USE OF SPECIAL FUNCTIONS IN THE PRODUCTION OF HEAT IN A CYLINDER

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We have considered the application of H-function, generalized hypergeometric function and Gauss's hypergeometric function in solving the fundamental differential equation of diffusion of heat in a cylinder. A few known results have also been derived as particular cases.

We have employed H-function, generalised hypergeometric function and Gauss's hypergeometric function to solve the fundamental differential equation of the diffusion of heat in a cylinder of radius y when there are sources of heat within it which lead to an axially symmetrical temperature distribution. The fundamental differential equation is then of the form[1].

$$\frac{\partial \phi}{\partial t} = \frac{k}{x} \frac{\partial}{\partial x}\left(x \frac{\partial \phi}{\partial x} \right) + \theta(x, t). \tag{1}$$

If we assume that the rate of generation of heat is independent of temperature and the cylinder is infinitely long so that the variation of z may be neglected. We shall in addition suppose that the surface $x=y$ is maintained at zero temperature and the initial distribution of temperature is also zero. We further suppose that

$$\theta(x, t) = \frac{k}{K} f(x)\, g(t), \tag{2}$$

where k is the diffusivity and K the conductivity of the material. Cases in which heat is produced in solids are becoming increasingly important in technical applications. Space research and nuclear reactors also give rise to different problems of heat transfer. It will be observed that the single function $f(x)$ can represent both sources and sinks embedded in the system. Whenever the product $f(x) g(t)$ gives a negative value, it shoud be treated as a sink. We shall characterise the heat sources by the behaviour of the function $g(t)$. Since the Gauss's hypergeometric function may be converted into Jacobi, Legendre, Gegenbauer, Tchebicheff polynomials, H-function and generalized hypergeometric function into a number of higher transcendental functions and polynomials, the results obtained in this paper are of general character. Some results recently obtained by Bajpai[2] and Bhonsle[3] follow as particular cases of our results.

The following formula due to author[4] is required in this paper.

$$\int_0^1 x^{\rho-1}(1-x)^{\beta-1} {}_2F_1\left(\begin{matrix}\alpha, \nu\\ \beta\end{matrix}; 1-x\right) {}_uF_v\left(\begin{matrix}\alpha_1, \ldots, \alpha_u\\ \beta_1, \ldots, \beta_v\end{matrix}; c(x^d)\right) H_{p,q}^{n,l}\left[z x^{m/h} \,\middle|\, \begin{matrix}\{(a_p, e_p)\}\\ \{(b_q, f_q)\}\end{matrix}\right] dx$$

$$= (2\pi)^{(1-h)} A\, h^{B}\, \Gamma(\beta) \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{u} (\alpha_j ; r)\, c^r}{\prod_{j=1}^{v} (\beta_j ; r)(r)!} .$$

$$. H_{ph+2,\, qh+2}^{nh,\, lh+2}\left[(zh^{\tau})^h \,\middle|\, \begin{matrix}(1-\rho-rd, m), (1+\alpha+\nu-\rho-\beta-rd, m), \{(\triangle(h, a_p), e_p)\}\\ \{(\triangle(h, bq), fq)\}, (1+\alpha-\rho-\beta-rd, m), (1+\nu-\rho-\beta-rd, m)\end{matrix}\right], \tag{3}$$

where m, d and h are positive integers, $(a; r) = a(a+1)\ldots\ldots, (a+r-1)$,

$$A = n + l - \tfrac{1}{2}p - \tfrac{1}{2}q,\; B = \sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j + \tfrac{1}{2}p - \tfrac{1}{2}q + 1,\; \tau = \sum_{j=1}^{p} e_j - \sum_{j=1}^{q} f_j,\; \{(\triangle(t,\delta_r), \gamma_r)\}$$

stands for $\left\{\left(\frac{\delta_r}{t}, \gamma_r\right)\right\}, \left\{\left(\frac{\delta_r+1}{t}, \gamma_r\right)\right\}, \ldots\ldots, \left\{\left(\frac{\delta_r+t-1}{t}, \gamma_r\right)\right\}$ and

