STRESS DISTRIBUTION IN A HOMOGENEOUS ELASTIC SPHERE CONTAINING A PENNY SHAPED CRACK OF PRESCRIBED SHAPE

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The problem of finding the distribution of surface stress $P(r) = -\sigma_{\theta\theta}(r, \frac{\pi}{2})$ necessary to maintain a penny shaped crack $0 \leqslant r \leqslant 1, \theta = \frac{\pi}{2}$ situated in a diametral plane of an elastic sphere in the shape $U_\theta(r, \frac{\pi}{2}) = \omega(r), 0 \leqslant r \leqslant 1$ is considered. The case when $\omega(r) = (1 - r^2)$ is investigated in detail.

In recent years, the interest in the problems of brittle fracture, specially in the theory of cracks, has grown appreciably due to various technical applications. A beautiful account of the theory of cracks has been recently given by Sneddon & Lowengrub[1], Srivastava & Dwivedi[2] have investigated the problem of finding stress distribution in an elastic sphere deformed by the application of known pressure to inner surface of a penny shaped crack situated in the diametral plane.

Recently Olesiak & Sneddon[3] have also considered the distribution of surface stress necessary to produce a penny shaped crack whose shape is given by $\omega(\rho) = \epsilon \left(1 - \frac{\rho^2}{a^2}\right)^\alpha$. Interest in this problem was revived by Barenblatt[4] who considered three possibilities for two dimensional problem, one of which had to be rejected since it was physically unrealistic. In three dimensional problem, the remaining two correspond to two cases (*i*) $\omega'(a) = -\infty$ and (*ii*) $\omega(a) = 0$. Case (*i*) corresponds to a finite stress intensity factor and in case (*ii*) stress intensity factor is zero and opposite faces of the crack touch smoothly at the rim. Barenblatt[5] argued that in nature it is case (*ii*) which always occurs. He explained that in case (*ii*) stress intensity factor is zero due to the existence of cohesive forces in the vicinity of the crack. Alternative suggestion of the same case was given by Dugdale[6] who suggested that stress intensity factor at the tip of Griffith crack is zero due to the existence of plastic zone surrounding the crack. Following this view Olesiak & Wnuk[7] calculated the widths of the plastic zones near the penny shaped cracks.

The present paper deals with the problem of finding the distribution of surface stress $P(r) = -\sigma_{\theta\theta}\left(r, \frac{\pi}{2}\right)$ necessary to maintain a penny shaped crack, $0 \leqslant r \leqslant 1, \theta = \frac{\pi}{2}$ situated at the centre of the diametral plane of an elastic sphere in the shape $U_\theta\left(r, \frac{\pi}{2}\right) = \omega(r), 0 \leqslant r \leqslant 1$. The case when $\omega(r) = (1 - r^2)$ is investigated in detail. The boundary value problem is reduced to the solution of ordinary integral equation involving Hankel kernel and Legendre polynomials. The values of stresses are calculated numerically.

FORMULATION OF THE PROBLEM

We consider the problem of determining the stresses that will be required to open out a penny shaped crack of unit radius, in an elastic sphere to a prescribed shape. The sphere is assumed to be isotropic and homogeneous. There is symmetry about Z-axis. The position of a typical point may be expressed in terms of spherical polar coordinates (r, θ, ϕ). The displacement vector has components $(U_r, U_\theta, 0)$ and only non-vanishing components of stress tensor are σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{\phi\phi}$ and $\sigma_{r\theta}$. The centre of crack is taken as origin and the plane in which the crack lies is a coordinate plane. The curved surface of the sphere is free from shear and is supported in such a way that the radial component of displacement vanishes on the boundary of the sphere. Such a situation will arise when sphere is resting in a spherical hollow of exactly the same radius in a rigid body. The shape of the crack is prescribed. We are required to find the stress that will preserve the prescribed shape of the crack. The boundary conditions for the problem under consideration are :

$$U_\theta \left(r, \frac{\pi}{2} \right) = f(r) \qquad , \quad 0 \leqslant r \leqslant 1 \tag{1}$$

$$= 0 \qquad , \quad r > 1$$

and due to symmetry the conditions to be satisfied on the surface of the sphere are :

$$U_r(c, \theta) = \sigma_{\theta r}(c, \theta) = 0 \, , \quad 0 \leqslant \theta \leqslant \frac{\pi}{2} \tag{2}$$

where c is radius of the sphere.

