DETERMINATION OF SLENDER BODIES OF MINIMUM TOTAL DRAG IN HYPERSONIC FLOW USING NEWTON-BUSEMANN PRESSURE COEFFICIENT LAW

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(*Received* 23 *June,* 1971)

A unified treatment is given to the problem of finding minimum total drag bodies—both two-dimensional as well as axisymmetric by using Newton-Busemann law under the assumption that the friction coefficient is constant. Particular cases have been discussed when two of the geometric quantities defining the body have prescribed values, and the results have been illustrated by means of graphs. In case of two dimensional bodies when the length is specified and in case of axisymmetric bodies when the surface area is known, the optimum shapes are independent of the friction coefficient.

Recently Miele[1,2] studied the nature of the body shapes— both two dimensional and axisymmetric —which have minimum pressure drag, using the Newton-Busemann law under various constraints on the geometrical quantities defining a body, namely length, diameter, enclosed area and moment of inertia in two dimensional case and length, diameter, surface area, and volume in axisymmetric case. But there exist practical configurations for which the friction drag has the same order of magnitude as the pressure drag and as such it is of interest to reinvestigate the minimum drag body problem from the point of view of minimising the total drag, i.e. sum of pressure drag and friction drag. As Newton-Busemann law is used to restudy the total minimum drag problem, taking constant friction coefficient and since from the literature it is known that many problems of optimum shapes admit power law solutions, it is worthwhile to investigate the existence of particular solutions having the form $y=(d/2)\ (x/l)^n$, where n is a constant. A general treatment is given here which holds good for both two-dimensional and axisymmetric bodies and then particular solutions have been deduced under various conditions on the geometrical quantities defining the body.

FORMULATION OF THE PROBLEM

If q denotes the free stream dynamic pressure; C_f the constant friction coefficient; x the abscissa, y' the first derivative dy/dx and y'' the second derivative d^2y/dx^2; then the general expression for the total drag may be written as

$$\frac{D}{4\pi^{\alpha}q} = \int_0^l y^{\alpha}\left(y'^3 + \frac{yy'y''}{\alpha+1} + \frac{C_f}{2}\right)dx \tag{1}$$

where α is a numerical constant, $\alpha = 0$ for two dimensional bodies and $\alpha = 1$ for axisymmetric bodies.

In case of two-dimensional bodies the enclosed area A and the moment of inertia M are given by

$$A = 2\int_0^l y\,dx, \quad M = 2\int_0^l y^2 dx \tag{2}$$

In case of axisymmetic bodies the surface area S and the volume V are given by

$$S = 2\pi\int_0^l y\,dx, \quad V = \pi\int_0^l y^2 dx \tag{3}$$

From (2) and (3), we observe that we can write

$$P = 2\pi^{\alpha}\int_0^l y\,dx, \quad Q = 2\pi^{\alpha}\int_0^l y^2 dx \tag{4}$$

where

$$\left.\begin{aligned} P &= A \\ Q &= M \end{aligned}\right\} \quad \text{When } \alpha = 0 \text{ (two-dimensional bodies)}$$

and

$$\left.\begin{aligned} P &= S \\ Q &= 2V \end{aligned}\right\} \quad \text{When } \alpha = 1 \text{ (axisymmetric bodies)}$$

Now we restrict ourselves to the study of the shapes of the form

$$y = (d/2)\,(x/l)^n \tag{5}$$

where d is the diameter, l the length of the body and n is a constant to be determined for the minimum drag body under various situations. This equation clearly satisfies the boundary conditions of the problem viz.

$$x_o = y_o = 0 \text{ and } x_f = l,\ y_f = d/2$$

SOLUTION OF THE PROBLEM

From (1), (4) and (5), we obtain

$$\frac{D}{4\pi^{\alpha}q} = \left(\frac{d}{2}\right)^{\alpha+3} \frac{1}{l^2}\,\frac{n^3}{(n\alpha + 3n - 2)} + \left(\frac{d}{2}\right)^{\alpha+3} \frac{1}{l^2}\,\frac{n^2(n-1)}{(n\alpha+3n-2)(\alpha+1)} + \frac{C_f}{2}\left(\frac{d}{2}\right)^{\alpha} \frac{l}{(n\alpha + 1)}, \tag{6}$$

$$P = \frac{\pi^{\alpha}\, dl}{(n+1)} \tag{7}$$

$$Q = \left(\frac{d}{2}\right)^{2} \frac{2\,\pi^{\alpha}\, l}{(2n+1)} \tag{8}$$

Now in what follows we examine different class of minimum drag bodies under various given conditions. The quantities which define a body are l, d, n, P and Q. These are five quantities and there are three equations (5), (7) and (8) connecting them and hence we have two quantities at our disposal to choose in advance and thus find the remaining three in such a way that the body may have minimum total drag.

We have to ensure that the value of n should be such that it does not make the pressure drag {represented by the first term on the right hand side of (6)} negative and this requires that

$$n > \frac{2}{\alpha + 3}$$

From (6), (7) and (8), it is obvious that for two-dimensional bodies, if the length is prescribed and for three dimensional bodies, if the surface area is prescribed then the optimum body profile is independent of the friction drag.