$$H_{p,q}^{n,l}\left[z \,\middle|\, \begin{matrix}\{(a_p, e_p)\}\\ (b_q, f_q)\}\end{matrix}\right] = H_{p,q}^{n,l}\left[z \,\middle|\, \begin{matrix}(a_1, e_1), \ldots\ldots, (a_p, e_p)\\ (b_1, f_1), \ldots\ldots, (b_q, f_q)\end{matrix}\right]$$

is the H-function defined by Fox[5] and can be reduced to Meijer's G-function if e_j $(j = 1, \ldots\ldots, p)$, f_h $(h = 1, \ldots\ldots, q)$ are positive integers, e.g.

$$H_{p,q}^{n,l}\left[x \,\middle|\, \begin{matrix}\{(a_p, e_p)\}\\ \{(b_q, f_q)\}\end{matrix}\right] = (2\Pi)^{D} \prod_{j=1}^{q} f_j^{(b_j-\frac{1}{2})} \prod_{j=1}^{p} e_j^{(\frac{1}{2}-a_j)} \cdot$$

$$\cdot G_{\sum_{j=1}^{p} e_j, \sum_{j=1}^{q} f_j}^{\sum_{j=1}^{n} f_j, \sum_{j=1}^{l} e_j}\left(\frac{\prod_{j=1}^{p} e_j^{e_j}}{\prod_{j=1}^{q} f_j^{f_j}}\, x \,\middle|\, \begin{matrix}\triangle(e_1, a_1), \ldots\ldots, \triangle(e_p, a_p)\\ \triangle(f_1, b_1), \ldots\ldots, \triangle(f_q, b_q)\end{matrix}\right), \tag{4}$$

where $D = \sum_{j=1}^{n} \frac{1-f_j}{2} - \sum_{j=n+1}^{q} \frac{1-f_j}{2} + \sum_{j=1}^{l} \frac{1-e_j}{2} - \sum_{j=l+1}^{p} \frac{1-e_j}{2}$ and $\triangle(n, a)$ represents a set of n parameters $\frac{a}{n}, \frac{a+1}{n}, \ldots\ldots, \frac{a+n-1}{n}$.

The formula (3) holds if $u \leqslant v$ (or $u = v + 1$ and $|c| < 1$), no one of $\beta_1, \ldots\ldots, \beta_v$ is zero or a negative integer, $R(\beta) > 0$, $R(\rho + \beta - \alpha - \nu) > 0$, $R\left(\rho + \frac{m\, b_j}{h\, f}\right) > 0$ $(j = 1, \ldots\ldots\ldots\ldots, n)$,

$$\tau = \sum_{j=1}^{p} e_j - \sum_{j=1}^{q} f_j \leqslant 0,\; \sum_{j=1}^{l} e_j - \sum_{j=l+1}^{p} e_j + \sum_{j=1}^{n} f_j - \sum_{j=n+1}^{q} f_j = M > 0 \text{ and } |\arg z| < \tfrac{1}{2}\pi M.$$

if we take $u = v = 0$, $d = 1$ in (3) and replace $1 - x$ by x, we find that

$$\int_0^1 x^{\beta-1} (1-x)^{\rho-1} e^{-cx} {}_2F_1\left(\begin{matrix}\alpha, \nu\\ \beta\end{matrix}; x\right) H_{p,q}^{n,l}\left[z(1-x)^{m/h} \,\middle|\, \begin{matrix}\{(a_p, e_p)\}\\ \{(b_q, f_q)\}\end{matrix}\right] dx$$

$$= (2\pi)^{(1-h)A} h^{B}\, \Gamma(\beta)\, e^{-c} \sum_{r=0}^{\infty} \frac{c^r}{(r)!} \cdot$$

