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Flow of a Dusty Gas Between Two Parallel Plates One Stationary and Other Oscillating

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Abstract. The solution for the flow of an incompressible viscous dusty gas, induced by two infinitely extended parallel plates when the lower plate is at rest and the upper plate begins to oscillate harmonically in its own plane, is obtained, It is found that (i) with the increase in the mass concentration both the velocities of the dusty gas and the particle decrease, (ii) the velocity of the dusty gas increases and that of the particle decreases with the increase in σ.

1. Introduction

Interest in problems of mechanics of systems with more than one phase has developed rapidly during the past few years. Such situations arise, for instance, in the movement of dust laden air, in fluidization, in the use of dust in gas cooling systems to enhance heat transfer processes, in environmental pollution, in hydrocyclones etc. Saffman[1] formulated the basic equations for the flow of a dusty gas. Since then there have been several papers in this field, and are well documented in a review by Marble[2].

Studies of the unsteady problems involved with helicopter rotors (such as oscillating cross-flow over the rotors, the time dependent oscillating flow over the airfoils etc) are of much interest for minimising the unsteady effects of the environment. The problem of laminar flow between parallel plates is of interest as it approximates to the flows that are commonly encountered in engineering and technological practices. It is well-known that the flow over a flat plate is a drastic simplification of the flow over a helicopter wing. To have an insight in such problems, we have considered the laminar unsteady flow of a dusty viscous incompressible gas between two infinitely extended parallel plates when the lower plate is at rest and the upper one starts oscillating harmonically in its own plane. The velocity fields for the dusty gas, dust particle and the clean gas are worked out using the technique of Laplace transform. The expressions for the drag on the lower fixed plate for the dusty and the clean gas are obtained. Initially, both the gas and the dust particles are assumed to be at rest.

Results obtained for two different time periods varying both l and σ are presented graphically. Finally, the drags on the lower fixed plate due to the oscillation of the upper plate have been calculated for the same set of values of T, l and σ, as done to obtain the velocity distributions. All the numerical calculations have been performed on EC — 1033 Computer and the results are discussed.

2. Fundamental Equations of Motion

The unsteady equations of motion of a dusty, viscous, incompressible gas[1] are

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\,\text{grad}\, p + \nu\, \nabla^2 \vec{u} + \frac{KN}{\rho}(\vec{v} - \vec{u}) \tag{1}$$

$$\text{Div}\, \vec{u} = 0 \tag{2}$$

$$m\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}\right] = K(\vec{u} - \vec{v}) \tag{3}$$

$$\frac{\partial N}{\partial t} + \text{div}\,(N\vec{v}) = 0 \tag{4}$$

where $\vec{u}$ and $\vec{v}$ denote the local velocity vectors of the gas and dust particle respectively, ρ the density of the gas, p the static gas pressure, ν the kinematic viscosity, N the number density of the dust particles, k the Stokes' resistance co-efficient (for spherical particle of radius a, it is $6\pi\mu a$), μ the gas viscosity and m the mass of a dust particle.

3. The Problem and its Solution

We assume the dusty gas to be confined between two infinitely extended parallel plates situated at $y = 0$ and at $y = h$. x-axis is taken along the lower fixed plate and y-axis is measured normal to it. The upper plate starts performing harmonic oscillation in its own plane. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as constant throughout the flow and let it be N_0. Under these assumptions, the flow will be a parallel flow in which the streamlines are parallel to x-axis and the velocities are functions of the distance from the lower plate and the time. Then the Eqns. (1) and (3) reduce to

$$\frac{\partial u}{\partial t} = \nu \cdot \frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(v - u) \tag{5}$$

$$m \cdot \frac{\partial v}{\partial t} = k(u - v) \tag{6}$$

where u and v represent the gas and dust velocity, respectively.

Let us reduce the Eqns. (5) and (6) to non-dimensional forms by putting

$$y^* = \frac{y}{h}, \quad t^* = \frac{\nu t}{h^2}, \quad u^* = \frac{uh}{\nu}, \quad v^* = \frac{vh}{\nu}.$$

Then, omitting the asterisks, the Eqns. (5) and (6) respectively reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}(v - u), \tag{7}$$

and

$$\sigma \cdot \frac{\partial v}{\partial t} = (u - v) \tag{8}$$

where $l \left(= \frac{mN_0}{\rho} \right)$ is the mass concentration of dust particles and $\sigma \left(= \frac{m\nu}{kh^2} \right)$ is the relaxation time parameter. Equations (7) and (8) are to be solved under the conditions

$$\left.\begin{aligned} &u = v = 0 \text{ for } 0 \leqslant y \leqslant 1, (t \leqslant 0) \\ &u = 0 \text{ at } y = 0 \\ &\qquad\qquad (t > 0) \\ &u = v \cdot e^{i\omega t} \text{ at } y = 1 \end{aligned}\right\} \tag{9}$$

where real part is being considered.

