A NOTE ON

UNSTEADY FLOW OF A DUSTY VISCOUS LIQUID IN A CHANNEL AND A PIPE

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The note considers the flow of a viscous liquid with uniform distribution of dust particles in a channel and a circular pipe under the influence of pressure gradient varying linearly with time. It is found that the velocity of fluid particles is more than that of dust particles.

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Much work has already been done on dusty gas flow[1-5]. Recently, Rao[6] has investigated the flow of a dusty viscous liquid in a circular pipe under the influence of exponential pressure gradient. The author's interest in this subject was aroused by Saffman's paper[7] in which Orr-Sommerfeld equation for small disturbances in plane parallel flow of a dusty gas was formulated. The laminar flow of a viscous liquid with uniform distribution of dust particles in a channel bounded by two parallel flat plates and a circular pipe under the influence of linear pressure gradient has been discussed in the present note and it is interesting to note that the velocity of fluid particles is more than that of dust particles.

EQUATIONS OF MOTION

The equations to represent the motion of a dusty viscous liquid, given by Saffman[7] are the following :

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\,\text{grad}\,p + \nu\,\nabla^2\vec{u} + \frac{KN}{\rho}\,(\vec{v}-\vec{u}), \tag{1}$$

$$\text{div}\,\vec{u} = 0, \tag{2}$$

$$m\,\frac{D\vec{v}}{Dt} = K\,(\vec{u}-\vec{v}), \tag{3}$$

$$\frac{\partial N}{\partial t} + \text{div}\,(N\vec{v}) = 0. \tag{4}$$

where $\vec{u}$ and $\vec{v}$ are velocities of liquid and dust respectively, N is the number density of dust particles, each of mass m, K is the Stokes resistance coefficient ($= 6\pi\mu E$ for spherical particles of radius E) and p, ρ, μ, are the pressure, density and viscosity of the liquid respectively. The time relaxation parameter τ is given from (3) by $\frac{m}{K}$.

FLOW IN A CHANNEL

Let us study the flow in a channel bounded by two parallel flat plates. The velocities of fluid and dust particles are

$$u_1 = u_1(y,t),\ u_2 = 0,\ u_3 = 0,$$
$$v_1 = v_1(y,t),\ v_2 = 0,\ v_3 = 0,$$
$$N = N_0 \text{ (a constant)},$$

where (u_1, u_2, u_3) and (v_1, v_2, v_3) are velocity components of fluid and dust particles.

Then the equations governing the motion are

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u_1}{\partial y^2} + \frac{l}{\tau}(v_1 - u_1), \quad (5)$$

$$\tau \frac{\partial v_1}{\partial t} = (u_1 - v_1), \quad (6)$$

where $l = \dfrac{m N_0}{\rho}$ (= constant) is the mass concentration of dust particles.

Eliminating v_1 from (5) and (6), we get

$$\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{1}{\rho}\frac{\partial p}{\partial x}\right) + \nu \frac{\partial}{\partial t}\left(\frac{\partial^2 u_1}{\partial y^2}\right) - \frac{(l+1)}{\tau}\frac{\partial u_1}{\partial t} + \frac{1}{\tau}\left(-\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u_1}{\partial y^2}\right). \quad (7)$$

The boundary conditions are

$$t > 0, \; u_1(\pm y_0) = 0, \; v_1(\pm y_0) = 0. \quad (8)$$

Since we have assumed pressure gradient to be linear function of time, we take

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = (a_0 + a\,t), \quad (9)$$

and

$$u_1 = \phi(y)\left[a_0 + a\,t\right] + a\,\psi(y). \quad (10)$$

Substituting (9) and (10) in (7), we get

$$(a_0 + a\,t)\left[1 + \nu \phi''\right] + a\left[\tau(1 + \nu\phi'') - (l+1)\phi + \nu\psi''\right] = 0, \quad (11)$$

where dashes denote differentiation w.r.t. y.

The expressions for ϕ and ψ are obtained from (11) by equating the coefficients of $a_0 + a\,t$ and a to zero and solving them with the conditions $\phi(\pm y_0) = 0$, $\psi(\pm y_0) = 0$.

Thus

$$u_1 = \frac{1}{2\nu}(y_0^2 - y^2)(a_0 + a\,t) - \frac{a(l+1)}{24\nu^2}(5y_0^2 - y^2)(y_0^2 - y^2). \quad (12)$$

Equation (6) then gives

$$v_1 = \frac{1}{2\nu}(y_0^2 - y^2)(a_0 + a\,t) - \frac{a(l+1)}{24\nu^2}(5y_0^2 - y^2)(y_0^2 - y^2) - \frac{1}{2\nu}(y_0^2 - y^2)\,a\,\tau. \quad (13)$$

FLOW IN A CIRCULAR PIPE

The equations governing the motion in the present case are

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r}\right) + \frac{l}{\tau}(v_1 - u_1), \quad (14)$$

and

$$\tau \frac{\partial v_1}{\partial t} = (u_1 - v_1) , \tag{15}$$

where u_1 and v_1 are velocity components of fluid and dust particles along the axis (z-axis) of the pipe.

The boundary conditions are

$$t > 0 , u_1 = 0 , v_1 = 0 \text{ when } r = r_0 ,$$
$$t > 0 , u_1 = \text{finite}, v_1 = \text{finite at } r = 0 .$$

Eliminating v_1 form (14) and (15), we get

$$\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \nu \frac{\partial}{\partial t}\left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r}\right) - \frac{(l+1)}{\tau}\frac{\partial u_1}{\partial t} +$$
$$+ \frac{1}{\tau}\left[-\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r}\right)\right]. \tag{16}$$

We now assume that

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = (a_0 + a t) , \tag{17}$$

and

$$u_1 = \phi (r) \left[a_0 + a t \right] + a \psi(r) . \tag{18}$$

Substituting (17) and (18) in (16), we get

$$(a_0 + a t)\left[1 + \nu\left(\phi'' + \frac{1}{r}\phi'\right)\right] + a\left[\tau\left\{1 + \nu\left(\phi'' + \frac{1}{r}\phi'\right)\right\} - (l+1)\phi + \nu\left(\psi'' + \frac{1}{r}\psi'\right)\right] = 0 , \tag{19}$$

where dashes denote differentiation w.r.t. r.

The expressions for ϕ and ψ can be easily obtained from (19) and thus

$$u_1 = \frac{1}{4\nu}(r_0^2 - r^2)(a_0 + a t) - \frac{a(l+1)}{64\nu^2}(3 r_0^2 - r^2)(r_0^2 - r^2) . \tag{20}$$

Equation (15) then gives

$$v_1 = \frac{1}{4\nu}(r_0^2 - r^2)(a_0 + a t) - \frac{a(l+1)}{64\nu^2}(3 r_0^2 - r^2)(r_0^2 - r^2) - \frac{1}{4\nu}(r_0^2 - r^2) a \tau . \tag{21}$$

CONCLUSION

Since $\tau > 0$, it is clear from the expressions (12), (13) and (20), (21) that the velocity of fluid particles is more than that of dust particles in the present set up, whereas Rao[6] has shown that the velocity of dust particles is more than fluid particles when the pressure gradient decreases exponentially with time. When the dust is very fine, τ decreases and when $\tau \to 0$; the velocity of dusty fluid becomes that of clean fluid in both the cases. If the masses of the dust particles are small, their influence on the fluid flow is reduced, and in the limit as $m \to 0$, the fluid becomes ordinary viscous and we get the laminar flow of a viscous liquid in a channel and a circular pipe under the influence of linear pressure gradient as obtained by Lal[8].

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