$$\cdot H_{ph+2,\, qh+2}^{nh,\, lh+2}\left[(z\, h^{\tau})^{h} \,\middle|\, \begin{matrix}(1-\rho-r, m), (1+\alpha+\nu-\rho-\beta-r, m), \{(\triangle(h, a_p), e_p)\}\\ \{(\triangle(h, b_q), f_q)\}, (1+\alpha-\rho-\beta-r, m), (1+\nu-\rho-\beta-r, m)\end{matrix}\right], \tag{5}$$

where $R(\beta) > 0$, $R(\rho + \beta - \alpha - \nu) > 0$, $R\left(\rho + \frac{m\, b_j}{h\, f_j}\right) > 0$ $(j = 1, \ldots\ldots, n)$, $\tau \leqslant 0$, $M > 0$ and $|\arg z| < \frac{1}{2}\pi M$.

FINITE HANKEL TRANSFORM

Let the finite Hankel transform[1] of $f(x)$ be

$$\bar{f}_J(w_i) = \int_0^y x f(x) J_0(x w_i)\, dx, \tag{6}$$

where w_i is the root of the transcendental equation

$$J_0(yw_i) = 0. \tag{7}$$

when $f(x) = x^{\rho-2}(y-x)^{\beta-1}\,{}_2F_1\left(\begin{matrix}\alpha, \nu\\ \beta\end{matrix}; 1-\frac{x}{y}\right){}_uF_v\left\{\begin{matrix}\alpha_1, \ldots, \alpha_u\\ \beta_1, \ldots, \beta_v\end{matrix}; c\left(\frac{x}{y}\right)^d\right\}, \quad (8)$

then in (3) set $e_j = f_h = 1$ $(j = 1, \ldots\ldots, p; h = 1, \ldots\ldots, q)$, apply the formula (4), put $n = h = 1$, $m = q = 2$, $l = p = b_1 = b_2 = 0$, $z = y^2 w_i^2/4$, replace x by x/y and use the formulae[7], we have

$$\bar{f}_J(w_i) = y^{\rho+\beta-1}\Gamma(\beta)\sum_{r=0}^{\infty}\frac{\prod_{j=1}^{u}(\alpha_j; r)\,\Gamma(\rho+rd)\,\Gamma(\rho+\beta+rd-\alpha-\nu)\,c^r}{\prod_{j=1}^{v}(\beta_j; r)\,\Gamma(\rho+\beta+rd-\alpha)\,\Gamma(\alpha+\beta+rd-\nu)\,(r)!}\,.$$

$$.\,{}_4F_5\left\{\begin{matrix}\Delta(2, \rho+rd), \Delta(2, \rho+\beta+rd-\alpha-\nu)\\ 1, \Delta(2, \rho+\beta+rd-\alpha), \Delta(2, \rho+\beta+rd-\nu)\end{matrix}; -\frac{y^2 w_i^2}{4}\right\}, \tag{9}$$

where d is a positive integer, $u \leqslant v$ (or $u = v + 1$ and $|c| < 1$), no one of $\beta_1, \ldots\ldots, \beta_v$ is zero or a negative integer, $R(\beta) > 0$, $R(\rho) > 0$ and $R(\rho+\beta-\alpha-\nu) > 0$. By virtue of the inversion theorem[1], from (9) we get

$$f(x) = x^{\rho-2}(y-x)^{\beta-1}\,{}_2F_1\left(\begin{matrix}\alpha, \nu\\ \beta\end{matrix}; 1-\frac{x}{y}\right){}_uF_v\left\{\begin{matrix}\alpha_1, \ldots\ldots, \alpha_u\\ \beta_1, \ldots\ldots, \beta_v\end{matrix}; c\left(\frac{x}{y}\right)^d\right\}$$