With the aid of the previous relationships, the optimum shapes can be determined for various combinations of the conditions on l, d, P and Q. The calculation of these shapes for several such combinations is now undertaken.

In what follows the drag coefficient C_D is defined as

$$C_D = (2/d)^{\alpha+1}\,(D/\pi^{\alpha}q)$$

Example 1—Given the diameter d and the length l.

From (6) the drag is given by

$$\frac{D}{4\pi^{\alpha}q} = \left(\frac{d}{2}\right)^{\alpha+3} \frac{1}{l^2}\,\frac{n^3(\alpha+2) - n^2}{(n\alpha + 3n - 2)(\alpha+2)} + \frac{C_f}{2}\left(\frac{d}{2}\right)^{\alpha} \frac{l}{(n\alpha + 1)} \tag{9}$$

In order that drag be minimum $dD/dn = 0$, i.e.

$$\frac{2n^3(\alpha^2+5\alpha+6)-n^2(7\alpha+15)+4n}{(\alpha+1)(n\alpha+3n-2)^2} - 4C_f\left(\frac{l}{d}\right)^3\frac{\alpha}{(n\alpha+1)^2} = 0 \tag{10}$$

This determines the optimum value of n to give the required shape of the body.

Again the drag coefficient may be written as

$$C_D = \left(\frac{d}{l}\right)^2\frac{n^3(\alpha+2)-n^2}{(n\alpha+3n-2)(\alpha+2)} + 4\,C_f\left(\frac{l}{d}\right)\frac{1}{(n\alpha+1)^2} = 0 \tag{11}$$

Case (*a*) : when $\alpha = 0$ (Two-dimensional body)

In this case from (10), we obtain $n = 0\cdot8644$. Knowing the value of n, we can obtain from (7), (8) and (11) the following quantities

Enclosed Area $\quad A = dl/(n+1)$

Moment of Inertia $\quad M = \dfrac{d^2}{2}\,\dfrac{l}{(2n+1)}$

Drag Coefficient $\quad C_D = \dfrac{n^2(2n-1)}{(3n-2)}\left(\dfrac{d}{l}\right)^2 + 4C_f\left(\dfrac{l}{d}\right)$

Fig. 1 illustrates relationship $n\,(d/l)$ and $C_D\,(d/l)$ for various values of C_f.

Case (*b*) : when $\alpha = 1$ (Axisymmetric body)

In this case the value of n is given by the relation

$$(12n^3-11n^2+2n)(n+1)^2 - 16\,C_f\,(l/d)^3(2n-1)^2 = 0 \tag{12}$$

Also from (7), (8) and (11), the following quantities are obtained

Surface Area $\quad S = \pi dl/(n+1)$

Volume $\quad V = \dfrac{\pi d^2}{4}\,\dfrac{l}{(2n+1)}$

Drag Coefficient $\quad C_D = \dfrac{n^2(3n-1)}{4(2n-1)}\left(\dfrac{d}{l}\right)^2 + 4\,C_f\left(\dfrac{l}{d}\right)\dfrac{1}{(n+1)}$

Fig. 2 gives the relationships $n\,(d/l)$ and $C_D\,(d/l)$ for given values of C_f.

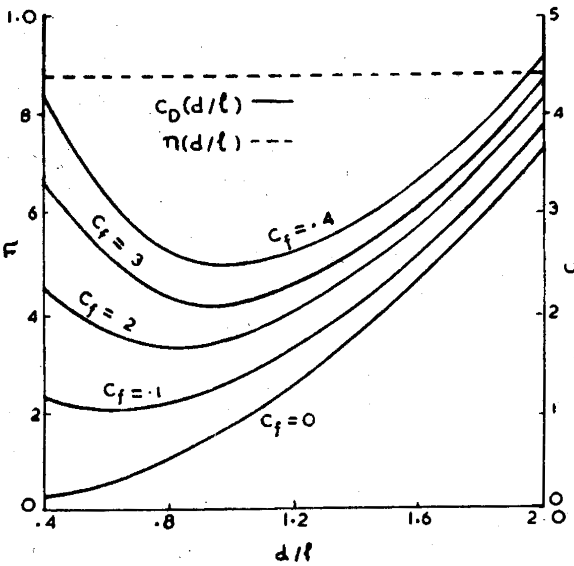

Fig. 1—n and C_D versus d/l for given values of C_f

(Two-dimensional body)

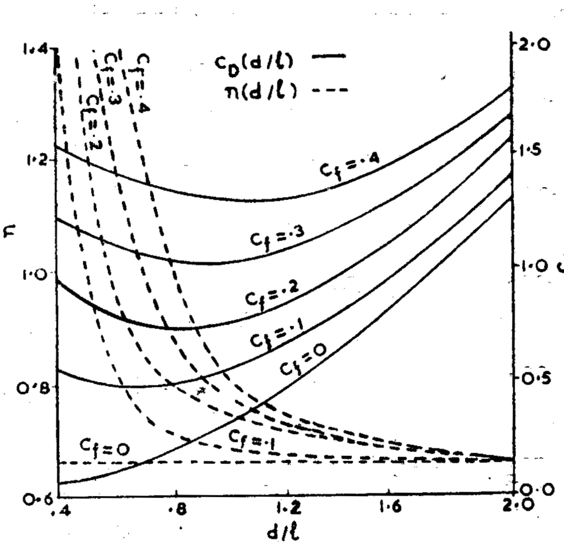

Fig. 2—n and C_D versus d/l for given values of C_f

(Three-dimensional body)

Example 2—Given the diameter d and the value of P.