4. Solution of the Equations

Let $\bar{u} = \int_0^\infty u \cdot e^{-st}\, dt$ and $\bar{v} = \int_0^\infty v \cdot e^{-st}\, dt (Re\, s > 0)$ be respectively the Laplace transforms of u and v. Applying these to Eqns. (7) and (8) and using Eqn. (9) we get,

$$\frac{d^2 \bar{u}}{dy^2} - p^2 \bar{u} = 0 \tag{10}$$

where

$$p^2 = \frac{s(1 + l + \sigma s)}{1 + \sigma s} \tag{11}$$

and

$$\bar{v} = \frac{1}{1 + \sigma s} \cdot \bar{u} \tag{12}$$

The boundary conditions are

$$\left.\begin{aligned} &\bar{u} = 0 \text{ at } y = 0 \\ &\bar{u} = \frac{v}{s - i\omega} \text{ at } y = 1 \end{aligned}\right\} \tag{13}$$

Applying these boundary conditions, the solution of Eqn. (10) is obtained as

$$\bar{u} = \frac{V}{s - i\omega} \cdot \frac{\sinh py}{\sinh p} \tag{14}$$

and so from Eqn. (12) we get

$$\bar{v} = \frac{V}{(1 + \sigma s)(s - i\omega)} \cdot \frac{\sinh py}{\sinh p} \tag{15}$$

Then by the inversion theorem (Carslaw and Jaegar[3]), we get from Eqn. (14),

$$u = \frac{V}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\lambda t} \cdot \frac{1}{(\lambda - i\omega)} \cdot \frac{\sinh \mu y}{\sinh \mu} \cdot d\lambda \tag{16}$$

where

$$\mu^2 = \frac{\lambda(1 + l + \sigma\lambda)}{1 + \sigma\lambda}. \tag{17}$$

Then applying the calculus of residue, we get from Eqn. (16)

$$u = V \cdot e^{i\omega t} \cdot \frac{\sinh\left[\sqrt{\frac{i\omega(1 + l + i\sigma\omega)}{1 + i\sigma\omega}} \cdot y\right]}{\sinh\left[\sqrt{\frac{i\omega(1 + l + i\sigma\omega)}{1 + i\sigma\omega}}\right]}$$

$$- 2\pi V \sum_{n=1}^{\infty} \frac{e^{\lambda_1 t}}{(\lambda_1 - i\omega)} \cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1 + \sigma\lambda_1)^2}{-\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} + \sigma^2\lambda_1^2}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{e^{\lambda_2 t}}{(\lambda_2 - i\omega)} \cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1 + \sigma\lambda_2)^2}{\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} - \sigma^2\lambda_2^2} \tag{18}$$

where

$$\left.\begin{aligned} 2\sigma\lambda_1 &= -[(1 + l) + \sigma n^2\pi^2] + \sqrt{\{(1 + l) + \sigma n^2\pi^2\}^2 - 4\sigma n^2\pi^2} \\ \text{and} \quad 2\sigma\lambda_2 &= -[(1 + l) + \sigma n^2\pi^2] - \sqrt{\{(1 + l) + \sigma n^2\pi^2\}^2 \, 4\sigma n^2\pi^2} \end{aligned}\right\} \tag{19}$$

Therefore, applying the convolution theorem, we get from Eqn. (12)

$$v = \frac{V}{(1 + \sigma^2\omega^2)} [\{\cos \omega t + \sigma\omega \cdot \sin \omega t - e^{-t/\sigma}\} + i \{\sin \omega t$$

$$- \sigma\omega(\cos \omega t - e^{-t/\sigma})\}] \cdot (A_i + iB_1)$$

$$- 2\pi V \sum_{n=1}^{\infty} \frac{(e^{\lambda_1 t} - e^{-t/\sigma})}{\lambda_1 - i\omega} \cdot$$

$$\cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1 + \sigma\lambda_1)}{-\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 + \sigma^2\lambda_1^2}}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{(e^{\lambda_2 t} - e^{-t/\sigma})}{\lambda_2 - i\omega} \cdot$$

$$\cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1 + \sigma\lambda_2)}{\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 - \sigma^2\lambda_2^2}} \tag{20}$$

