HEAT TRANSFER FOR LAMINAR FLOW THROUGH PARALLEL POROUS DISKS

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(*Received* 18 *May* 1978; *revised* 27 *December* 1979)

The problem of temperature distribution and heat transfer for laminar flow through two parallel porous disks, has been investigated when the flow is entirely due to injection or suction at the two disks. Viscous dissipation terms have been included in the energy equation and the uniform injection/suction velocities at the two disks, are assumed to be small. The boundaries are maintained at constant temperatures. The variation of temperature and Nusselt numbers at the two disks, is shown graphically, for various values of the injection/suction velocities.

The flow and heat transfer between two parallel disks, whether porous or non-porous, are of considerable practical interest in the design of thrust bearings, radial diffusers etc. Flow through porous boundaries with uniform suction or injection velocities, has been examined by various authors, namely Berman[1], Sellars[2], and Yuan[3]. Laminar flow between two parallel porous disks has been investigated by Elkouh[4] with uniform small suction or injection at the boundaries. Narayana and Rudraiah[5] studied the steady axisymmetric flow of a viscous incompressible fluid between two coaxial disks, one rotating and the other stationary, with uniform suction at the stationary disk. Regular perturbation technique has been adopted for small suction Reynolds number and the equations are numerically solved for an arbitrary suction Reynolds number. Viscous incompressible flow between two porous parallel rotating disks has been examined by Gaur[6] for small Reynolds number, defined in terms of the angular velocities of the disks. The problem of radial flow of a viscous incompressible fluid between two stationary uniformly porous disks has been investigated by Terrill and Cornish[7] The solution for small as well as large suction Reynolds numbers, has been obtained.

Inman[8] has discussed the effect of variation of the cross flow velocity on the temperature distribution and heat transfer for flow in an annulus with porous walls, with the assumption that the rate of injection of the fluid at one boundary is equal to the rate of suction of the fluid at the other. The effect of suction in the temperature distribution and heat transfer in plane couette flow and laminar flow in a circular pipe has been investigated by Verma and Bansal[9], while the problem of unsteady temperature distribution for laminar flow in a porous straight channel, has been studied by Gaur[10].

In the present investigation, we have obtained the solution of the energy equation, in cylinderical polar co-ordinates, when viscous dissipation has been accounted for. The velocity components have been taken from reference [7], when the suction or injection velocities at the disks are small and equal. Regular perturbation technique is used to obtain the temperature distribution when the disks are maintained at constant temperatures.

The present investigation can be made use of in porous bearings and self-impregnated bearings, used in defence equipments. The practical application that can be envisaged for this problem is where two porous disks are separated by a thin film of inert gas as in a gyro type of instrumentation. The velocity of gas being small, Mach no. effects can be neglected. And as is assumed in the numerical work done, the Prandtl number is of the order of unity for gases.

PROBLEM FORMULATION

The fluid is contained between two infinite parallel porous disks, situated at $z=-d$ and $z=+d$ respectively. The flow is due to small uniform injection or suction at the disks. The axisymmetric form of the energy equation in cylindrical polar co-ordinates (r,θ,z) is

$$\rho\, C_p \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = K \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \phi \tag{1}$$

where u and w are the velocities in the r and z directions respectively and the other notations have their usual meanings. The viscous dissipation function ϕ is given by

$$\phi = 2\mu \left[\left(\frac{\partial u}{\partial r} \right)^2 + \frac{u^2}{r^2} + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \mu \left(\frac{\partial u}{\partial z} \right)^2 \tag{2}$$

The boundary conditions, are

$$\left. \begin{array}{l} z = -d, \; T = T_1 \\ \text{and} \\ z = +d, \; T = T_2 \end{array} \right\} \tag{3}$$

where T_1 and T_2 are some constant temperatures.

