THE EFFECT OF VISCOSITY ON THE STABILITY OF A RADIALLY ACCELERATED, CYLINDRICAL SHELL OF PLASMA

P. K. BHAT, B. K. KAUSHIK & S. P. SRIVASTAVA

College of Military Engineering, Poona

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The effect of viscosity on the axisymmetric stability of a radially accelerated cylindrical shell of plasma in the presence of a solenoidal magnetic field has been investigated. It is found that when the Reynolds number as well as the dimensionless wave number is small, the system is unstable and the growth rate of instability depends upon its acceleration. This is in agreement with the experimental observations of Dickinson.

Considerable interest on the problem of stability of a radially accelerated cylindrical shell of plasma in the presence of high magnetic fields has been shown recently[1,2] as it leads towards the success of thermonuclear energy generation. The idealized theoretical investigations of several authors[3-7] show that such a system is unstable against small perturbations. They[6,7] attribute this instability to the neglect of viscosity and finite conductivity of the plasma shell.

This paper deals with the effect of viscosity on the stability of a radially accelerated cylindrical plasma shell of finite thickness, in the presence of solenoidal magnetic field, against small amplitude axisymmetric disturbances.

We have set the problem as an initial value problem. We assume that the shell is moving radially at a uniform rate before any disturbance is imposed on the system. In order to avoid cumbersome analysis we have considered only those disturbances for which the Reynolds number 'R_e' and the dimensionless wave number k_1, are both small and have discussed two cases of interest. In the first case, we have taken $R_e = k_1 = \epsilon << 1$, and have expanded the amplitudes of the disturbances in powers of ϵ, neglecting terms of the order of ϵ^2. In the second case we have assumed that R_e and k_1 are of different magnitudes of smallness and have expanded the amplitudes of perturbations in powers[8] of R_e and k_1 neglecting terms of the order higher than the first. We find that the imploding shell as well as the exploding shell is unstable in both cases. This is in agreement with the experimental observations of Dickinson et. al[9].

INITIAL STATE

Let at time $t = 0$, the inner and outer dimensionless radii of the shell be 1 and β respectively. The dimensionless magnetic field in the various regions of the system is

$$\vec{H} = \begin{bmatrix} (0\,,0\,,\ 1) & ; & 1 < r < \beta \\ (0\,,0\,,\ H_0) & ; & r < 1 \\ (0\,,0\,,\ H_0) & ; & r > \beta \end{bmatrix} \tag{1}$$

where r is the radial co-ordinate and β is the ratio of the outer radius to the inner radius. The regions $r < 1$ and $r > \beta$ denote respectively the inside and outside vacua. The dimensionless plasma pressure is given by

$$P = \left(\frac{H_0{}^2 - 1}{2} \right). \tag{2}$$

In passing we note that for non-dimensionalization we have made use of the following characteristic quantities:

Length = Inner radius of the steady shell

Velocity $= \dfrac{\text{Rate of implosion or explosion}}{\text{Inner radius}}$

Pressure $=$ Density $\times$ (velocity)^2

Megnetic field $=$ Uniform axial magnetic field in $1 < r < \beta$.

Starting with this initial state we impose a radial velocity $\frac{k}{r}$ to the system and calculate the physical and the dynamical state at any time $t \neq 0$. The dimensionless set of solutions consistant with the equations of motion and the boundary conditions at the boundaries of the system is :

(*a*) *Exploding shell* ($t > 0$)

Plasma ($R_i \leqslant r \leqslant R_0$) :

$$R_i^2 = 1 + 2\,t\,;\; R_0^2 = \beta^2 + 2\,t$$

$$\vec{v} = \left[\frac{1}{r}, 0, 0\right];\quad \vec{H} = [0, 0, 1]$$

$$\vec{E} = \left[0, \frac{1}{r}, 0\right];\quad \vec{J} = [0, 0, 0]$$

$$p = \left[\frac{1}{2\,R_i^3} - \frac{2}{R_e\,R_i^2} - \frac{1}{2}\left(1 - \frac{H_0^2}{R_i^4}\right) - \frac{1}{2\,r^3}\right]$$