DISPLACEMENT VECTOR AND STRESS TENSOR

The expressions for non-vanishing components of displacement and stress tensor appropriate for the problem under consideration have been derived by Srivastava & Dwivedi[2]. These expressions are :

$$U_\theta = \int_0^\infty B(\xi)\, e^{-\xi r \cos\theta} \Big[J_1(\xi r \sin\theta)(\xi r \cos\theta + 2\eta - 1)\cos\theta - J_0(\xi r \sin\theta) \cdot$$

$$\cdot (\xi r \cos\theta - 2\eta + 2)\sin\theta \Big] d\xi + \sum_{n=0}^{\infty} \Big\{ a_{2n}\, n\,(2n - 4\eta + 5)\, r^{2n+1} +$$

$$+ b_{2n}\, r^{2n-1} \Big\} \frac{d}{d\theta} P_{2n}(\cos\theta). \tag{3}$$

$$U_r = \int_0^\infty B(\xi)\, e^{-\xi r \cos\theta} \Big[J_1(\xi r \sin\theta)(\xi r \cos\theta + 2\eta - 1)\sin\theta + J_0(\xi r \sin\theta) \cdot$$

$$\cdot (\xi r \cos\theta - 2\eta + 2)\cos\theta \Big] d\xi + \sum_{n=0}^{\infty} \Big[a_{2n}(2n+1)(2n + 4\eta - 2)\, r^{2n+1} +$$

$$+ b_{2n}\, 2n\, r^{2n-1} \Big] P_{2n}(\cos\theta) \tag{4}$$

$$\sigma_{r\theta} = \int_0^\infty \frac{B(\xi)}{r}\, e^{-\xi r \cos\theta} \cos\theta \Big[J_0(\xi r \sin\theta)\, \xi^2 r^2 \sin 2\theta + J_1(\xi r \sin\theta)(1 - 2\eta -$$

$$- \xi r \cos\theta - \xi^2 r^2 \cos 2\theta) \Big] d\xi + \sum_{n=1}^{\infty} \Big[a_{2n}(4n^2 + 4n + 2\eta - 1)\, r^{2n} + b_{2n} \cdot$$

$$\cdot (2n - 1)\, r^{2n-2} \Big] \frac{d}{d\theta} P_{2n}(\cos\theta) \tag{5}$$

$$\sigma_{\theta\theta} = \int_0^\infty B(\xi) \cdot \xi . e^{-\xi r \cos\theta} \Big[\frac{J_1(\xi r \sin\theta)}{\xi r \sin\theta} \cos^2\theta (1 - 2\eta - \xi r \cos\theta + 2 \xi^2 r^2 \sin^2\theta) -$$

$$- J_0(\xi r \sin\theta)(1 - \xi r \cos\theta . \cos 2\theta) \Big] d\xi - \sum_{n=0}^{\infty} \Big\{ a_{2n} (4n^2 + 8n + 2\eta + 2) \cdot$$

$$\cdot (2n+1) r^{2n} + b_{2n} 4n^2 r^{2n-2} \Big\} P_{2n}(\cos\theta) + \Big[a_{2n} (2n - 4\eta + 5) r^{2n} + b_{2n} r^{2n-2} \Big] \cdot$$

$$\cdot \cot\theta \frac{d}{d\theta} P_{2n}(\cos\theta) \tag{6}$$

where $P_{2n}(\cos\theta)$ is a Legendre polynomial.

