ON THE INTERNAL BALLISTICS OF A SUPERGUN USING MULTITUBULAR POWDERS

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The basic equations of internal ballistics of the German Supergun using multitubular propellants have been set up. The equations have been expressed in terms of dimensionless variables and an analytical solution has also been given assuming a linear rate of burning and neglecting the co-volume correction.

The Hochdruckpumpe (HDP), Supergun was first invented by Coenders[1], which was designed to fire an arrow projectile across the channel at London but which was never realized. Recently multitubular propellants have become very popular. Germans use multitubular propellants very much on account of their progressivity. It has been found by Tavernier[2] and others that the form function for the Heptatubular powders, multitubular, modified multitubular and partially modified multitubular powders is always cubic.

In this paper, the basic equations of a Supergun (or any multipowder-chambers gun) using multitubular propellants have been set up. The equations have been reduced in terms of dimensionless variables and an analytical solution has also been given neglecting the co-volume correction $\left(\text{i.e. assuming } b_n = \dfrac{1}{\delta_n}\right)$ and using the linear law of burning. The results obtained by Jain and Sodha[3] are only particular cases of this treatment.

NOTATIONS

F_r	The force constant of the propellant in the r^{th} chamber.
C_r	The charge weight in the r^{th} chamber.
γ	The ratio of the specific heats at constant pressure and at constant volume of the propellant gases.
z_n	The fraction of the n^{th} charge burnt.
p	The mean pressure of the propellant gases.
A	The bore area.
V_r	The capacity of the r^{th} chamber.
δ_r	The density of the propellant in the r^{th} chamber.
x	The shot-travel.
b_r	The co-volume of the propellant gases in the r^{th} chamber.
W_n	$1.04\ \omega + \dfrac{1}{3} \sum\limits_{1}^{n} C_r$.
v	The shot-velocity.
R	The frictional force acting on the projectile at any instant.
H	The heat transferred per unit length of shot travel.
D_n	The web-size of the propellant in the n^{th} chamber.
f_n	The fraction of the initial thickness remaining at any instant t, for the first phase of combustion, while for the second phase of combustion defined as the ratio of the distance receded (from the beginning of the second phase up to the instant considered) to the initial thickness.
β_n	The rate of burning coefficient of the propellant in the n^{th} chamber.
α_n	The index of burning of the propellant in the n^{th} chamber.

BASIC EQUATIONS

The internal ballistics equations of a supergun after the n^{th} charge starts burning are

$$\sum_{1}^{n-1} \frac{F_r C_r}{\gamma-1} + \frac{F_n C_n z_n}{\gamma-1} = \frac{p}{\gamma-1} \left[Ax + \sum_{1}^{n} \left(U_r - \frac{C_r}{\delta_r} \right) - \sum_{1}^{n-1} \left(b_r - \frac{1}{\delta_r} \right) \right.$$

$$\left. C_r - \left(b_n - \frac{1}{\delta_n} \right) C_n z_n \right] + \tfrac{1}{2} W_n v^2 + \int_0^x R\, dx + \int_0^x H\, dx \tag{1}$$

$$D_n \frac{df_n}{dt} = -\beta_n p^{\alpha_n} \tag{2}$$

$$W_n v \frac{dv}{dx} = Ap - R \tag{3}$$

and

$$z_n = (1 - f_n)(a - bf_n - cf_n^2) \tag{4}$$

where a, b and c have values given by Tavernier[1], Kapur & Jain[4] in case of Heptatubular, Modified heptatubular and partially modified heptatubular propellants respectively.

To express the above equations in terms of dimensionless variables, we suppose

$$\left.\begin{aligned}
Al_n &= \sum_{1}^{n} \left(U_r - \frac{C_r}{\delta_r} \right) - \sum_{1}^{n-1} \left(b_r - \frac{1}{\delta_r} \right) C_r \\
B_n &= \left(b_n - \frac{1}{\delta_n} \right) \frac{C_n}{Al_n} \\
\xi_n &= 1 + \frac{x}{l_n} \\
\zeta_n &= \frac{pAl_n}{F_n C_n} \\
\eta_n &= \frac{vA D_n}{F_n C_n \beta_n} \left(\frac{F_n C_n}{Al_n} \right)^{1-\alpha_n} \\
M_n' &= \frac{A^2 D_n^2}{F_n C_n \beta_n^2 W_n} \left(\frac{F_n C_n}{Al_n} \right)^{2(1-\alpha_n)} \\
\rho_n &= \frac{R l_n}{F_n C_n} \\
h_n &= \frac{H l_n}{F_n C_n} \\
\text{and} \quad A_n &= \sum_{1}^{n-1} \frac{F_r C_r}{F_n C_n}
\end{aligned}\right\} \tag{5}$$