The following values of l and Q are obtained from (7) and (8) in terms of n, d and P:

$$l = \frac{P(n+1)}{\pi^{\alpha} d}, \quad Q = \frac{d}{2} \; \frac{P(n+1)}{(2n+1)} \tag{13}$$

Therefore from (6) we deduce that

$$\frac{D}{4\pi^{\alpha} q} = \pi^{2\alpha} \left(\frac{d}{2}\right)^{\alpha+3} \left(\frac{d}{P}\right)^{2} \frac{n^3(\alpha+2)-n^2}{(n+1)^2(n\alpha+3n-2)(\alpha+1)} + \frac{C_f}{2}\left(\frac{d}{2}\right)^{\alpha} \frac{P}{Q\,\pi^{\alpha}(n\alpha+1)} \tag{14}$$

In order that drag be minimum $dD/dn = 0$, i.e.

$$\frac{n^3(2\alpha^2+9\alpha+11)-n^2(7\alpha+15)+4n}{(\alpha+1)(n+1)^3(n\alpha+3n-2)^2} - 4\,C_f \, \frac{P^3(\alpha-1)}{\pi^{2\alpha}(n\alpha+1)^2 d^6} = 0 \tag{15}$$

Also

$$C_D = \frac{\pi^{2\alpha} d^4}{P^2} \; \frac{n^3(\alpha+2)-n^2}{(n\alpha+3n-2)(\alpha+1)(n+1)^2} + 4C_f \, \frac{P(n+1)}{(n\alpha+1)\, d^2 \pi^{\alpha}} \tag{16}$$

Case (a): when $\alpha = 0$ (Two-dimensional body)

In this case the optimum value of n is given by

$$11n^3 - 15n^2 + 4n + 4C_f (n+1)^3 (3n-2)^2 (A/d^2)^3 = 0 \tag{17}$$

Also from (13) and (16), we have the values of the following quantities:

Length $\quad l = \dfrac{A(n+1)}{d}$

Moment of Inertia $\quad M = \dfrac{d}{2} \; \dfrac{A(n+1)}{(2n+1)}$

Drag Coefficient $\quad C_D = \dfrac{n^2(2n-1)}{\left(\dfrac{A}{d^2}\right)^2 (n+1)^2 (3n-2)} + 4C_f (n+1) \dfrac{A}{d^2}$

Fig. 3 represents the relationships $n\,(A/d^2)$ and $C_D\,(A/d^2)$ for given values of C_f.

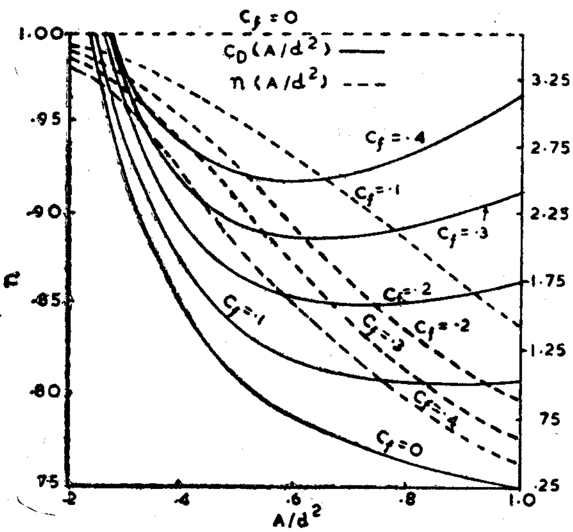

Fig. 3—n and C_D versus A/d^2 for known values of C_f

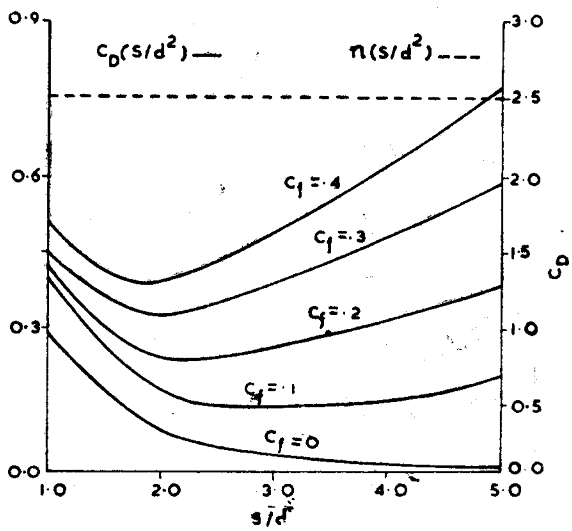

Fig. 4—n and C_D versus S/d^2 for known values of C_f

Case (*b*) : when $\alpha = 1$ (Axisymmetric body)

