APPLICATION OF GEOMETRIC PROGRAMMING FOR SPRING DESIGN

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(*Received* 12 *May* 1977 ; *revised* 13 *February* 1978)

The present paper discusses the design of conical helical springs and torsional helical springs using geometric programming technique. It is shown that the present method provides a saving of about 20 per cent of material by weight in a particular case which is substantial.

Springs are one of the most widely used components in different types of machines. They may be used to apply force and to control motion as in brakes and clutches, for measuring forces as in spring balances, for storing energy as in clock springs, for reducing shock or impact severity, as in automobiles, rail-cars, etc., for altering the vibration characteristics of a machine as in flexible mounts, etc. In some of these situations the spring may become a sensitive part and the varying spring rate may become a desirable quality.

Ever since the start of industrial revolution in Europe, the application of springs in different situations has increased considerably (Doughtie & Vallance)[1]. But in the recent past the interest in the design of springs has shifted towards optimization. Some authors have tried to provide optimum design of different types of springs notable amongst them are Hinkle & Morse[2], Rao[3], Suri & Kishor,[4,5] .

In the present paper geometric programming method of optimization has been used for the design of conical helical springs and torsional helical springs. The design of cylindrical helical springs can be done on the lines suggested for conical springs without any difficulty.

NOTATIONS

- C Spring index $=D/d$
- d Wire diameter
- D_1 Smallest mean diameter of the coil
- D_2 greatest mean diameter of the coil
- D_m mean coil diameter
- G torsional modulus of elasticity
- H Height of the cone
- i number of inactive coils
- K Wahl's correction factor
- P maximum spring load
- W Weight of the spring
- W_1 Optimum weight of the spring
- ρ density of wire material
- τ_{max} maximum shear stress in the spring
- δ deflection under maximum load
- M Bending moment
- a distance from the load line to the spring axis
- E modulus of elasticity
- n number of active coils.

DESIGN OF CONICAL HELICAL SPRINGS (MINIMUM WEIGHT)

The expression for weight of a conical helical spring is given by Suri & Kishor[4]

$$\frac{W\,H}{\frac{\rho\pi^2}{8}(H+h)} = Cnd^3 + Cid^3 \tag{1}$$

where

$$d = \left(\frac{8KPC}{\pi\tau_{max}}\right)^{1/2} \tag{2}$$

The deflection for conical spring is taken to be

$$\delta = \frac{2n\,C^3\,P}{d\,G\,K} \tag{3}$$

In the case of a conical spring when the deflection is to be accurately calculated then it is necessary to substitute $(D^2_1+D^2_2)$ for D^2 and $\pi n\,(D_1 + D_2)$ for $\pi n\, D$, D_1 and D_2 being smallest and greatest diameters of the coils. But for the case where the cross section dimension of the spring wire is small in comparison to mean coil diameter then above formula can be used to give fairly correct results.

Substituting the value of d and n from (2) and (3) in (1), we get

Therefore $$W\left(\frac{H}{H+h}\right) = \frac{4\,P\,K^3\,G\,\delta\,\rho}{\tau^2_{max}} + \frac{\rho\, i\, \pi^{1/2}\, 8^{1/2}\, P^{3/2}\, K^{3/2}\, C^{5/2}}{\tau^{3/2}_{max}} \tag{4}$$

K, the Wahl's correction factor can be represented approximately by the following equation for spring index greater than six as shown in Fig. 1