$$= 2y^{\rho+\beta-3}\Gamma(\beta)\sum_i\sum_{r=0}^{\infty}\frac{\prod_{j=1}^{u}(\alpha_j; r)\,\Gamma(\rho+rd)\,\Gamma(\rho+\beta+rd-\alpha-\nu)\,c^r J_0(x w_i)}{\prod_{j=1}^{v}(\beta_j; r)\,\Gamma(\rho+\beta+rd-\alpha)\,\Gamma(\rho+\beta+rd-\nu)\,(r)!\,[J_1(yw_i)]^2}\,.$$

$$.\,{}_4F_5\left\{\begin{matrix}\Delta(2, \rho+rd), \Delta(2, \rho+\beta+rd-\alpha-\nu)\\ 1, \Delta(2, \rho+\beta+rd-\alpha), \Delta(2, \rho+\beta+rd-\nu)\end{matrix}; -\frac{y^2 w_i^2}{4}\right\}, \tag{10}$$

where the sum is taken over all the positive roots of (7). The result (10) will be proved useful in the verification of the solutions.

SOLUTION OF THE PROBLEM

We apply finite Hankel transform (9) to obtain the solution of (1). Its solution obtained[1] is

$$\Phi(x, t) = 2y^{\rho+\beta-3}\Gamma(\beta)\frac{k}{K}\sum_i\sum_{r=0}^{\infty}\frac{\prod_{j=1}^{u}(\alpha_j; r)\,\Gamma(\rho+rd)\,\Gamma(\rho+\beta+rd-\alpha-\nu)\,c^r J_0(x w_i)}{\prod_{j=1}^{v}(\beta_j; r)\,\Gamma(\rho+\beta+rd-\alpha)\,\Gamma(\rho+\beta+rd-\nu)\,(r)!\,[J_1(yw_i)]^2}\,.$$

$$.\,{}_4F_5\left\{\begin{matrix}\Delta(2, \rho+rd), \Delta(2, \rho+\beta+rd-\alpha-\nu)\\ 1, \Delta(\rho+\beta+rd-\alpha), \Delta(2, \rho+\beta+rd-\nu)\end{matrix}; -\frac{y^2 w_i^2}{4}\right\}\Psi(w_i, t), \tag{11}$$

where d is a positive integer, $u \leqslant v$ (or $u = v + 1$ and $|c| < 1$), no one of $\beta_1, \ldots\ldots, \beta_v$ is zero or a negative integer, $R(\beta) > 0$, $R(\rho) > 0$, $R(\rho+\beta-\alpha-\nu) > 0$

and

$$\Psi (w_i, t) = \int_0^t g(T)\, e^{-k w_i^2 (t-T)} \, d T , \tag{12}$$

VERIFICATION OF THE SOLUTION

From (11) and reference 6, we have

$$\frac{k}{x} \frac{\partial}{\partial x} \left(x \frac{\partial \Phi}{\partial x} \right) =$$

$$\delta = -2y^{\rho+\beta-3} \Gamma(\beta) \frac{k^2}{K} \sum_i \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{u} (\alpha_j ; r) \Gamma(\rho + rd) \Gamma(\rho + \beta + rd - \alpha - \nu)\, c^r w_i^2 \, J_0 (x w_i)}{\prod_{j=1}^{v} (\beta_j ; r) \Gamma(\rho + \beta + rd - \nu) \Gamma(\rho + \beta + rd - \alpha) (r)! [J_1 (y w_i)]^2} \cdot$$

$$\cdot {}_4F_5 \left\{ \begin{matrix} \triangle (2, \rho + rd), \triangle (2, \rho + \beta + rd - \alpha - \nu) \\ 1, \triangle (2, \rho + \beta + rd - \alpha), \triangle (2, \rho + \beta + rd - \nu) \end{matrix} ; -\frac{y^2 w_i^2}{4} \right\} \int_0^t g(T)\, e^{-k w_i^2 (t-T)} dT. \tag{13}$$

