FLOW OF A DUSTY VISCOUS LIQUID THROUGH RECTANGULAR CHANNEL

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The Laminar flow of an unsteady liquid with uniform distribution of dust particles through a rectangular channel under the influence of exponential pressure gradient with respect to time has been investigated. The influence of the presence of the dust particles on fluid particles is discussed and graphs of velocity profile are drawn.

Michael & Miller[1] have discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Later Sambasiva Rao[2] has studied the flow of a dusty viscous liquid through circular cylinder by taking exponential pressure gradient with respect to time. With similar pressure gradient, we have investigated the case of a rectangular channel. Analytical expressions for the velocities of fluid and dust particles are obtained. Graphs of velocity profile are drawn and the influence of the dust particles on fluid particles has been studied.

EQUATIONS OF MOTION

The equations of motion of unsteady viscous liquid with uniform distribution of dust particles are given[1,3,4] by

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\bar{u} + \frac{kN_0}{\rho}(\bar{v}-\bar{u}) \tag{1}$$

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v}\cdot\nabla)\bar{v} = \frac{k}{m}(\bar{u}-\bar{v}) \tag{2}$$

$$\text{div}\ \bar{u}=0 \tag{3}$$

$$\text{div}\ \bar{v}=0 \tag{4}$$

where $\bar{u}, \bar{v}$ denote the velocity vectors of liquid and dust particles respectively, p the pressure, ρ the density of the fluid, ν the kinematic coefficient of viscosity, t the time, m the mass of a dust particle, N_0, the number density of dust particles which is a constant throughout the motion, k the Stokes resistance coefficient which for spherical particles of radius a is $6\pi\mu a$ and μ the coefficient of viscosity of fluid particles.

FORMULATION AND SOLUTION OF THE PROBLEM

In the present investigation we shall discuss the laminar flow of a viscous liquid, with uniform distribution of dust particles, through a rectangular channel whose cross section is given by $(x^2-a^2)(y^2-b^2)$, under influence of exponential pressure gradient with respect to time.

Choosing the z-axis along axis of the channel, the components of velocity of fluid and dust particles are respectively given by

$$u_1=0, \qquad u_2=0 \tag{5}$$

$$v_1=0, \qquad v_2=0 \tag{6}$$

where (u_1, u_2, u_3) and (v_1, v_2, v_3) denote the components of fluid and dust particles respectively.

Since the motion is assumed to be laminar using the relations (5) and (6), equations of motion can be written as

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{7}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{8}$$

$$\frac{\partial u_3}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 u_3 + \frac{kN_0}{\rho}(v_3-u_3) \tag{9}$$

$$\frac{\partial v_3}{\partial t} = \frac{k}{m}(u_3 - v_3) \tag{10}$$

From (7) and (8), it follows that $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right)$ is a function of t only. Since we have assumed the pressure gradient is exponential, we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = \alpha\, e^{-\lambda^2 t} \tag{11}$$

where α and λ are real constants.

In view of (11), we can express

$$u_3 = w_1(x, y)\, e^{-\lambda^2 t} \tag{12}$$

$$v_3 = w_2(x, y)\, e^{-\lambda^2 t} \tag{13}$$

Using (11) to (13) in (9) and (10), one obtains respectively

$$\nabla^2 w_1 + \frac{l}{\tau\nu}(w_2 - w_1) + \frac{\lambda^2}{\nu} w_1 + \frac{\alpha}{\nu} = 0 \tag{14}$$

$$-\lambda^2 w_2 = \frac{1}{\tau}(w_2 - w_1) \tag{15}$$

where

$$\tau = \frac{m}{k}$$

$$l = \frac{N_0\, m}{\rho}$$

Equation (15) can be written as

$$w_2 = w_1/(1 - \tau\lambda^2) \tag{16}$$

Eliminating w_2 from (14) and (16) we obtain the following equation

$$\nabla^2 w_1 + \frac{\lambda^2}{\nu}\frac{(l + 1 - \tau\lambda^2)}{(1 - \tau\lambda^2)}\left[w_1 + \frac{\alpha}{\lambda^2}\frac{(1 - \tau\lambda^2)}{(l + 1 - \tau\lambda^2)}\right] = 0 \tag{17}$$

which can be simplified as

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} + \beta^2(w_1 + \Omega) = 0 \tag{18}$$

where

$$\Omega = \frac{\alpha}{\lambda^2}\frac{(1 - \tau\lambda^2)}{(l + 1 - \tau\lambda^2)}$$

$$\beta^2 = \frac{\lambda^2}{\nu}\frac{(l + 1 - \tau\lambda^2)}{(1 - \tau\lambda^2)}$$