SOME USEFUL RESULTS

The results listed here shall be used in future calculations. These results are easily derived from standard results given by Sneddon[8]. These results for $x < r$ are :

$$\int_0^\infty \xi \cdot e^{-\xi r \cos\theta} J_0(\xi r \sin\theta) J_0(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+1)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+2}} P_{2m+1}(\cos\theta)$$

$$\int_0^\infty \xi^2 . e^{-\xi r \cos\theta} J_0(\xi r \sin\theta) J_0(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+2)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+3}} P_{2m+2}(\cos\theta)$$

$$\int_0^\infty \xi^3 . e^{-\xi r \cos\theta} J_0(\xi r \sin\theta) J_0(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+3)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+4}} P_{2m+3}(\cos\theta)$$

$$\int_0^\infty \xi . e^{-\xi r \cos\theta} J_1(\xi r \sin\theta) J_0(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+1)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+2}} \frac{d}{d\theta} P_{2m+1}(\cos\theta)$$

$$\int_0^\infty \xi^2 e^{-\xi r \cos\theta} J_1(\xi r \sin\theta) J_o(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+2)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+3}} \frac{d}{d\theta} P_{2m+2}(\cos\theta)$$

$$\int_0 \xi^3 e^{-\xi r \cos\theta} J_1(\xi r \sin\theta) J_0(\xi x) d\xi = \sum_{m=0}^{\infty} \frac{(-1)^m (2m+2)!}{2^{2m} m! m!} \frac{x^{2m}}{r^{2m+4}} \frac{d}{d\theta} P_{2m+3}(\cos\theta)$$

and

$$(2n+1)\cos\theta . P'_n = (n+1) P'_{n-1} + n P'_{n+1} ; \frac{d}{d\theta}\Big[\sin\theta \frac{d}{d\theta} P_n \Big] = - n(n+1) P_n .$$

With the help of these results and reccurence relations for Legendre polynomials, we obtain the following results :

$$\int_0^\infty \xi e^{-\xi c \cos\theta} [J_1(\xi c \sin\theta)(\xi c \cos\theta + 2\eta - 1) \sin\theta + J_0(\xi c \sin\theta) .$$

$$\cdot (\xi c \cos\theta - 2\eta + 2) \cos\theta] J_0(\xi x) d\xi$$

$$= \frac{1}{c^3} \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!}{2^{2n} n! n!} \Big(B_o(n) + B_1(n) \frac{x^2}{c^2} + B_2(n) \frac{x^4}{c^4} \Big) . \Big(\frac{x}{c} \Big)^{2n-4} P_{2n}(\cos\theta)$$

where

$$B_0(n) = \frac{4n^2 (4n-4)^2}{(4n-3)(4n-1)(n+\frac{1}{2})}$$

$$B_1(n) = -\frac{4n^2}{4n+1}\left\{1 - \frac{1}{4n+3} + \frac{1}{4n-1}\right\}$$

$$B_2(n) = \left\{1 - 2\eta + \frac{(2n+1)(2n+3)}{(4n+3)(4n+5)}\right\}$$

and

$$\frac{d}{d\theta}\left[\sin\theta \int_0^\infty \frac{\xi \, . \, e^{-\xi c \cos\theta}}{c} \left\{\xi^2 c^2 \sin^2\theta \, J_0(\xi c \sin\theta) + (\tfrac{1}{2} - \xi c \cos\theta - \xi^2 c^2 \cos 2\theta) \cdot \right.\right.$$

$$\left.\left. \cdot J_1(\xi c \sin\theta)\right\} J_o(\xi x) \, d\xi\right] = \sum_{n=1}^{\infty} \frac{(-1)^n \quad (2n+1)!}{2^{2n+1} \; (n)! \; (n)!} \left\{A_0(n) + A_1(n)\frac{x^2}{c^2} + A_2(n)\frac{x^4}{c^4} + \right.$$