From (2) and (10), we have

$$\theta (x, t) = 2y^{\rho+\beta-3} \Gamma(\beta) \frac{k}{K} \sum_i \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{u} (\alpha_j ; r) \; \Gamma(\rho + rd) \; \Gamma(\rho + \beta + rd - \alpha - \nu) \; c^r \; J_0 (x w_i)}{\prod_{j=1}^{v} (\beta_j ; r) \Gamma(\rho + \beta + rd - \alpha) \Gamma(\rho + \beta + rd - \nu) (r)! [J_1 (y w_i)]^2} \cdot$$

$$\cdot {}_4F_5 \left\{ \begin{matrix} \triangle (2, \rho + rd), \triangle (2, \rho + \beta + rd - \alpha - \nu) \\ 1, \triangle (2, \rho + \beta + rd - \alpha), \triangle (2, \rho + \beta + rd - \nu) \end{matrix} ; -\frac{y^2 w_i^2}{4} \right\} g(t) ; \tag{14}$$

and from (11), we get

$$\frac{\partial \Phi}{\partial t} = 2y^{\rho+\beta-3} \Gamma(\beta) \frac{k}{K} \sum_i \sum_{r=0}^{\infty} \frac{\prod_{j=1}^{u} (\alpha_j ; r) \Gamma(\rho + rd) \Gamma(\rho + \beta + rd - \alpha - \nu) \; c^r \; J_0 (x w_i)}{\prod_{j=1}^{v} (\beta_j ; r) \Gamma(\rho + \beta + rd - \alpha) \Gamma(\rho + \beta + rd - \nu) (r)! [J_1 (y w_i)]^2} \cdot$$

$$\cdot {}_4F_5 \left\{ \begin{matrix} \triangle (2, \rho + rd), \triangle (2, \rho + \beta + rd - \alpha - \nu) \\ 1, \triangle (2, \rho + \beta + rd - \alpha), \triangle (2, \rho + \beta + rd - \nu) \end{matrix} ; -\frac{y^2 w_i^2}{4} \right\} \Big[g(t) - k w_i^2 \cdot$$

$$\cdot \int_0^t g(T)\, e^{-k w_i^2 (t-T)} dT \Big], \tag{15}$$

substituting the above values in (1), the equation is satisfied.

The boundary condition $\Phi(y, t) = 0$ is satisfied because $J_0 (y w_i)$, which is present in every term of $\Phi (y, t)$, is zero. The initial condition is satisfied because $\Psi (w_i, 0) = 0$.

We see that (11) converges uniformly when $t > 0$ and so the function $\Phi (x, t)$ represented by it is continuous when $0 \leqslant x \leqslant y$. The term by term differentiations are justified because (13) and (15) are uniformly convergent when $t > 0$ and $0 \leqslant x \leqslant y$.

HEAT SOURCE

(i) Heat source of general character : Let the function $g(T)$ be

$$g(T) = g_0 T^{N-1} (t-T)^{P-1} e^{-ST} {}_2F_1\left(\begin{matrix} Q, b \\ N \end{matrix}; \frac{T}{t}\right) H^{n,l}_{p,q}\left[z\left(1-\frac{T}{t}\right)^{m/h} \Big| \begin{matrix} \{(a_p, e_p)\} \\ \{(b_q, f_q)\} \end{matrix} \right]. \quad (16)$$

then using (5) (with $x = \frac{T}{t}$ and $c = st$), we get

$$\psi(w_i, t) = g_0 (2\pi)^{(1-h)A} h^B t^{P+N-1} e^{-st} \Gamma(N) \sum_{\mu=0}^{\infty} \frac{(s - k w_i^2)^{\mu} t^{\mu}}{(\mu)!} .$$

$$. H^{nh,\, lh+2}_{ph+2,\, qh+2}\left[(zh^{\tau})^h \Big| \begin{matrix} (1-P-\mu, m), (1+Q+b-P-N-\mu, m), \{(\triangle(h, a_p), e_b)\} \\ \{(\triangle(h, b_q), f_q)\}, (1+Q-P-N-\mu, m), (1+b-P-N-\mu, m) \end{matrix} \right], \quad (17)$$

where m and h are positive integers, $R(N) > 0$, $R(P+N-Q-b) > 0$, $R\left(P + \frac{m\, b_j}{h f_j}\right) > 0$ $(j = 1, \ldots\ldots, n)$, $\tau \lesssim 0$, $M > 0$ and $|\arg z| < \frac{1}{2}\pi M$.