Introducing the following non-dimensional quantities

$$\bar{r} = \frac{r}{d}, \; \bar{z} = \frac{z}{d}, \; \bar{u} = \frac{\rho\, ud}{\mu}, \; \bar{w} = \frac{\rho\, wd}{\mu}$$

and

$$\bar{T} = \frac{T - T_1}{T_2 - T_1}, \tag{4}$$

equation (1) using (2) takes the form

$$\left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \frac{1}{P_r} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + $$
$$+ 2E \left[\left(\frac{\partial u}{\partial r} \right)^2 + \frac{u^2}{r^2} + \left(\frac{\partial w}{\partial z} \right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial z} \right)^2 \right] \tag{5}$$

where the bars, from the non-dimensional quantities have been dropped.

$P_r = \frac{\mu\, C_p}{K}$, is the Prandtl number and $E = \frac{\mu^2}{\rho^2 d^2 C_p (T_2 - T_1)}$, is the Eckert number.

The boundary conditions, now become

$$\left. \begin{array}{l} z = -1, \; T = 0 \\ z = +1, \; T = 1 \end{array} \right\} \tag{6}$$

and

METHOD OF SOLUTION

For the case of uniform injection at both the disks, the velocity components w and u are given by (Ref. [7])

$$w = h(z) = \frac{1}{2} V (z^3 - 3z) + \frac{V^2}{560} (z^7 - 21z^5 + 39z^3 - 19z) + 0 (V)^3$$

and

$$u = -\frac{r}{2} h'(z), \tag{7}$$

under the assumption that the uniform injection velocity V, is small. Prime denotes differentiation with respect to z.

Substituting (7) in (3), we have

$$-\frac{r}{2} h' \frac{\partial T}{\partial r} + h \frac{\partial T}{\partial z} = \frac{1}{P_r} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + E \left(3h'^2 + \frac{1}{4} r^2 h''^2 \right) \tag{8}$$

Equation (8) readily suggests that T should be of the form

$$T = r^2 T_2(z) + T_0(z) \tag{9}$$

Substituting (9) in (8) and comparing the coefficients of r^2 and r^0 from the two sides of the equation, we have

$$-h' T_2 + h T_2' = \frac{1}{P_r} T''_2 + \frac{E}{4} h''^2 \tag{10}$$

and

$$h T'_0 = \frac{1}{P_r} (4T_2 + T''_0) + 3 E h'^2 \tag{11}$$

The B. C's are

$$\left. \begin{array}{l} z = -1, \ T_2 = 0, \ T_0 = 0 \\ z = +1, \ T_2 = 0, \ T_0 = 1 \end{array} \right\} \tag{12}$$

Let

$$\left. \begin{array}{l} T_2 = T_{2,0} + V T_{2,1} + V^2 T_{2,2} + \ldots \\ T_0 = T_{0,0} + V T_{0,1} + V^2 T_{0,2} + \ldots \end{array} \right\} \tag{13}$$

and

$$h = V h_1 + V^2 h_2 + \ldots$$

Substituting (13) in (10) to (12) and comparing the coefficients of the various powers of V, we obtain the following two sets of differential equations.

I Set

$$T''_{2,0} = 0$$

$$-h'_1 T_{2,0} + h_1 T'_{2,0} = \frac{1}{P_r} T''_{2,1}$$

$$-h'_1 T_{2,1} - h'_2 T_{2,1} + h_1 T'_{2,0} + h_2 T'_{2,0} = \frac{1}{P_r} T''_{2,2} + \frac{E}{4} h''^2_1,$$

with the boundary conditions

$$\left. \begin{array}{l} z = -1; \ T_{2,0} = T_{2,1} = T_{2,2} = 0 \\ z = +1; \ T_{2,0} = T_{2,1} = T_{2,2} = 0 \end{array} \right\} \tag{14}$$

II Set

$$4 T_{2,0} + T''_{0,0} = 0$$

$$h_1 T'_{0,0} = \frac{1}{P_r} (4 T_{2,1} + T''_{0,1})$$

$$h_1 T'_{0,1} + h_2 T'_{0,0} = \frac{1}{P_r} (4 T_{2,2} + T''_{0,2}) + 3E h'^2_1$$

with the boundary conditions

$$\left.\begin{array}{l} z = -1;\ T_{0,0} = T_{0,1} = T_{0,2} = 0 \\ z = +1;\ T_{0,0} = 1,\ T_{0,1} = T_{0,2} = 0 \end{array}\right\} \quad (15)$$

