EFFECT OF IMAGE MOTION ON IMAGE QUALITY IN THE HIGH SPEED CAMERA

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The effects of image motion with aperture variation in a high speed camera have been described by Dubovic for special cases. In this paper a generalised approach based on the concept of transformation by two systems given by O'Neill is discussed.

The complex conditions under which an image in the high speed camera is formed can be enumerated as :

(1) Inherent movement of the image in the focal plane

(2) Movement of the image due to rapidly developing front of the event

(3) Movements due to after events in the instruments

(4) Rotation of the image about any axis

(5) Defocusing of the image due to the longitudinal velocity of the event

(6) Other optical effects common with still photography

In addition to these, there is variation of aperture during the exposure, which adds to the complexity of the situation.

In studying the effects of these factors the elementary approach of resolving power (Minimum angular or linear separation that is necessary to see that two point objects are distinctly seen as two), useful in simple systems, is not adequate. It will be shown that the simple criterion, followed very widely in high speed photography, for image arrest for ensuring image movements to be within the circle* of confusion, is not adequate. In many cases the mathematical identity of object and image distribution is not important, but what is important is the conservation of certain characteristics while the object image transformation takes place. For understanding the conservation of these characteristics the spatial frequency approach is superior. If at all we are able to conserve the spatial frequencies representing the characteristics, the image processing technique can be applied to extract the information even though the photograph quality is inadequate in the conventional sense.

Image quality studies with image movement and aperture variation during exposure have been reported by Dubovic[1] who has given the summary of earlier work and described the technique for such studies. Most of the authors who contributed in this field restricted themselves to the numerical methods for evaluation of the contrast transfer function (i.e. variation of amplitude and phase of spatial frequencies in the object image transformation) assuming certain conditions in respect of image motion and apperture variation during exposure. Simplicity of these method no longer remains when the assumptions do not hold. For example Dubovic has assumed that the image velocity is constant and studied the effect with various types of aperture variations.

*Diameter of the circular patch forming the image of a point scene.

It can be seen that for time dependant velocities his method will become quite complicated. Moreover the physical picture of these effects is not clear in the approach taken up by these authors and will become much obscure for complex image displacements.

In this paper an attempt is made to give the generalised treatment of the problem. Following O'Neill[2] it is suggested that the entire effect can be treated as a result of two systems doing object image transformation. It will be seen that no elaborate mathematical operations are necessary to find the effect and an intuitive assessment can be made if this approach is followed.

The effect of aperture variation during exposure will be to cause an intensity variation with time for each object point as well as the diffraction pattern. In addition if the image movement also takes place, the position and the intensity of the image will change from instant to instant during the exposure. The effect will also depend upon the direction of motion of the image. Since both the intensity and the position of the image are functions of time, the blur caused in this case will be quite different than when we have image movement with constant aperture during the exposure. We shall confine our attention to the problem of image motion combined with aperture variation without considering the diffraction effects caused by the aperture variation. The problem of diffraction effects can be tackled by the same approach. We will discuss the case of one dimension only.

We presume that the image motion can be described by a function of time such that an image point which had a coordinate ξ at time $t = 0$ will have co-ordinate equal to $\xi \pm \phi(t)$ at the time t. We also presume that the intensity variations caused by aperture variation can be represented by a function $\theta(t)$. Let us represent the object intensity distribution by $G(x)$, x being the identifying point in the image space. Let us also represent the system by $A(\xi)$ i.e. $A(\xi)$ is the intensity distribution in the image produced by the system of a point source at x. $A(\xi)$ will take into account the effect of imperfections of the optical system, which cause spread in the image of the point source. Then the intensity distribution $E(x)$ in the image of $G(x)$ can be respresented as

$$E(x) = \int_{-\infty}^{+\infty} \int_{t_1}^{t_2} G\left(x - \xi \pm \phi(t)\right) A(\xi)\, \theta(t)\, dt\, d\xi \tag{1}$$

where t_1 is the time of aperture opening and t_2 is the time of aperture closing. In order to get the expression for image quality in terms of brightness contrast which corresponds throughout the exposure, the right hand side expression in Eqn. (1) will have to be divided by a normalising factor of the form $\int_{t_1}^{t_2} \theta(t) = \frac{1}{N}$.