$$K = 1{\cdot}859\; C^{-0{\cdot}207}$$

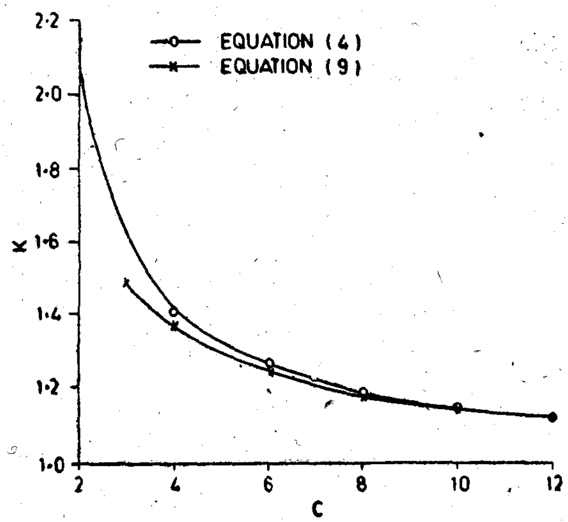

Fig. 1—Conical spring.

By substituting the value of K from (5) in (4), we get

$$W\left(\frac{H}{H+h}\right) = \frac{25{\cdot}64\,PG\,\delta\rho}{\tau^2_{max}}\; C^{-0{\cdot}621} + \frac{12{\cdot}65\; iP^{3/2}\,\rho}{\tau^{3/2}_{max}}\; C^{2{\cdot}09} \tag{6}$$

W function in equation (6) is the posynomial, which is to be minimized with the help of geometric programming. At the optimum point the normality conditions and the orthogonality equations are given by Duffin, Peterson & Zener[6]

$$\delta_1 + \delta_2 = 1 \tag{7}$$

$$-0{\cdot}621\;\delta_1 + 2{\cdot}09\;\delta_2 = 0 \tag{8}$$

Solving the equations (7) and (8), we get

$$\delta_1 = 0 \cdot 770 \quad (9)$$

$$\delta_2 = 0 \cdot 230 \quad (10)$$

Hence the optimum weight W_1 is given by

$$W_1 \left(\frac{H}{H+h} \right) = \left| \frac{25 \cdot 64\, P\, G\, \delta\, \rho}{\tau^2_{max}\; 0 \cdot 770} \right|^{0 \cdot 770} \times \left| \frac{12 \cdot 65\, i P^{3/2}\, \rho}{\tau^{3/2}_{max}\; 0 \cdot 230} \right|^{0 \cdot 230} \quad (11)$$

with the help of above equations, the following equation is obtained between W_1 and C.

$$W_1 \left(\frac{H}{H+h} \right) = \left| \frac{25 \cdot 64\, \rho\, P\, G\, \delta}{\tau^2_{max}\; 0 \cdot 770} \right| C^{-0 \cdot 621} \quad (12)$$

Fig. 2 gives the variation of W_1 with C for different values of h/H. This figure can be used to find out the optimum value of W_1 once the value of C is given and vice versa.

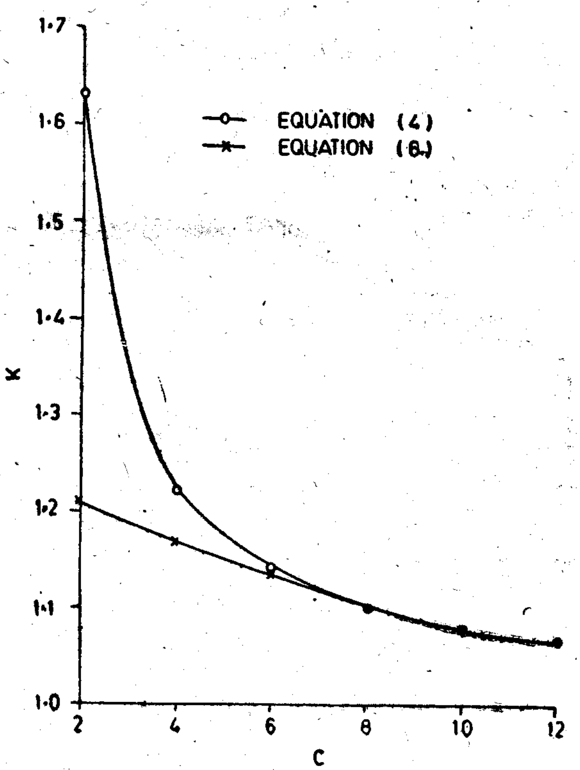

Fig. 2 Torsion spring.