Inner vacuum ($r < R_i$) :

$$\vec{H}_i = \left[0, 0, \frac{H_0}{R_i^2}\right]$$

$$\vec{E}_i = \left[0, \frac{H_0\,r}{R_i^4}, 0\right]$$

$$\left.\begin{aligned} J^* &= \left[0, \frac{H_0}{R_i^2} - 1, 0\right] \\ q^* &= 0 \end{aligned}\right\} \text{ at } r = R_i$$

Outer vacuum ($r > R_0$):

$$\vec{H}_0 = \left[0, \frac{\alpha_3(t)}{r}, H_0\right]$$

$$\vec{E}_0 = \left[0, \frac{H_0}{r}, \alpha'_3(t)\log\frac{r}{R_0} - \frac{\alpha_3(t)}{R_0^2}\right]$$

$$\left.\begin{aligned} \vec{J}^* &= \left[0, H_0 - 1, \frac{-\alpha_3(t)}{R_0}\right] \\ q^* &= 0 \end{aligned}\right\} \text{ at } r = R_0$$

where
$$\alpha_3^2(t) = 2\,R_0^2\left[\frac{1}{2\,R_0^3} - \frac{1}{2\,R_i^3} + \frac{2}{R_e\,R_i^2} + \right.$$
$$\left. + \frac{1}{2}\left(1 - \frac{H_0^2}{R_i^4}\right) - \frac{2}{R_e\,R_0^2} - \left(\frac{H_0^2 - 1}{2}\right)\right] \qquad (3)$$

where $\vec{v}, \vec{H}, \vec{E}, \vec{J}, p, \vec{J}^*, q^*$ denote the fluid velocity, magnetic field, electric field, current density, fluid pressure, surface current density, and surface charge density respectively. The subscripts i and 0 with a physical quantity denote its value in the inner and outer vacua respectively.

(b) *Imploding shell* ($t < 0$)

In this case, the solutions are obtained from (3) by changing dimensionless time 't' to '$-t$'.

PERTURBATION EQUATIONS

At any time $t = t_0 (\neq 0)$, we apply a small amplitude axisymmetric disturbance of the type $\tilde{X}(r,t) = \hat{X}(r,t)\, e^{ik_1 z}$ to the system where k_1 is the dimensionless axial wave numbers and z the dimensionless axial coordinate.

Let $\vec{\tilde{v}}$, $\tilde{p}$, $\vec{\tilde{H}}$, $\vec{\tilde{E}}$ and $\vec{\tilde{J}}$ denote the perturbations in the velocity, pressure, magnetic field, electric field and current density respectively. The linearised set of equations in dimensionless form determining them in various regions of the system is :

Plasma

$$\frac{\partial \vec{\tilde{v}}}{\partial t} + (\vec{\tilde{v}} \cdot \nabla)\, \vec{v^0} + (\vec{v} \cdot \nabla)\, \vec{\tilde{v}} = - \nabla \tilde{p} + + M \{ \text{Curl}\, \vec{H^0} \times \vec{\tilde{H}} + \text{Curl}\, \vec{\tilde{H}} \times \vec{H_0} \} + \frac{1}{R_e} \nabla^2 \vec{\tilde{v}} \tag{4}$$

$$\text{div}\, \vec{\tilde{v}} = 0 \tag{5}$$

$$\frac{\partial \vec{\tilde{H}}}{\partial t} = \text{Curl}\, (\vec{v^0} \times \vec{\tilde{H}}) + \text{Curl}\, (\vec{\tilde{v}} \times \vec{H^0}) \tag{6}$$

$$\text{div}\, \vec{\tilde{H}} = 0 \tag{7}$$

$$\vec{\tilde{E}} = - \{ \vec{\tilde{v}} \times \vec{H^0} + \vec{v^0} \times \vec{\tilde{H}} \} \tag{8}$$

where $M = \dfrac{\text{Magnetic pressure}}{\text{Plasma pressure}}$; $R_e = \dfrac{\text{Rate of implosion or explosion}}{\text{Coefficient of kinematic viscosity}}$

and $\vec{v^0}$ and $\vec{H^0}$ denote the unsteady state velocity and magnetic field respectively.