Using (5) in (1), (2) and (3), we get

$$A_n + z_n = \zeta_n (\xi_n - B_n z_n) + (\gamma - 1) \left[\frac{\eta_n^2}{2 M_n^1} + \int \rho_n \, d\xi_n + \int h_n \, d\xi_n \right] \tag{6}$$

$$\eta_n \frac{df_n}{d\xi_n} = -\zeta_n^{\alpha_n} \tag{7}$$

and

$$\eta_n \frac{d\eta_n}{d\xi_n} = M_n' (\zeta_n - \rho_n) \tag{8}$$

ANALYTICAL SOLUTION

An analytical solution of (4), (6), (7) and (8) can be obtained by assuming

(*i*) A linear law of burning, i.e. $\alpha_n = 1 = \alpha_r$

(*ii*) $b_n = \dfrac{1}{\delta_n}$ or $B_n = 0$

(*iii*) The heat transferred up to any instant to be proportional to the kinetic energy of the shot and the propellant gases,

i.e.

$$\int H dx = \frac{\chi}{2} W_n v^2$$

or

$$\int h_n d\xi_n = \chi \cdot \frac{\eta_n^2}{2M_n'}$$

(*iv*) The bore resistance to be proportional to Ap, i.e.

$$R = \sigma Ap$$

or

$$\rho = \sigma \zeta_n$$

Putting

$$M_n = M_n' (1-\sigma)$$

and

$$(\bar{\gamma}-1) = (\gamma-1)\left[1+\chi(1-\sigma)\right],$$

in equations (6), (7) and (8), we have

$$A_n + z_n = \zeta_n \xi_n + (\bar{\gamma}-1)\frac{\eta_n^2}{2M_n} \tag{9}$$

$$\eta_n \frac{df_n}{d\xi_n} = -\zeta_n \tag{10}$$

and

$$\eta_n \frac{d\eta_n}{d\xi_n} = M_n \zeta_n \tag{11}$$

From (10) and (11), we have

$$\frac{d\eta_n}{df_n} = -M_n \tag{12}$$

Integrating the equation (12) and using the conditions that

$$\eta_n = \eta_{n,0} \text{ when } f_n = 1,$$

we have

$$\eta_n = M_n\left(1-f_n+\frac{\eta_{n,0}}{M_n}\right) \tag{13}$$

Equations (4) and (13) give

$$z_n = \left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)\left[(a-b-c)+(b+2c)\left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)-c\left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)^2\right] \tag{14}$$

Equations (9), (11) and (14) give

$$A_n+\left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)\left[(a-b-c)+(b+2c)\left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)-c\left(\frac{\eta_n-\eta_{n,0}}{M_n}\right)^2\right] = \frac{\eta_n\xi_n}{M_n}\frac{d\eta_n}{d\xi_n}+\frac{(\bar{\gamma}-1)\eta_n^2}{2M_n}.$$

or

$$\xi_n\eta_n\frac{d\eta_n}{d\xi_n} = P_1+Q_1\eta_n-R_1\eta_n^2-S_1\eta_n^3 \tag{15}$$

where

$$\left.\begin{aligned}
P_1 &= A_n M_n-(a-b-c)\,\eta_{n,0} + (b+2c)\frac{\eta_{n,0}^2}{M_n}+c\cdot\frac{\eta_{n,0}^3}{M_n^2}\\
Q_1 &= (a-b-c)-\frac{2\eta_{n,0}(b+2c)}{M_n} - 3c\frac{\eta_{n,0}^2}{M_n^2}\\
R_1 &= \frac{\bar{\gamma}-1}{2}-\frac{(b+2c)}{M_n} - 3c\frac{\eta_{n,0}}{M_n^2}\\
&\text{and}\\
S_1 &= \frac{c}{M_n^2}
\end{aligned}\right\} \tag{16}$$

From (15) and (16), we have

$$\int_{\xi_{n,0}}^{\xi_n}\frac{d\xi_n}{\xi_n} = \int_{\eta_{n,0}}^{\eta_n}\frac{\eta_n\,d\eta_n}{P_1+Q_1\eta_n-R_1\eta_n^2-S_1\eta_n^3}$$

or

$$\xi_n = \xi_{n,0}\cdot e^{\int_{\eta_{n,0}}^{\eta_n}\frac{\eta_n\,d\eta_n}{P_1+Q_1\eta_n-R_1\eta_n^2-S_1\eta_n^3}} \tag{17}$$