In this case from (15) we obtain $n = 0{\cdot}7611$. Also from (13) and (16), we have

Length $\qquad l = \dfrac{S\,(n+1)}{\pi d}$

Volume $\qquad V = \dfrac{Sd\,(n+1)}{4\,(2n+1)}$

Drag Coefficient $\qquad C_D = \dfrac{\pi^2 n^2\,(3n-1)}{4(S/d^2)^2\,(2n-1)\,(n+1)^2} + \dfrac{4C_f}{\pi}\left(\dfrac{S}{d^2}\right)$

The relation $C_D\,(S/d^2)$ and $n\,(S/d^2)$ are drawn in Fig. 4 for given values of C_f.

Example 3—Given the length l and the value of P.

In this case, we can obtain from (7) and (8) the following values of the unknown quantities d and Q in terms of l, P and n:

$$d = \frac{P\,(n+1)}{\pi^\alpha\, l}\,,\; Q = \frac{P^2\,(n+1)^2}{2\pi^\alpha\, l\,(2n+1)} \tag{18}$$

Also from (6) and (18), the drag is given by

$$\frac{D}{4\pi^\alpha\, q} = \left(\frac{P}{2\pi^\alpha}\right)^{\alpha+3} \frac{(n+1)^{\alpha+1}\,\{n^3\,(\alpha+2) - n^2\}}{(\alpha n + 3n - 2)\,(\alpha+1)} + \frac{C_f}{2}\left(\frac{P}{2\pi^\alpha}\right)^{\alpha} \frac{(n+1)^{\alpha}}{l^{\alpha-1}} \tag{19}$$

For drag to be minimum $dD/dn = 0$, i.e.

$$\frac{n^4\,(\alpha^3+10\,\alpha^2+31\,\alpha+30) - n^3\,(\alpha^2+13\,\alpha+24) - 5\,n^2\,(\alpha+1) + 4n}{(\alpha+1)\,(n\alpha+3\,n-2)^2} + 4\,C_f\,\frac{n\alpha\,(\alpha-1)\,\pi^{3\alpha}\,l^6}{(n\,\alpha+1)^2\,(n+1)^3\,P^3} = 0 \tag{20}$$

Also the drag coefficient is expressed by

$$C_D = \frac{P^2}{\pi^{2\alpha}\,l^4}\,\frac{(n+1)^2\,\{n^3\,(\alpha+2) - n^2\}}{(n\alpha+3n-2)\,(\alpha+1)} + 4\,C_f\,\frac{\pi^\alpha\,l^2}{P\,(n\,\alpha+1)\,(n+1)} \tag{21}$$

Case (*a*) : when $\alpha = 0$ (Two-dimensional body)

In this case the optimum body profile is obtained for $n = 0{\cdot}8000$. Also from (18) and (21) the values of the following quantities are known.

Diameter $\qquad d = \dfrac{A\,(n+1)}{l}$

Moment of Inertia $\quad M = \dfrac{A^2\,(n+1)^2}{2l(2n+1)}$

Drag Coefficient $\qquad C_D = \dfrac{n^2\,(2n-1)\,(n+1)^2}{\left(\dfrac{l^2}{A}\right)^2\,(3n-2)} + 4\,C_f\,\dfrac{l^2}{A}\;\dfrac{1}{(n+1)}$

Fig. 5 represents the relationships $C_D\,(l^2/A)$ and $n\,(l^2/A)$ for different values of C_f.

Case (*b*) : when $\alpha = 1$ (Axisymmetric body)

Here the optimum value of $n = 0{\cdot}6056$. Also from (18) and (21), we obtain

Diameter $\qquad d = \dfrac{S\,(n+1)}{\pi l}$

Volume $\quad V = \dfrac{S^2\,(n+1)}{4\pi l\,(2n+1)}$

Drag Coefficient $\quad C_D = \dfrac{n^2\,(3n-1)\,(n+1)^2}{4\pi^2\,(l^2/S)^2\,(2n-1)} + 4\pi C_f \left(\dfrac{l^2}{S}\right) \dfrac{1}{(n+1)^2}$

Fig. 6 shows the relationships $C_D\,(l^2/S)$ and $n\,(l^2/S)$ for specified values of C_f.

Example 4—Given the diameter d and the value of Q.