$$\left. + A_3(n)\frac{x^6}{c^6}\right\} \left(\frac{x}{c}\right)^{2n-6} \frac{\sin\theta}{c^3} P_{2n}(\cos\theta)$$

where

$$A_o(n) = \frac{8\{2n(2n-2)(2n-4)\}^2}{(4n-5)(4n-3)(4n-1)}\left[\frac{2n-3}{2n-1} + \frac{2n}{4n-1}\right]$$

$$A_1(n) = \frac{2\{n(n-1)\}^2}{(4n-1)(4n+1)}\left[\frac{2n-1}{4n-3} + \frac{2n+2}{4n+3}\right]$$

$$A_2(n) = \frac{2n^2}{4n-1}\left\{1 - \frac{2(2n-1)(2n+1)(8n+3)}{(4n+1)(4n+3)}\right\} - \frac{64n^3(2n+1)(2n+2)}{(4n+1)(4n+3)(4n+5)}$$

$$A_3(n) = \frac{-2n(2n+2)}{(4n+3)}\left\{1 - \frac{2(2n+1)(8n+11)}{(4n+5)(4n+7)}\right\}$$

SOLUTION OF THE PROBLEM

From the boundary conditions (1) we get

$$\int_0^\infty B(\xi) J_o(\xi r) \, d\xi = -\frac{f(r)}{2(1-\eta)} \quad , \quad 0 \geqslant r \leqslant 1 \tag{7}$$

$$= 0 \quad , \quad r > 1$$

The inversion theorem for Hankel transform gives

$$B(\xi) = -\frac{\xi}{2(1-\eta)} \int_0^1 r f(r) J_o(\xi r) \, dr \tag{8}$$

The boundary conditions (2) give

$$\sum_{n=0}^{\infty} \left\{a_{2n}(2n+1)(2n+4\eta-2) c^{2n+1} + b_{2n} 2n c^{2n-1}\right\} \cdot P_{2n}(\cos\theta) =$$

$$= - \int_0^\infty B(\xi)\, e^{-\xi c \cos\theta} \Big[J_1(\xi c \sin\theta)(\xi c \cos\theta + 2\eta - 1)\sin\theta +$$

$$+ J_o(\xi c \sin\theta)(\xi c \cos\theta - 2\eta + 2)\cos\theta \Big] d\xi \qquad (9)$$

and

$$\sum_{n=1}^{\infty} \Big\{ a_{2n}(4n^2 + 4n + 2\eta - 1)c^{2n} + b_{2n}(2n-1)c^{2n-2} \Big\} \cdot \frac{d}{d\theta} P_{2n}(\cos\theta) =$$

$$- \int_0^\infty \frac{B(\xi)}{c} e^{-\xi c \cos\theta} \cos\theta \Big[J_o(\xi c \sin\theta)\, \xi^2 c^2 \sin 2\theta + J_1(\xi c \sin\theta)$$

$$(1 - 2\eta - \xi c \cos\theta - \xi^2 c^2 \cos 2\theta \Big] d\xi \qquad (10)$$

Substituting the value of $B(\xi)$ from (8) into (9) and (10) we get

$$\sum_{n=0}^{\infty} \Big\{ a_{2n}(2n+1)(2n + 4\eta - 2)c^{2n+1} + b_{2n}\, 2n\, c^{2n-1} \Big\} P_{2n}(\cos\theta) =$$

$$= \frac{1}{2(1-\eta)} \int_0^1 x f(x) \Big[\int_0^\infty \xi e^{-\xi c \cos\theta} J_o(\xi x) \Big\{ J_1(\xi c \sin\theta)(\xi c \cos\theta + 2\eta - 1)\sin\theta +$$

$$+ J_o(\xi c \sin\theta)(\xi c \cos\theta - 2\eta + 2)\cos\theta \Big\} d\xi \Big] dx \qquad (11)$$