TORSION HELICAL SPRING (MINIMUM WEIGHT)

The expression for weight of a torsion helical spring is given by Suri & Kishor[4]

$$W = \rho \frac{\pi^2}{4} d^2 D_m + \frac{i \pi^2}{4} d^2 D_m \rho \quad (1)$$

where

$$d^3 = \frac{K \cdot 32\, M}{\pi\, \tau_{max}} \quad (2)$$

The deflection, δ, for torsion helical spring is given by

$$\delta = \frac{64\, M D_m\, an}{E\, d^4} \quad (3)$$

Substituting the value of D_m and d^3 in (1), we get

$$W = \frac{4 M \delta \rho}{\tau^2_{max}\ a} K^2 + \frac{8 i \rho}{\tau_{max}} \pi M K C \tag{4}$$

It is found that K can be represented approximately by the following equation for spring index greater than six as shown in Fig. 3.

$$K = 1 \cdot 302\ C^{-0 \cdot 0775} \tag{5}$$

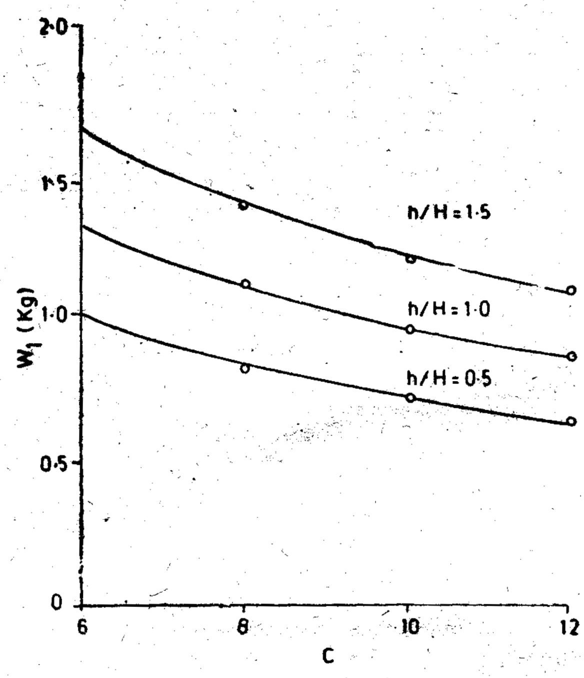

Fig. 3—Conical spring.

By substituting the value of K from (5) in (4), we get

$$W = \left| \frac{6.76 M \delta E \rho}{\tau_{max}\ a} \right| C^{-0 \cdot 155} + \left| \frac{10 \cdot 41\ i M \pi \rho}{\tau_{max}} \right| C^{0 \cdot 922} \tag{6}$$

The above expression is minimized using geometric programming method of solution. At the optimum point the normality conditions and orthogonality equations are given by Duffin, Peterson & Zener[6]

$$\delta_1 + \delta_2 = 1 \tag{7}$$

$$-0 \cdot 155\ \delta_1 + 0 \cdot 922\ \delta_2 = 0 \tag{8}$$

By solving (7) and (8), we get

$$\delta_1 = 0 \cdot 595 \tag{9}$$

$$\delta_2 = 0 \cdot 405 \tag{10}$$

Hence the optimum weight is given by

$$W_1 = \left| \frac{6 \cdot 76\ M \delta E \rho}{0.595\ \tau^2_{max}\ a} \right|^{0 \cdot 595} \times \left| \frac{10 \cdot 41\ i M \pi \rho}{\tau_{max} \cdot 0 \cdot 405} \right|^{0 \cdot 405} \tag{11}$$

The value of C is found out by the following equation.

$$W_1 \frac{a}{M} = \left| \frac{6 \cdot 76\ E \delta \rho}{0 \cdot 595\ \tau^2_{max}} \right| C^{-0 \cdot 155} \tag{12}$$

Fig. 4 shows the variation of W_1 with C for different values of a/M. The designer can use this graph for obtaining the value of either W_1 or C, once the value of one of them is specified.