Inner and Outer Vacua

$$\text{div}\, \vec{\tilde{H}} = 0 \tag{9}$$

$$\text{Curl}\, \vec{\tilde{H}} = 0 \tag{10}$$

$$\text{Curl}\, \vec{\tilde{E}} = - \frac{\partial \vec{\ddot{H}}}{\partial t} \tag{11}$$

$$\text{div}\ \vec{\tilde{E}} = 0 \tag{12}$$

The set of boundary conditions to be satisfied by the perturbations in the linearised form is as follows:

$$\vec{n^0} \left[\vec{\tilde{H}} \right] + \vec{\tilde{n}} \left[\vec{H^0} \right] = 0 \tag{13}$$

$$\vec{n^0} \times \left[\vec{\tilde{H}} \right] + \vec{\tilde{n}} \times \left[\vec{H^0} \right] = \vec{\tilde{J}}^* \tag{14}$$

$$\vec{n^0} \times \left[\vec{\tilde{E}} \right] + \vec{\tilde{n}} \times \left[\vec{E^0} \right] = u \left[\vec{H^0} \right] + u^0 \left[\vec{\tilde{H}} \right] \tag{15}$$

$$\vec{n^0} \cdot \left[\overleftrightarrow{\tilde{p}} \right] + \vec{\tilde{n}} \cdot \left[\overleftrightarrow{p^0} \right] = \vec{\tilde{J}}^* \times \vec{H^0} + \vec{J^{0*}} \times \vec{\tilde{H}} + \tilde{q}^* \vec{E_0} + q^{0*} \vec{\tilde{E}} \tag{16}$$

$$\vec{n^0} \cdot \left[\vec{\tilde{E}} \right] + \vec{\tilde{n}} \cdot \left[\vec{E^0} \right] = \tilde{q}^* \tag{17}$$

$$\vec{n^0} \cdot \vec{\tilde{v}} + \vec{\tilde{n}} \cdot \vec{v^0} = \tilde{u} \tag{18}$$

where $\vec{u^0}$ and $\overleftrightarrow{p}$ stand for the velocity of the boundary in the undisturbed state and the stress tensor, respectively. Further, the superscript '0' with a physical quantity denotes its value in the unperturbed state.

The equations to the disturbed boundaries are

$$r = R_{i,\,0} + \left[\delta_r (t) \right]_{i,\,0} e^{ik_1 z} \tag{19}$$

where (δ_r) stands for the displacement of the boundary. The perturbation in the unit normal to the surface is

$$\vec{\tilde{n}} = \left[0\,,\, 0\,,\, -i\, k_1\, (\delta r)_{i,\,0}\, e^{ik_1 z} \right] \tag{20}$$

PART 'A'

SOLUTIONS FOR $R_e = k_1 << 1$

Here we shall solve the partial differential equations (4) to (12) under the assumption that $R_e = k_1 = \epsilon << 1$, as this type of disturbance is of particular interest in such problems. We set

$$\hat{X} = \hat{X}_0 + \epsilon\, \hat{X}_1 + 0\, (\epsilon^2) \tag{21}$$

and evaluate $\hat{X}_0$ and $\hat{X}_1$ from (4) to (12) and (21).

(a) Solutions for Exploding Shell

The zeroth order set of solutions in this case is as follows:

***Plasma*:**

$$\left.\begin{aligned} \overset{\Delta}{\vec{v}}_0 &= \left[\,0\,,0\,,F_3(t)\,\right] \\ \overset{\Delta}{\vec{H}}_0 &= \left[\,0\,,0\,,0\,\right] \\ \overset{\Delta}{\vec{E}}_0 &= \left[\,0\,,0\,,0\,\right] \\ \overset{\Delta}{p}_0 &= \phi_5(t) \end{aligned}\right\} \tag{22}$$

where $F_3(t)$ and $\phi_5(t)$ are arbitrary functions of integration.