Maximum Pressure

From (11), we have

$$\zeta_n = \frac{\eta_n}{M_n} \cdot \frac{d\,\eta_n}{d\,\xi_n}$$

$$= \frac{P_1 + Q_1\,\eta_n - R_1\,\eta_n^2 - S_1\,\eta_n^3}{M_n\,\xi_n} \qquad (18)$$

Equation (18) gives on differentiation

$$\frac{1}{\zeta_n} \cdot \frac{d\,\zeta_n}{d\,\eta_n} = -\frac{1}{\xi_n} \cdot \frac{d\,\xi_n}{d\,\eta_n} + \frac{Q_1 - 2R_1\,\eta_n - 3S_1\,\eta_n^2}{P_1 + Q_1\,\eta_n - R_1\,\eta_n^2 - S_1\,\eta_n^3}$$

$$= \frac{Q_1 - (2R_1 + 1)\,\eta_n - 3S_1\,\eta_n^2}{P_1 + Q_1\,\eta_n - R_1\,\eta_n^2 - S_1\,\eta_n^3}$$

For maximum pressure $\frac{d\zeta_n}{d\eta_n} = 0$. The shot velocity at the instant of maximum pressure is given by

$$3\,S_1\,\eta_n^2 + (2\,R_1 + 1)\,\eta_n - Q_1 = 0 \qquad (19)$$

or

$$\eta_n = \frac{-(2R_1 + 1) \pm \sqrt{(2R_1 + 1)^2 + 12\,Q_1\,S_1}}{6\,S_1} \qquad (20)$$

Using (17), (18) and (20), the maximum pressure can be evaluated.

At Rupture

At the point of rupture of the grain, we have $f_n = 0$.

Putting $f_n = 0$ in (13), the shot velocity at the rupture is given by

$$\eta_{n,r*} = M_n \left(1 + \frac{\eta_{n,0}}{M_n} \right) \qquad (21)$$

From (18), the shot travel at the rupture of the grain is given by

$$\xi_{n,r_*} = \xi_{n,0} \,.\, e^{\int_{\eta_{n,0}}^{\eta_{n,r*}} \frac{\eta_n\,d\,\eta_n}{P_1 + Q_1\,\eta_n - R_1\,\eta_n^2 - S_1\,\eta_n^3}} \qquad (22)$$

where the letters with suffix r^* denote conditions at rupture.

After All Burnt

Putting $z=1$ in (9) and using (11), we have

$$(1 + A_n) = \frac{\xi_n}{M_n} \cdot \frac{\eta_n\,d\,\eta_n}{d\,\xi_n} + (\bar{\gamma} - 1)\,\frac{\eta_n^2}{2M_n} \cdot$$

or

$$(1 + A_n)\,\xi_n^{(\bar{\gamma}-2)} = \xi_n^{(\bar{\gamma}-1)} \cdot \frac{\eta_n}{M_n} \cdot \frac{d\eta_n}{d\,\xi_n} + (\bar{\gamma} - 1)\,\xi_n^{(\bar{\gamma}-2)}\,\frac{\eta_n^2}{2M_n} \qquad (23)$$

Integrating (23), we have

$$\left[\frac{\eta_n^2}{2M_n} \cdot \xi_n^{\bar{\gamma}-1} \right]_{\eta_{n,b*},\,\xi_{n,b*}}^{\eta_n,\,\xi_n} = \frac{1 + A_n}{\bar{\gamma} - 1} \left[\xi_n^{\bar{\gamma}-1} - \xi_{n,b^*}^{\bar{\gamma}-1} \right]$$

or

$$\eta_n^2 = 2M_n\,\xi_n^{(1-\bar{\gamma})} \left[\frac{\eta_{n,b*}^2}{2M_n} \cdot \xi_{n,b^*}^{\bar{\gamma}-1} + \left(\frac{1 + A_n}{\bar{\gamma} - 1} \right) \cdot \left(\xi_n^{\bar{\gamma}-1} - \xi_{n,b^*}^{\bar{\gamma}-1} \right) \right] \qquad (24)$$

where the letters with suffix b^* denote conditions at all burnt.

Putting $a=1$, $b= -\theta_n$ and $c=0$, in the above treatment, we get all the results for a supergun using powders with quadratic form function as obtained by Jain and Sodha[2].

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