From (11) and (17) the solution is

$$\Phi(x, t) = 2 g_0 (2\pi)^{(1-h)A} h^B y^{\rho+\beta-3} t^{P+N-1} \Gamma(\beta) \Gamma(N) e^{-st} \frac{k}{K} \sum_{i} \sum_{r=0}^{\infty} \sum_{\mu=0}^{\infty} .$$

$$. \frac{\prod_{j=1}^{u} (\alpha_j; r) \Gamma(\rho + rd) \Gamma(\rho+\beta+rd-\alpha-\nu)\, c^r (s - k w_i^2)^{\mu} t^{\mu} J_0(x w_i)}{\prod_{j=1}^{v} (\beta_j; r) \Gamma(\rho+\beta+rd-\alpha) \Gamma(\rho+\beta+rd-\nu) (r)!! [J_1(y w_i)]^2 (\mu)!} .$$

$$. {}_4F_5 \left\{ \begin{matrix} \triangle(2, \rho+rd), \triangle(2, \rho+\beta+rd-\alpha-\nu) \\ 1, \triangle(2, \rho+\beta+rd-\alpha), \triangle(2, \rho+\beta+rd-\nu) \end{matrix} ; -\frac{y^2 w_i^2}{4} \right\} H^{nh,\, lh+2}_{ph+2,\, qh+2} .$$

$$. \left[(z h^{\tau})^h \Big| \begin{matrix} (1-\rho-\mu, m), (1+Q+b-P-N-\mu, m), \{(\triangle(h, a_p), e_p)\} \\ \{(\triangle(h, b_q) f_q)\}, (1+Q-P-N-\mu, m), (1+b-P-N-\mu, m) \end{matrix} \right]. \quad (18)$$

obviously $\Phi(x, 0) = 0$.

If we put $e_j = f_h = 1$, $(j = 1, 2, \ldots\ldots, p;\ h = 1, \ldots\ldots, q)$ in (17) and apply the formula (4) we get a heat source recently obtained by Bajpai[2].

Set $u = v = 0$, $d = 1$, $e_j = f_h = 1$ $(j = 1, \ldots\ldots\ldots, p;\ h = 1, \ldots\ldots\ldots, q)$ and use the formula (4), (18) yields a known solution due to Bajpai[2].

(ii) *Heat source of hypergeometric character* :—In (16) and (17) put $e_j = f_h = 1$ $(j = 1, \ldots\ldots\ldots, p;\ h = 1, \ldots\ldots\ldots, q)$ use the formula (4), again put $s = k w_i^2$, $n = q = m = h = 1$, $l = p = b_1 = 0$, replace z by $(k w_i^2 - z)\, t$ and use the relation[7] we have

$$g(T) = g_0 T^{N-1} (t-T)^{P-1} e^{-zT} {}_2F_1\left(\begin{matrix} Q, b \\ N \end{matrix}; \frac{T}{t}\right) \quad (19)$$

and

$$\psi(w_i, t) = g_0 t^{P+N-1} e^{-zt} \frac{\Gamma(N)\Gamma(P)\Gamma(N+P-Q-b)}{\Gamma(P+N-Q)\Gamma(P+N-b)} {}_2F_2\left\{ \begin{matrix} P, P+N-Q-b \\ N+P-Q, N+P-b \end{matrix} (z - k w_i^2)\, t \right\}, \quad (20)$$

provided $R(N) > 0$, $R(P) > 0$ and $R(P+N-Q-b) > 0$. Substituting in (11) the value of $\psi(w_i, t)$ from (20), the solution becomes