***Inner vacuum*:**

$$\left.\begin{aligned} \left(\overset{\Delta}{\vec{H}}_0\right)_i &= \left[\,0\,,0\,,\frac{c_1}{2t+1}\,\right] \\ \left(\overset{\Delta}{\vec{E}}_0\right)_i &= \left[\,0\,,\frac{c_1\,r}{(1+2t)^2}\,,0\,\right] \\ \left(\delta R_i\right)_0 &= 0 \end{aligned}\right\} \tag{23}$$

***Outer vacuum*:**

$$\left(\overset{\Delta}{\vec{H}}_0\right)_0 = \left[\,0\,,0\,,0\,\right];\ \left(\overset{\Delta}{\vec{E}}_0\right)_0 = \left[\,\frac{\Omega_{11}(t)}{r}\,,0\,,0\,\right];\ (\delta R_0)_0 = 0 \tag{24}$$

where $\Omega_{11}(t)$ is an arbitrary function of integration. $(\delta R_i)_0$ and $(\delta R_0)_0$ are the zeroth order displacements of the inner and outer boundary respectively.

(b) Solutions for Imploding Shell

The solutions for imploding case are obtained from (22) to (24) after changing t to $-t$. The first order set of solutions after making use of (22) to (24) is:

Plasma:

$$\left.\begin{aligned} \overset{\Delta}{\vec{H}}_1 &= \left\{\frac{\lambda_{11}}{r}\,,0\,,0\right\} \\ \overset{\Delta}{\vec{v}}_1 &= \left[\left\{-\frac{i\,r}{2}F_3(t) + \frac{1}{r}\phi_1(t)\right\},\ \frac{\phi_4(t)}{r}\,,\right. \\ &\qquad \left.,\left\{\frac{r^2}{4}F'_3(t) + \phi_2(t)\log r + \phi_3(t)\right\}\right] \\ \overset{\Delta}{\vec{E}}_1 &= \left\{-\frac{\phi_4(t)}{r}\,,-\frac{i\,r}{2}F_3(t) + \frac{1}{r}\phi_1(t)\,,0\right\} \end{aligned}\right\} \tag{25}$$

where $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$ and $\phi_4(t)$ are arbitrary functions of integration and λ_{11} is a pure constant, *Inner vacuum:*

$$\left.\begin{aligned} \left(\hat{\vec{H}}_1\right)_i &= \left\{ -\frac{i\, c_1\, r}{2\,(1+2\,t)} \,,\, 0 \,,\, l_1\,(t) \right\} \\ \left(\hat{\vec{E}}_1\right)_i &= \left\{ 0 \,,\, -\frac{r}{2}\,\frac{d\,l_1}{d\,t} \,,\, l_2\,(t) \right\} \end{aligned}\right\} \tag{26}$$

Outer vacuum:

$$\left.\begin{aligned} \left(\hat{\vec{H}}_1\right)_0 &= \left\{ \frac{E_{11}}{r} \,,\, 0 \,,\, E_{22} \right\} \\ \left(\hat{\vec{E}}_1\right)_0 &= \left[\frac{m_5\,(t)}{r} \,,\, \frac{m_4\,(t)}{r} \,,\, m_3\,(t) \right] \end{aligned}\right\} \tag{27}$$

where $l_1\,(t)$; $l_2\,(t)$, $m_3\,(t)$, $m_4\,(t)$, $m_5\,(t)$ are arbitrary functions of integration and E_{11} and E_{22} are pure constants.

DETERMINATION OF ARBITRARY FUNCTIONS OF INTEGRATION

After applying the boundary conditions (13) to (18) upto this order of approximation we have the following set of equations from which we determine the arbitrary functions of time occuring in the solutions (22) to (27) :

$$\left.\begin{aligned} \lambda_{11} &= -\frac{i\,c_{11}}{2} \,;\, l_2\,(t) = 0,\ \Omega_{11}\,(t) = 0 \\ \phi_4\,(t) &= 0 \,,\, E_{11} = -\frac{i\,c_1}{2} \,;\, E_{22} = 0 \\ \phi_2\,(t) &= 0 \,,\, F_3\,(t) = s_1 \,;\, m_5\,(t) = 0 \\ \phi_3\,(t) &= 0 \end{aligned}\right\} \tag{28}$$

$$\phi_5\,(t) + i\,s_1 + 2\,\frac{\phi_1\,(t)}{R_i^2} = \frac{c_1}{1+2\,t} \tag{29}$$