and

$$\sum_{n=1}^{\infty} \Big\{ a_{2n}(4n^2 + 4n + 2\eta - 1)c^{2n} + b_{2n}(2n-1)c^{2n-2} \Big\} \cdot \frac{d}{d\theta} P_{2n}(\cos\theta) =$$

$$= \Big[\frac{1}{2(1-\eta)} \int_0^1 \frac{x f(x)}{c} \Big\{ \int_0^\infty \xi e^{-\xi c \cos\theta} \cos\theta \Big\{ J_o(\xi c \sin\theta)\xi^2 c^2 \sin 2\theta + J_1(\xi c \sin\theta) \cdot$$

$$\cdot (1 - 2\eta - \xi c \cos\theta - \xi^2 c^2 \cos 2\theta \Big\} \cdot J_o(\xi x)\, d\xi \Big\} dx \qquad (12)$$

Multiply (12) by $\sin\theta$ and differentiate with respect to θ. By substituting the values of the inner integrals and using the orthogonal property for the Legendre polynomials, after some calculations, we get

$$c^{2n}(2n+1)(3n+1)(4n-1)\, a_{2n} = \int_0^1 \Big[\alpha_o(n) + \alpha_1(n)\frac{x^2}{c^2} + \alpha_2(n)\frac{x^4}{c^4} + \alpha_3(n)\frac{x^6}{c^6} \Big] \left(\frac{x}{c}\right)^{2n-6} x f(x) dx \qquad (13)$$

$$c^{2n-2}(2n)(3n+1)(4n-1)\, b_{2n} = \int_0^1 \Big[\beta_o(n) + \beta_1(n)\frac{x^2}{c^2} + \beta_2(n)\frac{x^4}{c^4} +$$

$$+ \beta_3(n)\frac{x^6}{c^6} \Big] \frac{x^{2n-6}}{c^{2n-3}}\, x f(x)\, dx \qquad (14)$$

where

$$\alpha_o(n) = \frac{(-1)^n (2n+1)!\, 512 n^2 (n-1)^2 (n-2)^2}{2^{2n+1}\, n!\, n!\, (4n-5)(4n-3)(4n-1)} \left[\frac{2n-3}{2n-1} + \frac{2n}{4n-1} \right]$$

$$\alpha_1(n) = -\frac{(-1)^n (2n+1)!\ (n-1)^2\, 2n^2}{2^{2n+1}\ n!\ \ n!\ \ (4n-1)} \left[\frac{128(2n-1)}{4n-3} - \frac{1}{4n+1} \left\{ \frac{2n-1}{4n-3} + \frac{2n+2}{4n+3} \right\} \right]$$

$$\alpha_2(n) = -\frac{(-1)^n (2n+1)!\, 2n}{2^{2n+1}\, n!\ \ n!} \left[\frac{8(2n^2)(2n+1)(2n+2)}{(4n+1)(4n+3)(4n+5)} - \frac{4n(4n+1)(4n^2-1)}{(4n+3)\ (4n-1)} - \right.$$

$$\left. - \frac{n}{4n-1} \left\{ 1 - \frac{2(2n-1)(2n+1)(8n+3)}{(4n+1)(4n+3)} \right\} \right]$$

$$\alpha_3(n) = -\frac{(-1)^n\, 2(2n+1)!}{2^{2n+1}\, n!\, n!\, (4n+3)} \left[\frac{(2n-1)(2n+1)^2(2n+3)}{4n+5} + \right.$$

$$\left. + 2n(n+1) \left\{ 1 - \frac{2(2n+1)(2n+3)(8n+11)}{(4n+5)(4n+7)} \right\} \right]$$

$$\beta_o(n) = \frac{(-1)^n (2n+1)!\, 512 (2n-1)\, n^2 (n-1)^2 (n-2)^2}{2^{2n+1}\, n!\, n!\, (4n-1)(4n-3)(4n-5)} \left[\frac{2n-3}{2n-1} + \frac{2n}{4n-1} \right]$$