From (7) and (8) we deduce the following value of l and P in terms of n and the two given quantities d and Q as

$$l = \frac{2Q\,(2n+1)}{\pi^{\alpha}\,d^2}, \quad P = \frac{2Q\,(2n+1)}{d\,(n+1)} \tag{22}$$

Therefore from (6), we obtain

$$\frac{D}{4\pi^{\alpha} q} = \frac{d^{\alpha+7}\,\pi^{2\alpha}}{2^{\alpha+5}\,Q^3}\;\frac{n^3\,(\alpha+2)-n^2}{(2n+1)^2\,(n\alpha+3n-2)\,(\alpha+1)} + C_f\,\frac{d^{\alpha-2}\,Q}{2^{\alpha}\,\pi^{\alpha}}\;\frac{(2n+1)}{(n\alpha+1)}$$

For the drag to be minimum $dD/dn=0$, i.e.

$$\frac{2n^3\,(\alpha^2+4\alpha+5)-n^2\,(7\alpha+15)+4n}{(\alpha+1)\,(2n+1)^3\,(n\alpha+3n-2)^2} - 32C_f\,\frac{Q^3}{d^9} \times \frac{(\alpha-2)}{\pi^{3\alpha}\,(n\alpha+1)} = 0 \tag{23}$$

Here the drag coefficient is expressed by

$$C_D = \frac{\pi^{2\alpha} d^6}{4Q^2} \times \frac{n^3\,(\alpha+2)-n^2}{(n\alpha+3n-2)\,(\alpha+1)\,(2n+1)^2} + 8C_f\,\frac{Q}{\pi^{\alpha}\,a^3}\;\frac{(2n+1)}{(n^{\alpha}+1)} \tag{24}$$

Case (*a*) : when $\alpha=0$ (Two-dimensional body)

In this case the optimum value of n is obtained from

$$10n^3 - 15n^2 + 4n + 64C_f\,(2n+1)^3\,(3n-2)^2\,(M/d^3)^3 = 0 \tag{25}$$

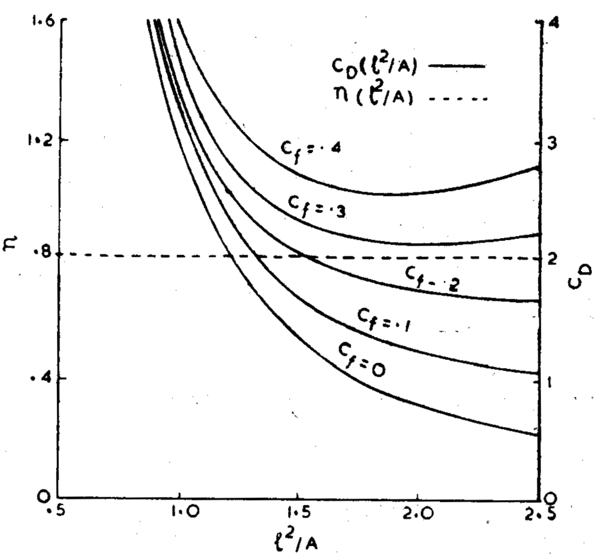

Fig. 5—n and C_D versus l^2/A for given values of Cf

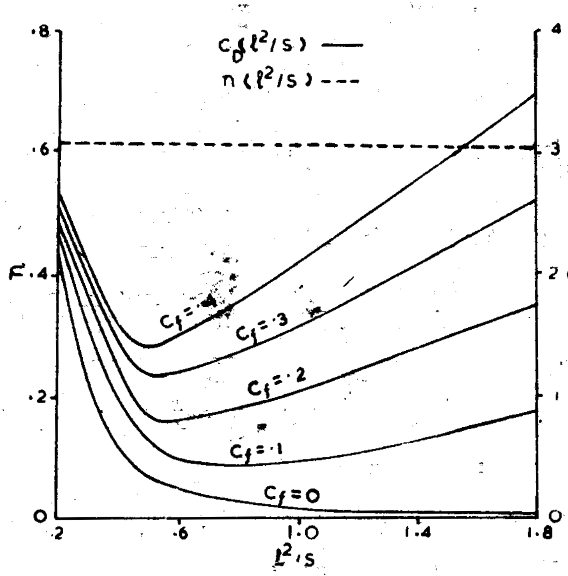

Fig. 6—n and C_D versus l^2/S for given value of C_f

After having calculated the value of n, we can calculate the following quantities

Length $$l = \frac{2M\,(2n+1)}{d^2}$$

Enclosed Area $$A = \frac{2M\,(2n+1)}{d\,(n+1)}$$

Drag Coefficient $$C_D = \frac{n^2\,(2n-1)}{4\left(\frac{M}{d^3}\right)^2 (2n+1)^2 (3n-2)} + 8C_f(2n+1)\left(\frac{M}{d^3}\right)$$

The relations $n(M/d^3)$ and $C_D\,(M/d^3)$ for known values of C_f are illustrated in Fig 7.

Case (*b*) : when $\alpha=1$ (Axisymmetric body)

In this case the required value of optimum n is given by

$$n(n+1)^2\,(10n^2-11n+2)\pi^3+1024C_f\,(2n+1)^3\,(2n-1)^2\,(V/d^3)^3 = 0 \tag{26}$$

Knowing n we can easily calculate the following quantities

Length $$l = \frac{2V(2n+1)}{\pi d^2}$$

Surface Area $$S = \frac{4V\,(2n+1)}{\pi d\,(n+1)}$$

Drag Coefficient $$C_D = \frac{n^2\,(3n-1)\,\pi^2}{64\,(V/^3d)^2\,(2n-1)\,(2n+1)^2} + 16\,C_f\,\frac{(2\,n+1)}{\pi\,(n+1)}\,\frac{V}{d^3}$$

Fig. 8 gives the relationships $n(V/d^3)$ and $C_D\,(V/d^3)$ for given values of C_f.