As h_1 and h_2 are already known from (7), solving, (14) and then (15), we are able to get the values of T_2 and T_0, the substitution of which in (9), gives

$$T = r^2 \frac{3EP_r}{16} V^2 (1 - z^4) + \frac{1}{2} (1 + z) + \frac{VP_r}{80} (1 - z^2) (9z - z^3) +$$

$$+ V^2 (1 - z^2) \left[\frac{EP_r}{40} (113 - 37z^3 + 8z^4) + \frac{P_r}{1120 \times 360} (613z - 527z^3 + 175z^5 - 5z^7) + \right.$$

$$\left. + \frac{P^2_r}{160 \times 2520} (1391z - 9949z^3 + 2525z^5 - 175z^7) \right] \quad (16)$$

which is correct upto the terms of the order of V^2.

The rate of heat transfer is expressed in terms of Nusselt number which in terms of the dimensional quantities, is given by

$$Nu = \frac{2d\, Q^*}{K (T_2 - T_1)},$$

where

$$Q^* = \frac{1}{\pi (r^2 - r_0^2)} \int_{r_0}^{r} (2\pi\, r\, q)\, dr,$$

and

$$q = - K \frac{\partial T}{\partial z},$$

where the meanings of Q^* and q are evident from the expressions and r_0, is the distance of some given point from the centre of either disk. Now calculating Q^* for $z = -d$ and $z = +d$, Nusselt numbers for the lower and the upper disks can separately be obtained.

Using the non-dimensional quantities, as defined in (4) and dropping bars, the Nusselt number at the lower disk $[(Nu)_{-1}]$ and the Nusselt number at the upper disk $[(Nu)_{+1}]$ are given by

$$(Nu)_{-1} = - \frac{3}{4} E\, P_r\, V^2\, r_0^2\, (\lambda^2 + 1) -$$

$$- 2 \left\{ \frac{1}{2} - \frac{1}{5} V P_r + 2V^2 \left(\frac{21}{10} EP_r - \frac{1}{35 \times 45} P_r + \frac{97}{6300} P_r^2 \right) \right\}$$

$$(Nu)_{+1} = \frac{3}{4} EP_r\, V^2\, r_0^2\, (\lambda^2 + 1) - 2 \left\{ \frac{1}{2} - \frac{1}{5} V P_r + \right.$$

$$\left. + 2V^2 \left(- \frac{21}{10} EP_r - \frac{1}{35 \times 45} P_r + \frac{97}{6300} P_r^2 \right) \right\}$$

where $\lambda = r/r_0$.

NUMERICAL DISCUSSION

Temperature profiles for $P_r = 1.0$, $E = 0.01$ and $r = 100$ have been drawn for various values of V (Fig. 1). For $V=0.2$, that is, when there is small injection at both the disks, cooling effect is observed

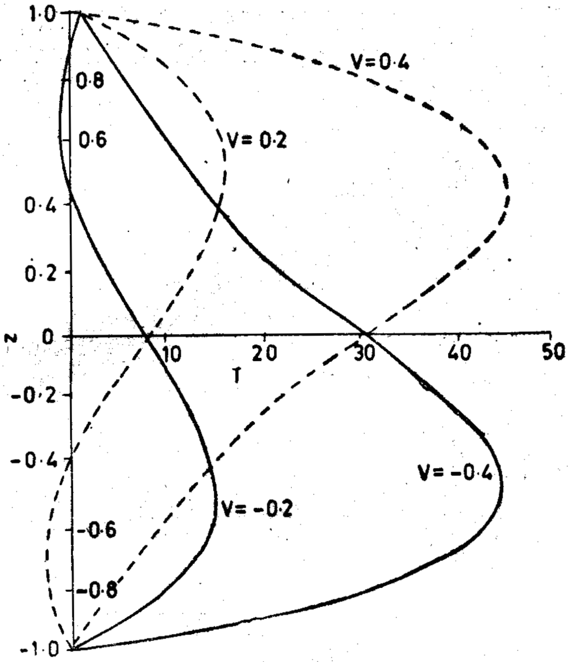

Fig. 1—Temperature profiles for $E=0.01$, $Pr=1.0$, $r=100$ and for values of V indicated.