$$\frac{d\,x_{12}}{d\,t} + \frac{x_{12}}{R_i^2} = -\frac{i\,R_i\,s_1}{2} + \frac{1}{R_i}\,\phi_1\,(t) \tag{30}$$

$$\phi_5\,(t) + i\,s_1 + \frac{2\,\phi_1\,t)}{R_0^2} = 0 \tag{31}$$

$$\frac{d\,y_{12}}{d\,t} + \frac{y_{12}}{R_0^2} = -\frac{i\,R_0\,s_1}{2} + \frac{\phi_1\,(t)}{R_o} \tag{32}$$

$$m_3\,(t) = \frac{\alpha_3\,(t)}{R_0}\left\{ -\,i\,R_0\,F_3\,(t) + \frac{1}{R_0}\,\phi_1\,(t) \right\} \tag{33}$$

$$m_4\,(t) = H_0\,R_0\left\{ -\frac{i\,R_0}{2}\,F_3\,(t) + \frac{1}{R_0}\,\phi_1\,(t) \right\} \tag{34}$$

where $x_{12} = (\delta\,R_i)_1$ and $y_{12} = (\delta\,R_0)_1$ are the first order displacements of the inner and outer boundary respectively.

From (29) and (31) we get

$$\phi_1\,(t) = \frac{H_1\,c_1}{2\,(\beta^2-1)}\,(\beta^2 + 2\,t) \tag{35}$$

Again from (31) and (35) we have

$$\phi_5(t) = -i\,s_1 - \frac{H_1 c_1}{\beta^2 - 1} \tag{35a}$$

Making use of (35) in (30) we get

$$x_{12} = \frac{1}{\sqrt{1+2t}}\left[s_2 + \frac{c_1}{2(\beta^2-1)}(\beta^2 t + t^2) - \frac{i}{2}(t+t^2)s_1 \right] \tag{36}$$

where s_2 is an arbitrary constant of integration. (32) and (35) yield

$$y_{12} = \frac{1}{\sqrt{\beta^2+2t}}\left[s_3 + \frac{c_1}{2(\beta^2-1)}(\beta^2 t + t^2) - \frac{i}{2}\,s_1(\beta^2 t + t^2) \right] \tag{37}$$

where s_3 is an arbitrary constant of integration. Similarly from (33) and (34) we have

$$m_3(t) = \left[\frac{1}{(\beta^2+2t)^{3/2}} - \frac{1}{(1+2t)^{3/2}} + \frac{4}{R_e(1+2t)} + \left(1 - \frac{H_0^2}{(1+2t)^2} \right) - \frac{4}{R_e(\beta^2+2t)} - \right.$$

$$\left. -\,(H_0^2 - 1) \right]^{1/2} \cdot \left[-i\sqrt{\beta^2+2t}\;s_1 + \frac{c_1}{2(\beta^2-1)}\sqrt{\beta^2+2t} \right] \tag{38}$$

and

$$m_4(t) = H_0\left[-\frac{i\,s_1}{2}(\beta^2+2t) + \frac{c_1}{2(\beta^2-1)}(\beta^2+2t) \right] \tag{39}$$

CONCLUSIONS

(a) *Exploding Case*

In this case t can take any value > 0, we find that the zeroth order solutions (22) to (24) are bounded for large time. The first order perturbation in the magnetic field inside the plasma shell is steady and therefore remains finite as $t \to \infty$. Further the first order velocity field and the first order electric field become unbounded as $t \to \infty$. The electromagnetic fields in the inner and outer vaccum increase as time advances. Thus the exploding shell is unstable against small wave number disturbances at small Reynolds numbers and the growth rate of instability depends on the rate of explosion. This is in agreement with the experimental results[9].

(b) *Imploding Case*

In this case the inner vaccum of the shell is extinct when $t \to \frac{1}{2}$. We find that the perturbations in the velocity, field, electromagnetic fields inside the plasma, and the plasma pressure remain bounded as $t \to \frac{1}{2}$. The perturbations in electromagnetic fields in the inner and outer vacuum become infinitely large as $t \to \frac{1}{2}$. This shows that the imploding shell is unstable and the rate of growth of instability depends on the rate of implosion.