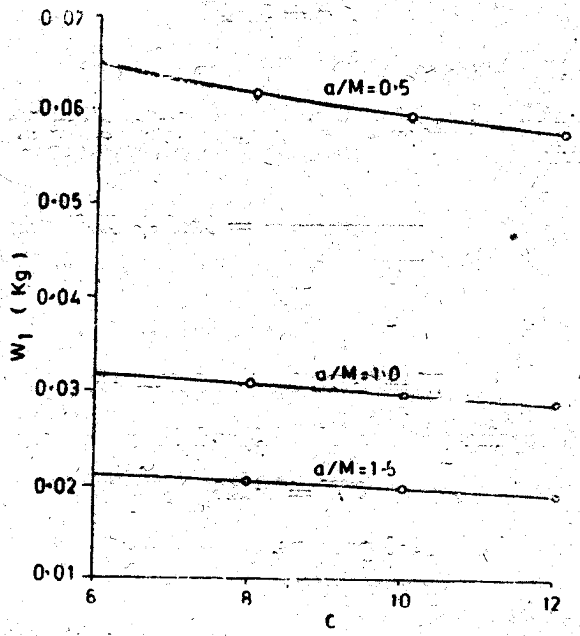

Fig. 4—Torsion spring.

The method presented in the paper has been illustrated with the help of an example given below.

Numerical Example

A conical spring is to be designed for the following data

$$P = 40 \text{ kg}$$

$$G = 0 \cdot 8 \times 10^6 \text{ kg/cm}^2$$

$$\delta = 1 \cdot 5 \text{ cm}$$

$$\tau_{max} = 2500 \text{ kg/cm}^2$$

$$\rho = 7 \cdot 8 \text{ gm/cm}^2$$

$$i = 2$$

Solution

On substitution of the values of $\rho, G, \delta, P, \tau_{max}$ and $h/H = 0 \cdot 5$ in (11), we get

$$W_1 = 0 \cdot 588 \text{ kg} \tag{13}$$

Using the above mentioned value of W_1 in (12) along with the values of other parameters, we get

$$C = 7 \cdot 09$$

Therefore, we adopt the value of $C = 7 \cdot 0$.

The value of K is obtained from (5) as

$$K = 1 \cdot 240$$

The value of d is obtained from (2) as

$$d = 0 \cdot 597 \text{ cm}$$

The value of n is obtained from (3) as

$$n = 31 \cdot 4$$

We adopt the value of $n = 32$.

The weight of a conical spring using the unoptimized expression as given by (1) is obtained by substituting values of C, n, i, and d in it.

$$\frac{W\,H}{\frac{\rho\,\pi^2}{8}\,(H+h)} = Cnd^3 + Cid^3$$

Therefore

$$W = 0 \cdot 734 \text{ kg (for } h/H\text{—}0 \cdot 5) \tag{14}$$

The difference in weight of the spring as calculated in (13) and (14) is

$$\delta\,W = 0 \cdot 146$$

Hence the saving in the material is 19·8 per cent.

CONCLUSION

There is a great necessity to optimise the weight of a machine part so as to use the minimum weight of the material for a given application. In the present paper the same has been obtained for conical helical and torsional helical springs with the help of geometric programming.

In order to help the designer for obtaining the required values of weight etc. graphs have been plotted between the weight and the spring index for different value of h/H. The method of solution has been illustrated with the help of numerical example. The calculations show that this method affects a saving in the spring weight of about 20 per cent. This saving is substantial and can significantly affect the price of a machine employing heavy and large number of springs.

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