The expression for velocity of the fluid particles is obtained if the solution of the differential equation (18) is obtained subject to the following boundary conditions.

$$w_1(\pm a, y) = 0 \quad (-b \leqslant y \leqslant b) \tag{19}$$

$$w_1(x, \pm b) = 0 \quad (-a \leqslant x \leqslant a) \tag{20}$$

Since there is no slip of the fluid particles on the walls of the channel.

The boundary condition (20) can be satisfied by taking w_1 in the following form

$$w_1(x, y) = \sum_{0}^{\infty} F(x) \cos\left(\frac{2n + 1}{2b}\right)\pi y \tag{21}$$

which simplifies the first boundary condition (19) to

$$F(\pm a) = 0 \tag{22}$$

Using (21) and taking[5] Ω, as

$$\Omega = \frac{4\,\Omega}{\pi} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1)} \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{23}$$

differential equation (18) can be expressed as

$$\frac{d^2 F}{d x^2} - P^2 F = -\frac{4\,\Omega}{\pi} \frac{\beta^2 (-1)^n}{(2n+1)} \tag{24}$$

where

$$P^2 = \frac{(2n+1)^2 \pi^2}{4 b^2} - \beta^2$$

The general solution of the differential equation (24) is given by

$$F(x) = A \cosh P x + B \sinh P x + \frac{4\,\Omega\,\beta^2 (-1)^n}{\pi P^2 (2n+1)} \tag{25}$$

where A and B are arbitrary constants to be determined subject to the boundary conditions (22).

Using the boundary conditions (22) in (25) one obtains the following values for A and B.

$$A = \frac{-4\,\Omega\,\beta^2 (-1)^n}{\pi P^2 (2n+1) \cosh P a} \tag{26}$$

$$B = 0 \tag{27}$$

Using the above values of A and B, (25) can be expressed as

$$F(x) = \frac{4\,\Omega\,\beta^2 (-1)^n}{\pi (2n+1)} \left(1 - \frac{\cosh P x}{\cosh P a}\right) \tag{28}$$

Which on substitution in (21) yields the following expression:

$$w_1(x, y) = \frac{4\,\beta^2\,\Omega}{\pi} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1) P^2} \left(1 - \frac{\cosh P x}{\cosh P a}\right) \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{29}$$

Using above relation in (16), we obtain

$$w_2(x, y) = \frac{4\,\beta^2\,\Omega}{\pi (1 - \tau \lambda^2)} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1) P^2} \left(1 - \frac{\cosh P x}{\cosh P a}\right) \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{30}$$

Using (29) and (30) in (12) and (13) we obtain respectively the following relations:

$$u_3(x, y, t) = \frac{4\,e^{-\lambda^2 t} \beta^2\,\Omega}{\pi} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1) P^2} \left(1 - \frac{\cosh P x}{\cosh P a}\right) \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{31}$$

$$v_3(x, y, t) = \frac{4\,e^{-\lambda^2 t} \beta^2\,\Omega}{\pi (1 - \tau \lambda^2)} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1) P^2} \left(1 - \frac{\cosh P x}{\cosh P a}\right) \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{32}$$

Relations (31) and (32) express respectively the velocity of the fluid and dust particles.

Following Drakes[6] approximation, we shall discuss the case of large values of P.