Thus the normalised intensity distribution $E_N(x)$ in the image is

$$E_N(x) = N \int_{-\infty}^{+\infty} \int_{t_1}^{t_2} G\left(x - \xi \pm \phi(t)\right) A(\xi)\, \theta(t)\, dt\, d\xi \tag{2}$$

At this stage we deviate from the approach adopted for numerical calculation, which consist in selecting a suitable form for $G(x)$ which with the help of assumptions in respect of $\theta\ (t)$ and $\phi\ (t)$ enables to assess the image quality. We presume that the time and image displacement are described by inversible functions i.e. if image displacement is given by $\eta = \phi\ (t)$ then $t=f\ (\eta)$. The variable t in the Eqn. (2) can be changed to η. For this purpose the limits t_1 and t_2 will be changed to $-\frac{L}{2}$ to $+\frac{L}{2}$ where L is the total displacement during the time t_1 and t_2 i.e. the exposure period. The Eqn. (2) can therefore be written

$$E_N(x) = N \int_{-\infty}^{+\infty} \int_{-L/2}^{+L/2} G\,(x-\xi \pm \eta)\,A\,(\xi)\,f^1\,(\eta)\,\psi\,(\eta)\,d\eta\ d\xi \tag{3}$$

Where $f^1\,(\eta)$ is the derivative of $f\,(\eta) = t$ with respect to η and $\Psi(\eta)$ is the function obtained after substituting $t=f\,(\eta)$ in $\theta\ (t)$. In this expression it will be seen that the physical space of η and ξ is the same but their abstract spaces are different, the reason being that η is varied while ξ remains constant, in much the same way as ξ is varied while η remains constant. Hence Eqn. (3) is as much two dimensional as Eqn. (2) from which it is derived. Further Eqn. (3) can be put in standard two dimensional convolution form. The Eqn. (3) is different from the standard two dimensional convolution form only in that the limits of the first integral are from $-\frac{L}{2}$ to $+\frac{L}{2}$ instead of $-\infty$ to $+\infty$. By suitable defination Eqn. (3) can be converted to the standard form and can be written as

$$E\,(x) = N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G\,(x-\xi \pm \eta)\,A\,(\xi)\,R\,(\eta)\,f'\,(\eta)\,\psi\,(\eta)\,d\eta\,d\xi \tag{4}$$

where $R\,(\eta)$ is a rectangle function of unit height within limits $-\frac{L}{2}$ to $+\frac{L}{2}$ and $f^1(\eta)$ is derivative of $f(\eta)$ with respect to η.

From this convolution Eqn. (4) we can easily conclude that the combined effect of shutter operation and image motion will be given by a new transform function which will be the product of the fourier transforms of $A(\xi)$ and the fourier transform of the product $R\,(\eta)\,f^1\,(\eta)\,\Psi\,(\eta)$.

Thus the contrast transfer function as a result of image motion and aperture variation only is contrast transfer function $F\,\{R(\eta)\,\Psi(\eta)f'(\eta)\}$ at normalised frequencies. It can be shown that this Fourier transform is at the same frequencies as the fourier transform of A (ξ) since the physical space for η and ξ is the same. Thus the effect of image motion and the shutter operation can be considered as equivalent to a system having contrast transfer function $F\{R(x)\Psi(x)\,f'(x)\}$. We can consider the image as being formed by two systems viz. one the optical system having a transfer function $F\{A(x)\}$ as for still photography while the other having a transfer function $F\{R(x)\,\Psi(x)f^1\,(x)\}$.

The case when there is no aperture variation and the image has a constant velocity is considered by O' Neill. The transfer function $F\{R(x)\Psi(x)f^1(x)\}$ correctly describes this. In this case $f^1\,(x) = 1$ and $\Psi\,(\eta)=1$ since image velocity and aperture is constant with respect to time throughout the exposure. Thus the response function will be $R(x)$ which converts a point source into a line and the corresponding contrast transfer function will be a sin $\gamma\,L$, where γ is the spatial frequency. In this case a moving point source will be recorded as line of constant density. The photograph can be considered as being produced by two systems. The first system converts the point source into a line image and second system photographs it. We call the first system as an intermediate system.

With aperture variation and image movement during exposure, the intermediate system converts the point source into a line of varying density. For the first case considered by Dubovic we have $f^1(x)=1$, because of constant velocity and $\Psi(x)=x$, for first half of exposure period, while for the next half $\Psi(x)=x-1$. The response function $R(x)\ f^1(x)\ \Psi(x)$ in this case is a triangle function between $\pm\frac{L}{2}$ and the contrast transfer function will be $\sin^2\frac{\gamma L}{2}$ where γ is normalised frequency. This is the same as found by Dubovic but by different approach.

Once we accept that the contrast transfer function is a result of two systems viz one having transfer function $FA(x)$ as for still photography and the other $F\{R(x)\,f^1(x)\Psi(x)\}$ to account for image motion and aperture variation, we can examine the fundamental question regarding the criterian for image arrest. In the case of constant image velocity and linear variation of aperture the transfer function will have $\sin^2\frac{\gamma\phi}{2}$ as one of the factors where ϕ is the radius of circle of confusion. This factor will attenuate the contrast at high frequencies considerably.

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REFERENCES

1. DUBOVIC., 'Photographic Recording of High Speed Process' (Devbovic Pergamon Press, Oxford, London), 1968, p. 317.
2. O'NEILL., 'Introduction to Statistical Optics Addison', (Wesly Publishing Co., Massachusets), 1963, p. 27.