Example 5—Given the length l and the value of Q. Making use of (7) and (8) the values of d and P in terms of n and the given quantities l and Q are

$$d = 2\left[\frac{Q\,(2n+1)}{2\pi^{\alpha}l}\right]^{\frac{1}{2}},\quad P = \frac{2\pi^{\alpha}l}{(n+1)}\left[\frac{Q\,(2n+1)}{2\pi^{\alpha}l}\right]^{\frac{1}{2}} \tag{27}$$

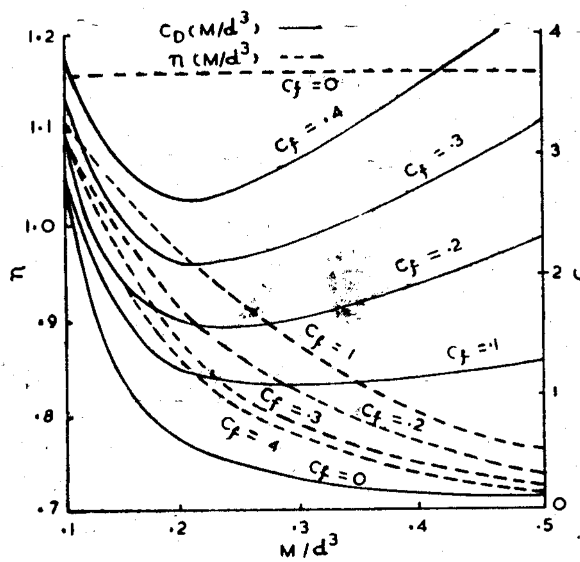

Fig. 7—n and C_D versus M/d^3 for various values of C_f

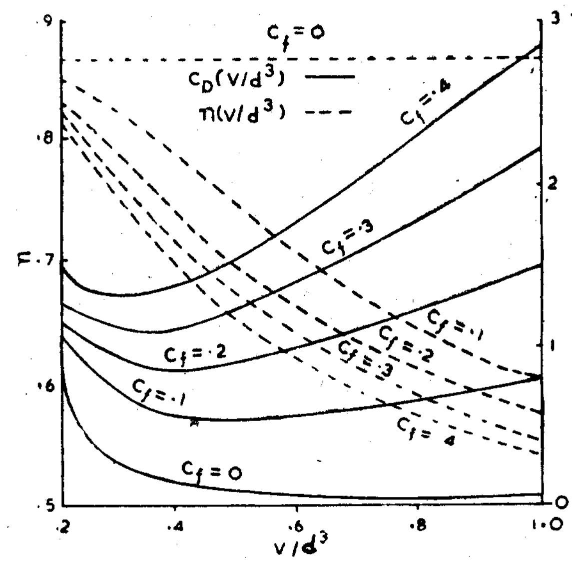

Fig. 8—n and C_D versus V$/d^3$ for various value of C_f

With the aid of (6), we obtain

$$\frac{D}{4\pi^{\alpha} q}=\left[\frac{Q\,(2n+1)}{2\pi^{\alpha} l}\right]^{\frac{\alpha+3}{2}} \frac{1}{l^2}\ \frac{n^3\,(\alpha+2)-n^2}{(n\alpha+3n-2)\,(\alpha+1)}+\frac{C_f}{2}\left[\frac{Q\,(2n+1)}{2\pi^{\alpha} l}\right]^{\frac{\alpha}{2}} \frac{l}{(n\alpha+1)}$$

If the drag is to be minimum, $dD/dn=0$, i.e.

$$\frac{n^4\,(\alpha^3+12\alpha^2+41\alpha+42)-n^3\,(\alpha^2+20\alpha+39)-n^2\,(5\alpha+1)+4n}{(\alpha+1)\,(n\alpha+3n-2)^2}$$

$$-\frac{C_f}{2}\left[\frac{2\pi^{\alpha}\,l^3}{Q\,(2n+1)}\right]^{3/2} \frac{n\alpha\ (2-\alpha)}{(n\alpha+1)^2}=0 \qquad (28)$$

Also the drag coefficient is given by

$$C_D=\frac{2Q}{\pi^{\alpha}\,l^3}\ \frac{(2n+1)\,\{n^3\,(\alpha+2)-n^2\}}{(n\alpha+3n-2)\ (\alpha+1)}+\frac{2\,C_f}{(n\alpha+1)}\left[\frac{2\pi^{\alpha}\,l^3}{Q\,(2n+1)}\right]^{\frac{1}{2}} \qquad (29)$$

Case (*a*) : when $\alpha=0$ (Two-dimensional body)

From (28) we deduce that $n=0\cdot 8141$. It is now easy to calculate from (27) and (29) the following quantities for the optimum body

Diameter $\quad d=\left[\dfrac{2M\ (2n+1)}{l}\right]^{1/2}$

Enclosed Area $\quad A=\dfrac{l}{(n+1)}\left[\dfrac{2M\ \ (2n+1)}{l}\right]^{1/2}$

Drag Coefficient $C_D=\dfrac{2n^2\ (2n-1)\ (2n+1)}{(l^3/M)\ \ (3n-2)}+2C_f\sqrt{\dfrac{2\,l^3}{M\,(2\,n+1)}}$

The relationships $C_D\ (l^3/M)$ and $n(l^3/M)$ are in Fig. 9.