For large values of P the velocity of fluid and dust particles can be simplified as

$$u_3(x, y, t) = \frac{4\,e^{-\lambda^2 t} \beta^2\,\Omega}{\pi} \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1) P^2} \left[1 - e^{-(a-x) P}\right] \cos\left(\frac{2n+1}{2b}\right)\pi y \tag{33}$$

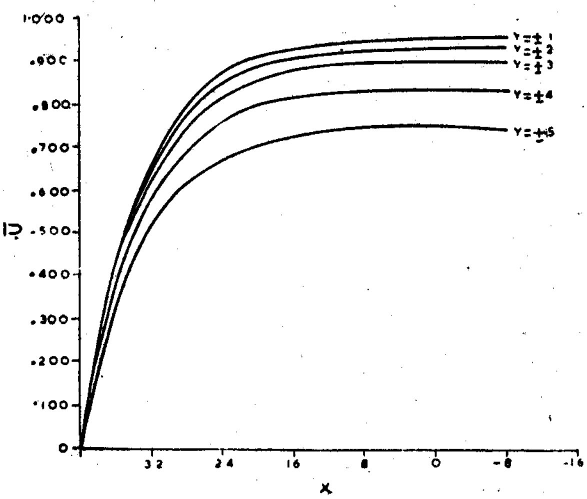

Fig 1—Variation of $\overline{U}$ with x (x=40 to —10) when $a = 40$; $b = 10$, $y = \pm 1$ to ± 5.

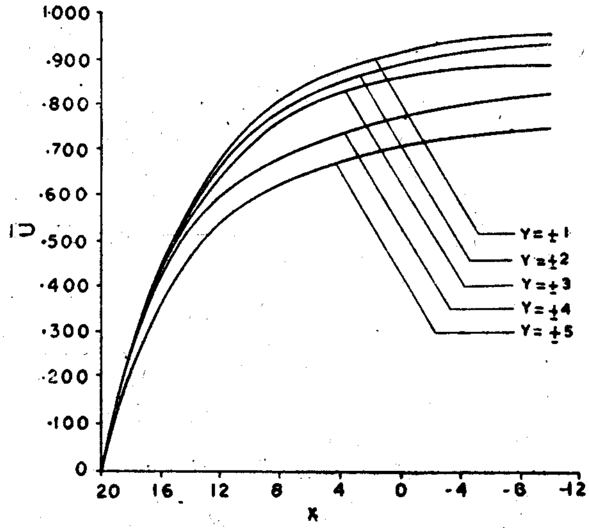

Fig 2—Variation of $\overline{U}$ with x (x = 20 to — 10) when $a = 20$; $b = 10$, $y = \pm 1$ to ± 5.

$$v_3(x, y, t) = \frac{4\, e^{-\lambda^2 t} \beta^2\, \Omega}{\pi (1 - \tau \lambda^2)} \sum_0^\infty \frac{(-1)^n}{(2n+1) P^2} \left[1 - e^{-(a-x)P} \right] \cos\left(\frac{2n+1}{2b} \right) \pi y \qquad (34)$$

From (33) and (34) it is seen that both the fluid and dust particles which are nearer to the axis of cylinder, move with the greater velocity. Since τ, λ^2 are positive, the velocity of the dust particles is more than that of the fluid particles, when the dust is very fine, the relaxation time of dust particles decreases and ultimately as $\tau \to 0$ the velocity of dust and fluid particles will be the same.

If the masses of dust particles are small, their influence on the fluid flow is reduced, and in the limit as $m \to 0$ the fluid becomes ordinary viscous, and we get the solution of the laminar flow of a viscous liquid through rectangular channel under the influence of exponential pressure gradient, with respect to time.

Denoting $\dfrac{u_3\, \pi\, e^{\lambda^2 t}}{4\, \beta^2\, \Omega}$ by $\overline{U}$, (33) can be expressed as

$$\overline{U} = \sum_0^\infty \frac{(-1)^n}{(2n+1) P^2} \left[1 - e^{-(a-x)P} \right] \cos\left(\frac{2n+1}{2b} \right) \pi y \qquad (35)$$

Graphs are drawn between $\overline{U}$ and x

(i) for $a = 40$, $b = 10$, $y = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ and x varies from 40 to — 10 (Fig. 1), and

(ii) for $a = 20$, $b = 10$, $y = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ and x varies from 20 to — 10 (Fig. 2).

From Fig. 1 and 2 the variation of the velocity of the fluid particles and hence the variation velocity of the dust particles can be studied.

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