)^{1-\gamma} \right\} \right] \tag{34}$$

where

$$M_0 = e^{\frac{M_1}{1+\theta_1}} + \frac{M_1(\alpha_0^2-1)}{2(1+\theta_1)} e^{-\frac{M_1}{1+\theta_1}}$$

The mean pressure is given by

$$\bar{p} = \frac{\omega V^2}{2Ad}$$

Hence to determine Piezometric efficiency we are to consider three cases :

Case 1. $\theta_1 = \frac{M_1}{2}$

Piezometric efficiency

$$= \frac{l}{d} \frac{\theta_1}{(1+\theta_1)^2} e M_1 \left[\alpha_0^2 - \frac{2M_0(1+\theta_1)}{M_1(1-\gamma)} e^{-\frac{M_1}{1+\theta_1}} \left\{ 1 - \left(\frac{1+d/l}{M_0} \right)^{1-\gamma} \right\} \right] \tag{35}$$

Case 2. $\theta_1 = 0$

Piezometric efficiency

$$= \frac{l}{d} \frac{M_1(M_1-2\theta_1)}{4(1+\theta_1)^2} \frac{\left[\alpha_0^2 - \frac{2l\beta_1^2\omega}{AD_1^2(1-\gamma)} p_{B1} M_0' \left\{ 1 - \left(\frac{1+d/l}{M_0'} \right)^{1-\gamma} \right\} \right]}{\left\{ \frac{M_1+2}{2(1+\theta_1)} \right\}^{-\left(\frac{4\theta_1}{M_1-2\theta_1}\right)} - \left\{ \frac{M_1+2}{2(1+\theta_1)} \right\}^{-\frac{M_1+2\theta_1}{M_1-2\theta_1}}} \tag{36}$$

In the above expression we are to put $\theta_1 = 0$.

Case 3. $\theta_1 \neq \frac{M_1}{2}$

Piezometric efficiency

$$= \frac{l}{d} \frac{M_1(M_1-2\theta_1)}{4(1+\theta_1)^2} \frac{\left[\alpha_0^2 - \frac{2l\beta_1^2\omega}{AD_1^2(1-\gamma)} p_{B1} M_0' \left\{ 1 - \left(\frac{1+d/l}{M_0'} \right)^{1-\gamma} \right\} \right]}{\left(\frac{M_1+2\theta_1}{4\theta_1} \right)^{-\left(\frac{4\theta_1}{M_1-2\theta_1}\right)} - \left(\frac{M_1+2\theta_1}{4\theta_1} \right)^{-\frac{M_1+2\theta_1}{M_1-2\theta_1}}} \tag{37}$$

Now for composite charge there are three possibilities:

(i) $\frac{x_{B1}}{d} > 1$ i.e. shot leaves the gun before all burnt of the first component such that constant pressure phase has not yet been reached. We are not concerned with this stage as we are to consider the cases under constant pressure phase with the second component.

(ii) $\frac{x_{B1}}{d} < 1$ and $\frac{x_{B2}}{d} > 1$ i.e. the shot leaves the gun after all burnt of the first component. In this stage no analytical expression for PE is possible and as such during this stage PE is to be calculated numerically.

(iii) $\frac{x_{B1}}{d} < 1$ and $\frac{x_{B2}}{d} < 1$ i.e. the shot leaves the gun after all burnt of the second component. The expressions (35), (36) and (37) give the PE during this stage of burning. However in a recent paper Bhattacharyya[3] discussed the calculation of MV and PE in an orthodox gun with single charge.

NUMERICAL CALCULATIONS

The numerical calculation is based on the principle as discussed in author's previous paper.

(i) For $l/d = 0{\cdot}5$, $\theta = 0$, $M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·093	0·8114	65	1·694	0·7629	85	1·132	0·4219
		70	1·635	0·6805	90	1·068	0·3738
		75	1·592	0·5913	95	1·012	0·3649
		80	1.325	0·5012	98	1·005	0·3216

(ii) For $l/d = 0{\cdot}3$, $\theta = 0$, $M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·370	0·7656	50	2·759	0·8771	85	2·135	0·8089
		60	2·712	0·8551	90	1·963	0·7999
		70	2·521	0·8421	95	1·956	0·7625
		75	2·467	0·8379			

(iii) For $l/d = 0{\cdot}2$, $\theta = 0$, $M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·562	0·6632	40	3·235	0·8985	80	2·713	0·8711
		50	3·130	0·8952	85	2·543	0.8679
		60	3·050	0·8901	90	2·480	0·8611
		70	2·912	0·8875	95	2·430	0·8592

(iv) For $l/d = 0{\cdot}1, \quad \theta = 0, \quad M = 1$

Single Charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·833	0·3207	30	5·010	0·8939	70	3·729	0·7929
		40	4·925	0·8775	80	3·525	0·7667
		50	4·552	0·8425	90	3·282	0·6948
		60	4·012	0·8135	95	3·151	0·6782

(v) For $l/d = 0{\cdot}5, \quad \theta = 1, \quad M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·330	0·7522	65	3·012	0·7625	90	1·625	0·3629
		80	2·013	0·5236	95	1·459	0·3125
		88	1·721	0·4293			

(vi) For $l/d = 0{\cdot}2, \quad \theta = 1, \quad M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·709	0·4920	70	3·523	0·5129	90	2·312	0·3886
		80	3·125	0·4559	95	2·216	0·3725
		85	2·562	0·4229			

(vii) For $l/d = 0{\cdot}1, \quad \theta = 1, \quad M = 1$

Single charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·944	0·4525	75	3·859	0·6016	90	3·450	0·5225
		80	3·529	0·5662	95	3·392	0·5038
		85	3·422	0·5449			

(viii) For $l/d = 0{\cdot}1, \quad \theta = -0{\cdot}5, \quad M = 1$

Single Charge		Composite charge					
MV	PE	C_1/C (%)	MV	PE	C_1/C (%)	MV	PE
1·787	0·8400	80	3·526	0·3609	90	2·927	0·4865
		85	3·223	0·4205	95	2·734	0·5490

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