Case (*b*) : when $\alpha=1$ (Axisymmetric body)

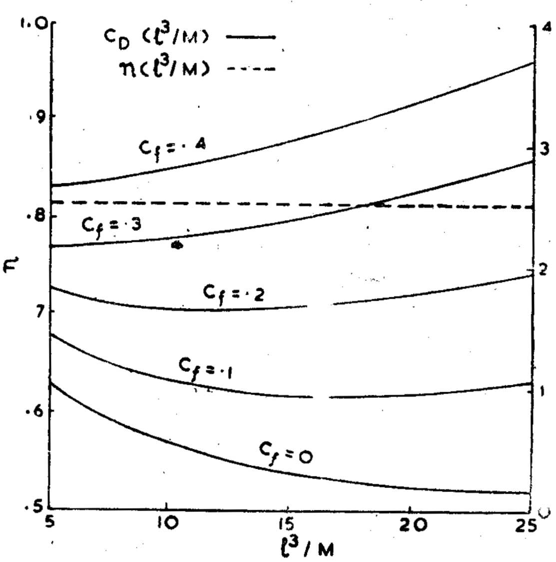

Fig. 9—n and C_D versus l^3/M for known values of C_f

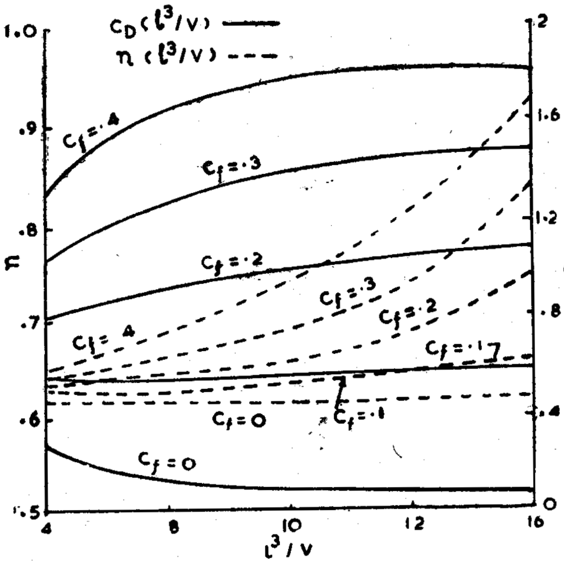

Fig. 10—n and C_D versus l^3/V for known values of C_f

From (28) we deduce that the required value of the exponent n for the minimum drag body is given by

$$(2n+1)^{3/2}(n+1)^2(48n^3-30n^2-3n+2)-2\pi^{3/2}C_f(2n-1)^2(l^3/V)^{3/2}=0 \tag{30}$$

Having calculated the value of n, we can calculate the following quantities for the optimum body

Diameter $\quad d=\left[\dfrac{4V(2n+1)}{\pi l}\right]^{\frac{1}{2}}$

Surface Area $\quad S=\dfrac{1}{(n+1)}\left[4\pi l V(2n+1)\right]^{\frac{1}{2}}$

Drag Coefficient $\quad C_D=\dfrac{n^2(2n+1)(3n-1)}{\pi(2n-1)(l^3/V)}+\dfrac{2C_f}{(n+1)}\sqrt{\dfrac{\pi}{2n+1}\dfrac{l^3}{V}}$

The relationships $n\,(l^3/V)$ and $C_D\,(l^3/V)$ for the known values of C_f are shown in Fig. 10.

Example 6: Given the values of P and Q.

In this case making use of (7) and (8) the unknown quantities l and d in terms of n and the known quantities P and Q may be deduced as

$$l=\frac{P^2(n+1)^2}{2\pi^{\alpha}Q(2n+1)},\quad d=\frac{2Q}{P}\frac{(2n+1)}{(n+1)} \tag{31}$$

Making use of (6), we have

$$\frac{D}{4\pi^{\alpha}q}=\frac{4Q^{\alpha+5}\pi}{P^{\alpha+7}}\frac{(2n+1)^{\alpha+5}\{n^3(\alpha+2)-n^2\}}{(n+1)^{\alpha+7}(n\alpha+3n-2)(\alpha+1)}+\frac{C_f}{4}\frac{Q^{\alpha-1}}{P^{\alpha-2}}\frac{(n+1)^{2-\alpha}}{\pi^{\alpha}(2n+1)^{1-\alpha}(n\alpha+1)}$$

For drag to be minimum $dD/dn=0$, i.e.