$$\Phi(x,t) = 2\,g_0\, y^{\rho+\beta-3}\, t^{P+N-1}\, e^{-zt} \frac{\Gamma(\beta)\,\Gamma(N)\,\Gamma(P)\,\Gamma(P+N-Q-b)}{\Gamma(P+N-Q)\,\Gamma(P+N-b)} \frac{k}{K} \sum_i \sum_{r=0}^{\infty} \cdot$$

$$\cdot \frac{\prod_{j=1}^{u} (\alpha_j\,;\,r)\,\Gamma(\rho+rd)\,\Gamma(\rho+\beta+rd-\alpha-\nu)\,c^r\,J_0(x\,w_i)}{\prod_{j=1}^{v} (\beta_j\,;\,r)\,\Gamma(\rho+\beta+rd-\alpha)\,\Gamma(\rho+\beta+rd-\nu)\,(r)\,!\,[J_1(y\,w_i)]^2}$$

$$\cdot\ {}_2F_2 \left\{ \begin{matrix} P, P+N-Q-b \\ P+N-Q, P+N-b \end{matrix} ; (z-k w_i^2)\, t \right\} \cdot$$

$$\cdot\ {}_4F_5 \left\{ \begin{matrix} \triangle(2,\rho+rd), \triangle(2,\rho+\beta+rd-\alpha-\nu) \\ 1, \triangle(2,\rho+\beta+rd-\alpha), \triangle(2,\rho+\beta+rd-\nu) \end{matrix} ; -\frac{y^2 w_i^2}{4} \right\}, \quad (21)$$

obviously $\Phi(x, 0) = 0$, with $c = 0$ in (21), we get a next known result obtained in reference[2].

In (19) and (20), take $Q = b = 0, P = 1$, we get the heat source of exponential character given recently by Bhonsle[3]. Further with $N = 1, z = 0$ and use[6] viz. ${}_1F_1\left(\frac{1}{2}\,;z\right) = \frac{e^z - 1}{z}$, we obtain heat source of a finite interval of time given by Bhonsle[3].

Behaviour of $f(x)$

From (8), we have

$$f(x) = x^{\rho-2}(y-x)^{\beta-1}\ {}_2F_1\left(\begin{matrix}\alpha, \nu \\ \beta\end{matrix} ; 1-\frac{x}{y}\right) {}_uF_v \left\{ \begin{matrix} \alpha_1, \ldots\ldots, \alpha_u \\ \beta_1, \ldots\ldots, \beta_v \end{matrix} ; c\left(\frac{x}{y}\right)^d \right\}. \quad (22)$$

Let $\nu = \beta$, $c = 1$, $\alpha_1 = -1$, replace α by $-\alpha$, then we have

$$f(x) = x^{\rho+\alpha-2}\,(y-x)^{\beta-1}\,y^{-\alpha}\left\{1-\lambda\left(\frac{x}{y}\right)^d\right\},$$

$$\text{where } \lambda = \frac{\alpha_2\,\alpha_3 \ldots\ldots \alpha_u}{\beta_1\,\beta_2 \ldots\ldots \beta_v}\ .$$

Now we have

(i) $f(x) = 0$, when $x = 0$ and $\rho + \alpha > 2$; $x = y$ and $\beta > 1$; $x = y\left(\frac{1}{\lambda}\right)^{\frac{1}{d}}$,

(ii) $f(x) =$ negative, when $0 < x < y\left(\frac{1}{\lambda}\right)^{\frac{1}{d}}$

and

(iii) $f(x) =$ positive, when $y\left(\frac{1}{\lambda}\right)^{\frac{1}{d}} < x < y$.

The value of λ will determine the radius of the inner cylinder. If $g(t) > 0$, then the inner circular cylinder will enclose the sinks, while the volume between the two concentric cylinders will contain sources. If $g(t) < 0$, then sinks and sources will interchange their roles.

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