Collecting the real parts of Eqn. (18), we get

$$u = V(A_1 \cos \omega t - B_1 \sin \omega t)$$

$$- 2\pi V \sum_{n=1}^{\infty} \frac{\lambda_1 \cdot e^{\lambda_1 t}}{\lambda_1^2 + \omega^2} \cdot \frac{n(-1)^n \sin(n\pi y) \cdot (1+\sigma\lambda_1)^2}{-\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 + \sigma^2\lambda_1^2}}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{\lambda_2 \cdot e^{\lambda_2 t}}{\lambda_2^2 + \omega^2} \cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1+\sigma\lambda_2)^2}{\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 - \sigma^2\lambda_2^2}} \tag{21}$$

where

$$\left.\begin{aligned} A_1 &= \frac{\cos(y\mu_1) \cdot \sinh(y\mu_2) \cdot \cos\mu_1 \cdot \sinh\mu_2 + \sin(y\mu_1) \cdot \cosh(y\mu_2) \cdot \sin\mu_1 \cosh\mu_2}{\cos^2\mu_1 \sinh^2\mu_2 + \sin^2\mu_1 \cdot \cosh^2\mu_2} \\ B_1 &= \frac{\sin(y\mu_1) \cdot \cosh(y\mu_2) \cdot \cos\mu_1 \sinh\mu_2 - \cos(y\mu_1) \cdot \sinh(y\mu_2) \cdot \sin\mu_1 \cdot \cosh\mu_2}{\cos^2\mu_1 \cdot \sinh^2\mu_2 + \sin^2\mu_1 \cosh^2\mu_2} \end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned} \mu_1 &= \left(\frac{\sqrt{A^2 + B^2} - A}{2}\right)^{1/2} \\ \mu_2 &= \left(\frac{\sqrt{A^2 + B^2} + A}{2}\right)^{1/2} \end{aligned}\right\} \tag{23}$$

and

$$\left.\begin{aligned} A &= \frac{\sigma\omega^2 l}{1 + \sigma^2\omega^2} \\ B &= \frac{\omega(1 + l + \sigma^2\omega^2)}{1 + \sigma^2\omega^2} \end{aligned}\right\} \tag{24}$$

Again, collecting the real parts of Eqn. (20), we get

$$v = \frac{V}{(1 + \sigma^2\omega^2)} \cdot \left[\left(A_1 + B_1\sigma\omega\right) \cdot \cos \omega t - (B_1 - A_1\sigma\omega) \cdot \sin \omega t - (A_1 + B_1\sigma\omega) \cdot e^{-t/\sigma}\right] - 2\pi V \sum_{n=1}^{\infty} \frac{\lambda_1(e^{\lambda_1 t} - e^{-t/\sigma})}{\lambda_1^2 + \omega^2} \cdot \frac{n(-1)^n \cdot \sin (n\pi y) \cdot (1 + \sigma\lambda_1)}{-\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} + \sigma^2\lambda_1^2}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{\lambda_2(e^{\lambda_2 t} - e^{-t/\sigma})}{\lambda_2^2 + \omega^2} \cdot \frac{n(-1)^n \cdot \sin (n\pi y) \cdot (1 + \sigma\lambda_2)}{\sigma n^2\pi^2 + \sqrt{(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} - \sigma^2\lambda_2^2}. \tag{25}$$

Now, for a clean gas, where $l = 0$, the velocity of the gas is similarly obtained as

$$u_c = V(A_2 \cos \omega t - B_2 \sin \omega t) + 2\pi^3 V \sum_{n=1}^{\infty} n^3 \cdot (-1)^n \cdot \frac{\sin (n\pi y) \cdot e^{-n^2\pi^2 t}}{(n^4\pi^4 + \omega^2}, \tag{26}$$

where

$$\left.\begin{aligned} A_2 &= \frac{\cos (\alpha y) \cdot \sinh (\alpha y) \cdot \cos \alpha \cdot \sinh \alpha + \sin (\alpha y) \cdot \cosh (\alpha y) \cdot \sin (\alpha) \cdot \cosh \alpha,}{\cos^2 \alpha \cdot \sinh^2 \alpha + \sin^2 \alpha \cdot \cosh^2 \alpha} \\ B_2 &= \frac{\sin (\alpha y) \cdot \cosh (\alpha y) \cdot \cos \alpha \cdot \sinh \alpha - \cos (\alpha y) \cdot \sinh (\alpha y) \cdot \sin \alpha \cdot \cosh \alpha}{\cos^2 \alpha \cdot \sinh^2 \alpha + \sin^2 \alpha \cdot \cosh^2 \alpha} \end{aligned}\right\} \tag{27}$$

and $\alpha = \sqrt{\frac{\omega}{2}}$.