near the lower disk. Increase in the injection velocity ($V=0.4$) results in the increase in temperature at both the disks. However, it can be seen that the maximum value of the temperature is located in the upper half of the region for injection. Instead for negative V, that is suction at both the disks, a phenomenon opposite to that of injection is observed. It is interesting to note that for equal suction or injection the temperature in the central region remains unaltered.

Fig. 2 shows the variation of the Nusselt number at the lower disk against λ, for the same fixed values of P_r, E and r. It is observed that $(Nu)_{-1}$ remains negative for suction as well as injection. Increase in the

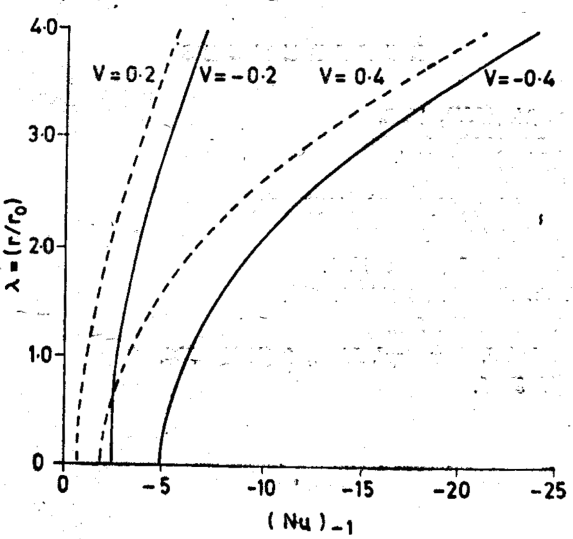

Fig. 2—Nusselt number at the lower disk for $E=0.01$, $P_r=1.0$, $r_0=10$ and for values of V indicated.

uniform injection velocity at the lower disk results in the decrease of this Nusselt number as we move away from the centre of the disk. However, this decrease is faster in case of suction.

The variation of Nusselt number at the upper disk against λ, for the same fixed values of E, P_r and r_0, has been depicted in Fig. 3. It is seen that for small injection $(Nu)_{+1}$ remains negative near the centre of the disk but changes sign and becomes positive as we move away from the centre. With the increase in the

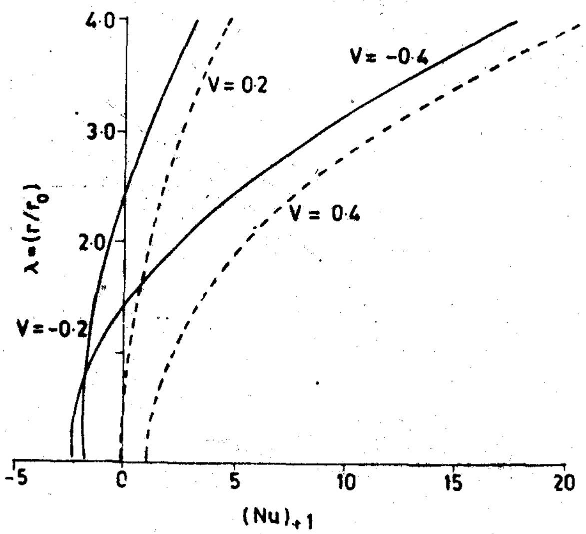

Fig. 3—Nusselt number at the upper disk for $E=0.01$, P_r-1.0, $r_0=10$ and for values of V indicated.

injection velocity, the Nusselt number also increases sharply and becomes throughout positive. However, increase in the suction velocity at the upper disk decreases the Nusselt number near the centre of the disk, while increase is observed as we proceed away from the centre.

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