PART 'B'

SOLUTIONS FOR $R_e \neq k_1$ WHEN R_e AND k_1 ARE SMALL

In this section we shall discuss the perturbations for which both R_e and k_1 are small but of different orders of magnitude of smallness. This approximation helps us to separate out the effects of small k_1 and small R_e. Thus accordingly we set

$$\hat{X} = \hat{X}_0 + k_1\,\hat{X}_1 + R_e\,\hat{X}'_1 + 0\,(R_e^2,\,k_1^2,\,R_e\,k_1) \tag{40}$$

and evaluate $\hat{X}_0$, $\hat{X}_1$ and $\hat{X}'_1$.

After making use of (40) in equations (4) to (12) and separating the various order terms of the same order and solving the resulting partial differential equations we obtain the following sets of various order solutions.

Exploding case: The zeroth order set of solutions for the three regions of the system is given by (22) to (24).

Solutions of Order k_1

Plasma :

$$\left.\begin{aligned} \vec{\hat{v}}_1 &= \left\{ -\frac{ir}{2} F_3(t) + \frac{F_6(t)}{r} , \frac{F_{10}(t)}{r} , F_7(t) \log r + F_8(t) \right\} \\ \vec{\hat{H}}_1 &= \left\{ \frac{\alpha_{11}}{r} , 0 , r \int^t F_3(t)\, dt \right\} \\ \vec{\hat{E}}_1 &= \left\{ -\frac{F_{10}(t)}{r} , -\frac{ir}{2} F_3(t) + \frac{F_6(t)}{r} + \frac{i}{r} \int^t F_3(t)\, dt , 0 \right\} \end{aligned}\right\} \quad (41)$$

where $F_3(t)$, $F_6(t)$, $F_7(t)$, $F_8(t)$ and $F_{10}(t)$ are arbitrary functions of time and α_{11} is a pure constant.

Inner and outer vacua :

The solutions are the same as (26) and (27) respectively.

Solutions of Order R_e

Plasma :

$$\left.\begin{aligned} \vec{\hat{v}}'_1 &= \left\{ \frac{F_{11}(t)}{r} , 0 , 0 \right\} \\ \vec{\hat{H}}'_1 &= \left\{ \frac{A_1}{r} , 0 , 0 \right\} \\ \hat{p}_0 &= \left\{ F_{12}(t) - \frac{F_{11}(t)}{r} - \frac{dF_{11}}{dt} \log r \right\} \\ \vec{\hat{E}}'_1 &= \left\{ 0 , \frac{F_{11}(t)}{r} , 0 \right\} \end{aligned}\right\} \quad (42)$$

Inner vacuum :

$$\left.\begin{aligned} \left(\vec{\hat{H}}'_1\right)_i &= \left\{ 0 , 0 , \mu_{11}(t) \right\} \\ \left(\vec{\hat{E}}'_1\right)_i &= \left\{ 0 , -\frac{r}{2} \frac{d\mu_{11}}{dt} , \mu_{12}(t) \right\} \end{aligned}\right\} \quad (43)$$

Outer vacuum :

$$\left.\begin{aligned} \left(\vec{\hat{H}}'_1\right)_0 &= \left\{ \frac{A_{11}}{r} , 0 , B_{11} \right\} \\ \left(\vec{\hat{E}}'_1\right)_0 &= \left\{ \frac{\lambda_{13}(t)}{r} , \frac{\lambda_{14}(t)}{r} , \lambda_{15}(t) \right\} \end{aligned}\right\} \quad (44)$$

where $F_{11}(t)$, $F_{12}(t)$, $\mu_{11}(t)$, $\mu_{12}(t)$, $\lambda_{13}(t)$, $\lambda_{14}(t)$ and $\lambda_{15}(t)$ are arbitrary functions of time and A_1, A_{11} and B_{11} are pure constants.

The solutions for imploding case are obtained from (41) to (44) after changing t into $(-t)$.