$$\beta_1(n) = -\frac{(-1)^n (2n+1)!\, 2n^2 (n-1)^2}{2^{2n+1}\, n!\, n!\, (4n-1)} \left[\frac{32(8n^3+8n^2-n)}{(4n-3)(2n+1)} - \frac{2n-1}{4n+1} \left\{ \frac{2n-1}{4n-3} + \frac{2n+2}{4n+3} \right\} \right]$$

$$\beta_2(n) = -\frac{(-1)^n (2n+1)!\, 2n}{2^{2n+1}\, n!\, n!} \left[\frac{(8n^3+8n^2-n)\, 4n(4n+1)}{(4n-1)(4n+3)} - \right.$$

$$\left. - \frac{8(2n)(2n-1)(2n+1)(2n+2)}{(4n+1)(4n+3)(4n+5)} - \frac{n(2n-1)}{4n-1} \left\{ 1 - \frac{2(2n-1)(2n+1)(8n+3)}{(4n+1)(4n+3)} \right\} \right]$$

$$\beta_3(n) = -\frac{(-1)^n (2n+1)!\, 2n}{2^{2n+1}\, n!\, n!\, (4n+3)} \left[2(n+1) \left\{ 1 - \frac{2(2n+1)(2n+3)(8n+11)}{(4n+5)(4n+7)} \right\} + \right.$$

$$\left. + \frac{(8n^2+8n-1)\ (2n+1)\ (2n+3)}{4n+5} \right]$$

In the above calculations we have assumed that $\eta = 1/4$

Thus the unknown function $B\ (\xi)$ and unknown coefficients a_{2n} and b_{2n} are now known in terms o prescribed function $f\ (x)$. The normal stress necessary for producing a crack of the prescribed shape can be calculated. The required stresses are gienn by the expression :

$$\sigma_{\theta\theta}\left(r\,,\ \frac{\pi}{2} \right) = -\int_0^\infty \xi\, B(\xi).\ J_o(\xi r)\, d\xi - \sum_{n=0}^{\infty} \left\{ a_{2n} (4n^2 + 8n + 5/2)(2n+1)\, r^{2n} + \right.$$

$$\left. + b_{2n}\, 4n^2\, r^{2n-2} \right\} P_{2n}(0) \tag{15}$$

On substituting the value of $B\ (\xi)$ from (8) into (15) we get

$$\sigma_{\theta\theta}\left(r\,, \frac{\pi}{2} \right) = \frac{1}{2(1-\eta)} \int_0^\infty \xi^2.\ J_o(\xi\, r) \left[\int_0^1 x\, J_o(\xi\, x)\, f(x)\, dx \right] d\xi - \sum_{n=0}^{\infty} \left\{ a_{2n} (4n^2 + 8n + 5/2)\ . \right.$$

$$\left. \cdot\, (2n+1)\, P_{2n}(0) + b_{2n+2} \cdot 4(n+1)^2\, P_{2n+2}(0) \right\} r^{2n} \tag{16}$$

where the coefficients a_{2n} and b_{2n} have the values given in (13) and (14).

TABLE 1

NUMERICAL VALUES OF $\sigma_{\theta\theta}\left(r, \frac{\pi}{2}\right)$ FOR SOME VALUES OF $1/c$ AND r

r	$1/c$					
	0·2	0·4	0·5	0·6	0·7	0·8
0	1·338843	1·369310	1·402590	1·451690	1·521265	1·615200
0·2	1·292171	1·322170	1·355230	1·404849	1·475380	1·574379
0·4	1·125635	1·154200	1·186571	1·237749	1·310904	1·424789
0·5	1·016164	1·043640	1·075432	1·127630	1·202267	1·325738
0·6	0·833966	0·860090	0·891080	0·944592	1·019499	1·150323
0·7	0·580750	0·605250	0·635147	0·688510	0·772040	0·890101
0·8	0·266967	0·289538	0·317868	0·372598	0·435957	0·633031