$$\frac{n^4(\alpha^3+14\alpha^2+49\alpha+52)-n^3(\alpha^2+27\alpha+54)-n^2(5\alpha-3)+4n}{(\alpha+1)(n\alpha+3n-2)^2}$$
$$-\frac{C_f}{16}\frac{P^9(n+1)^9(3n\alpha-2n-n\alpha^2)}{Q^6\pi^{3\alpha}(2n+1)^6(n\alpha+1)^2}=0 \tag{32}$$

Here the drag coefficient of the body is given by

$$C_f=\frac{16Q^4\pi^{2\alpha}}{P^6}\frac{(2n+1)^4\{n^3(\alpha+2)-n^2\}}{(n+1)^6(n\alpha+3n-2)(\alpha+1)}+C_f\frac{P^3}{Q^2}\frac{(n+1)^3}{\pi^2(n\alpha+1)(2n+1)^2} \tag{33}$$

Case (*a*) : when $\alpha=0$ (Two-dimensional body)

In this case the relation (32) reduces to

$$8(2n+1)^6(52n^3-54n^2+3n+4)+C_f(n+1)^9(3n-2)^2(A^3/M^2)^3=0 \tag{34}$$

This gives the required value of n. Having known n we can obtain the following quantities of the optimum body.

Length $\quad l=\dfrac{A^2(n+1)^2}{2M(2n+1)}$

Diameter $\quad d=\dfrac{2M(2n+1)}{A(n+1)}$

Drag Coefficient $\quad C_f=\dfrac{16n^2(2n+1)^4(2n-1)}{(A^3/M^2)^2(n+1)^6(3n-2)}+C_f\dfrac{(n+1)^3}{(2n+1)^3}\dfrac{A^3}{M^2}$

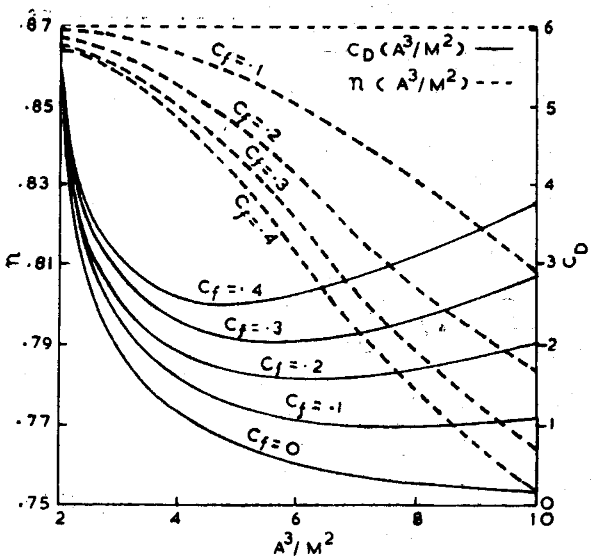

Fig. 11—n and C_D versus A^3/M^2 for given values of C_f

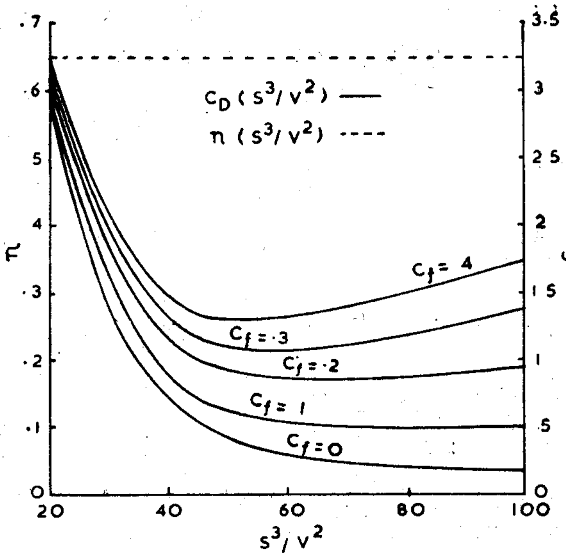

Fig. 12—n and C_D versus S^3/V^2 for given values of C_f

The relationships $n\ (A^3/M^2)$ and $C_D\ (A^3/M^2)$ for known values of C_f are illustrated in Fig. 11.

Case (*b*) : when $\alpha = 1$ (Axisymmetric body).

In this case from (32), we see that $n = 0{\cdot}6523$. We can therefore obtain the following quantities of the optimum body.

Length $$l = \frac{S^2 (n+1)^2}{4\pi V (2n+1)}$$

Diameter $$d = \frac{4V\ (2n+1)}{S\ (n+1)}$$

Drag coefficient $$C_D = \frac{64\,\pi^2 (2n+1)^4 n^2 (3n-1)}{(S^3/V^2)^2 (n+1)^6 (2n-1)} + \frac{C_f}{4\pi}\ \frac{S^3}{V^2}\ \frac{(n+1)^2}{(2n+1)^2}$$

Fig. 12 represents the relationship $C_D\ (S^3/V^2)$ and $n\ (S^3/V^2)$ for given values of C_f.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Dr. R. R. Aggarwal, P.Sc.O. for his encouragement in the preparation of this paper. Their thanks are also due to Dr. P. L. Kapur, the then Director, Defence Science Laboratory, Delhi for his permission to publish this work.

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