DETERMINATION OF ARBITRARY FUNCTIONS OF INTEGRATION

Applying the boundary conditions (13) to (18) at the perturbed boundaries (19) to (41) —(44) we have

$$\left.\begin{aligned} \alpha_{11} &= -\frac{i\,c_1}{2}\;;\; l_2(t) = 0\,,\; F_{10}(t) = 0 \\ E_{11} &= -\frac{i\,c_1}{2}\,,\; m_4(t) = E_{22} = 0\,,\; F_7(t) = 0 \\ m_5(t) &= 0, F_3(t) = 0\,,\; F_6(t) = 0 \\ x_{11} &= \frac{c_5}{\sqrt{1+2t}}\;;\; y_{11} = \frac{c_6}{\sqrt{1+2t}}\,, \\ l_1(t) &= \frac{c_4}{\sqrt{1+2t}}\;; \end{aligned}\right\} \tag{45}$$

where c_4, c_5, c_6 are arbitrary constants of integration and

$$x_{11} = (\delta R_i)_1\,,\; y_{11} = (\delta R_0)_1$$

Further

$$\left.\begin{aligned} A_1 &= 0\,,\; A_{11} = 0\,, \\ B_{11} &= 0\,,\; \mu_{12}(t) = 0 \end{aligned}\right\} \tag{46}$$

$$\frac{d\,\mu_{11}}{d\,t} + \frac{2\,\mu_{11}}{R_i^2} = -\frac{2\,H_0}{R_i^4}\,F_{11}(t) \tag{47}$$

$$\frac{d\,F_{11}(t)}{d\,t}\log R_i - \frac{F_{11}(t)}{R_i^2} = F_{12}(t) + \frac{c_1}{2\,(1+2\,t)}\left\{\frac{H_0}{R_i^2} + H_0\right\} \tag{48}$$

$$\frac{d\,x_{13}}{d\,t} + \frac{x_{13}}{R_i^2} = \frac{F_{11}(t)}{R_i} \tag{49}$$

$$\lambda_{15} = -\frac{F_{11}(t)}{R_0^2} \times \alpha_3(t) \tag{50}$$

$$\frac{H_0}{R_0}\,F_{11}(t) + \frac{\mu_{11}(t)}{R_0} = \frac{\lambda_{14}(t)}{R_0} \tag{51}$$

$$\frac{d\,F_{11}}{d\,t}\log R_0 - \frac{F_{11}(t)}{R_0^2} = F_{12}(t) \tag{52}$$

$$\frac{d\,y_{13}}{d\,t} + \frac{y_{13}}{R_0^2} = \frac{F_{11}(t)}{R_0} \tag{53}$$

where $x_{13} = (\delta R'_i)_1$ and $y_{13} = (\delta R'_0)_1$

Eliminating $F_{12}(t)$ from (48) and (52) we get

$$\frac{d\,F_{11}}{d\,t} + \left(\frac{1}{R_i^2} - \frac{1}{R_0^2}\right) \cdot \frac{F_{11}(t)}{\log \dfrac{R_0}{R_i}} = -\frac{H_0\,l_1\,(1+t)}{(1+2\,t)^2 \log \dfrac{R_0}{R_i}} \tag{54}$$

Solving (54) for $F_{11}(t)$, we get

$$F_{11}(t) = \left\{\log \frac{\beta^2 + 2\,t}{1+2\,t}\right\}\left[-\frac{H_0\,l_1}{2}\left\{\log(1+2\,t) - \frac{1}{1+2\,t}\right\} + T_1\right] \tag{55}$$

where T_1 is an arbitrary constant of integration. After making use of $F_{11}(t)$ in (47) to (51) and (53) we can obtain the values of

$$\mu_{11}(t)\,;\, F_{12}(t)\,;\, x_{13}\,,\, \lambda_{15}(t)\,,\, \lambda_{14}(t) \text{ and } y_{13}(t)$$

CONCLUSION

From our solutions, we note that in the exploding case, the velocity, the plasma pressure and the electromagnetic fields in the outside vacua grow as t becomes large. In the imploding case when $t \to \frac{1}{2}$, the disturbance becomes infinitely large. Therefore, the exploding shell as well as the imploding shell is unstable. The growth rate of instability in both cases depends on the acceleration of the shell.

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