We now calculate the stresses when the shape of the crack is given by $f(x) = 1 - x^2$. Changing the order of integration and evaluating the integrals we have

$$\sigma_{\theta\theta}\left(r, \frac{\pi}{2}\right) = I(r) + B(r)$$

$$I(r) = \frac{2}{3}\int_0^\infty \xi^2 . \ J_o(\xi r)\left[\int_0^1 x J_o(\xi x)(1 - x^2)\, dx\right] d\xi = \frac{8}{3\pi}\Big[(1 + r) E(k) + (1 - r) K(k) - K(r)\Big]$$

$$B(r) = -\sum_{n=0}^{\infty}\left\{ a_{2n}(4n^2 + 8n + 5/2)(2n + 1) P_{2n}(0) + b_{2n+2}\, 4(n + 1)^2 P_{2n+2}(0)\right\} r^{2n}$$

where $K(k)$, $K(r)$ and $E(k)$ are elliptic integrals of first and second kind and $k = \dfrac{2 r^{\frac{1}{2}}}{1 + r}$

and

$$a_{2n} = \frac{1}{(2n + 1)(3n + 1)(4n - 1)}\left\{\frac{\alpha_o(n)}{(n - 1)(2n - 4)} + \frac{\alpha_1(n)\, c^{-2}}{n(2n - 2)} + \frac{\alpha_2(n)\, c^{-4}}{n(2n + 2)} + \right.$$
$$\left. + \frac{\alpha_3(n)\, c^{-6}}{(n + 1)(2n + 4)}\right\} c^{-4n+3}$$

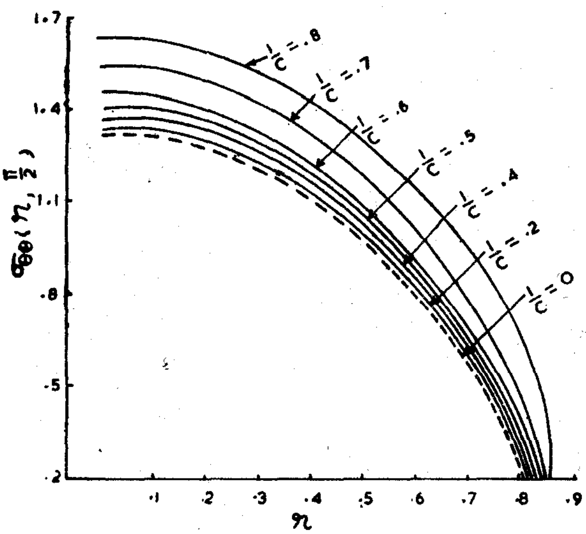

Fig. 1—Relationship between r and $\sigma_{\theta\theta}\left(r, \frac{\pi}{2}\right)$ for various values of $1/c$

$$b_{2n} = \frac{1}{n(3n + 1)(4n - 1)}\left[\frac{\beta_o(n)}{(2n - 4)(2n - 2)} + \right.$$
$$+ \frac{\beta_1(n)\, c^{-2}}{2n(2n - 2)} + \frac{\beta_2(n)\, c^{-4}}{2n(2n + 4)} +$$
$$\left. + \frac{\beta_3(n)\, c^{-6}}{(2n + 2)(2n + 4)}\right] c^{-4n+5}$$

The numerical values of the stresses were calculated. These calculations were made for $1/c = 0·2, 0·4, 0·5, 0·6, 0·7, 0·8$ and for $r = 0·2, 0·4, 0·5, 0·6, 0·7$ and $0·8$. The values of $\sigma_{\theta\theta}\left(r, \frac{\pi}{2}\right)$ are given in Table 1 and represented graphically in Fig. 1. The dotted curve represents the stress required to preserve the shape of the crack in an infinite solid.

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