Now, as $t \to \infty$, $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ both vanish, for λ_1 and λ_2 are the roots of the quadratic equation

$$\sigma\lambda^2 + \lambda [(1 + l) + \sigma n^2\pi^2] + n^2\pi^2 = 0 \tag{28}$$

with no change of sign and so λ_1 and λ_2 are negative. Thus, there is no contribution to the velocities in Eqns. (21), (25) and (26) from the summation terms. Hence, the velocity fields for the asymptotic case $t \to \infty$ are

$$u = V(A_1 \cos \omega t - B_1 \sin \omega t) \tag{29}$$

$$v = \frac{V}{1 + \sigma^2\omega^2}\left[(A_1 + B_1\sigma\omega) \cdot \cos \omega t - (B_1 - A_1\sigma\omega) \cdot \sin \omega t\right], \tag{30}$$

and

$$u_c = V(A_2 \cos \omega t - B_2 \sin \omega t) \tag{31}$$

5. Particular Case

Let us now discuss the particular case when the upper plate begins to move with a constant velocity i.e. $\omega = 0$. Therefore, from Eqns. (21) and (25), the gas and the dust velocities are respectively obtained as

$$u = Vy - 2\pi V \sum_{n=1}^{\infty} \frac{e^{\lambda_1 t}}{\lambda_1} \cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1+\sigma\lambda_1)^2}{-\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} + \sigma^2\lambda_1^2}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{e^{\lambda_2 t}}{\lambda_2} \cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1+\sigma\lambda_2)^2}{\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} - \sigma^2\lambda_2^2} \tag{32}$$

and

$$v = Vy(1 - e^{-t/\sigma}) - 2\pi V \sum_{n=1}^{\infty} \frac{(e^{\lambda_1 t} - e^{-t/\sigma})}{\lambda_1} \cdot$$

$$\cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1+\sigma\lambda_1)}{-\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} - \sigma^2\lambda_1^2}$$

$$+ 2\pi V \sum_{n=1}^{\infty} \frac{(e^{\lambda_2 t} - e^{-t/\sigma})}{\lambda_2} \cdot$$

$$\cdot \frac{n(-1)^n \cdot \sin(n\pi y) \cdot (1+\sigma\lambda_2)}{\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2} - \sigma^2\lambda_2^2} \tag{33}$$

Thus, the exact solutions for the Couette motion of dusty gas are obtained.

6. Calculation of Drag

The expression for the skin-friction drag per unit length on the lower fixed plate $y = 0$ due to the dusty gas is (by Eqn. (21))

$$D_d = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu V \left[\frac{\mu_2 \cos\mu_1 \cdot \sinh\mu_2 + \mu_1 \sin\mu_1 \cosh\mu_2}{\cos^2\mu_1 \cdot \sinh^2\mu_2 + \sin^2\mu_1 \cosh^2\mu_2} \cdot \cos\omega t\right.$$

$$\left. - \frac{\mu_1 \cos\mu_1 \cdot \sinh\mu_2 - \mu_2 \sin\mu_1 \cosh\mu_2}{\cos^2\mu_1 \sinh^2\mu_2 + \sin^2\mu_1 \cosh^2\mu_2} \cdot \sin\omega t\right]$$

$$- 2\mu\pi^2 V \sum_{n=1}^{\infty} \frac{\lambda_1 \cdot e^{\lambda_1 t}}{\lambda_1^2 + \omega^2} \cdot$$

(*equation continued on p.* 218)

$$\cdot \frac{n^2 \cdot (-1)^n \cdot (1+\sigma\lambda_1)^2}{-\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 + \sigma^2\lambda_1^2}}$$

$$+ 2\mu\pi^2 V \sum_{n=1}^{\infty} \frac{\lambda_2 \cdot e^{\lambda_2 t}}{\lambda_2^2 + \omega^2} \cdot \frac{n^2 \cdot (-1)^n \cdot (1+\sigma\lambda_2)^2}{\sigma n^2\pi^2 + \sqrt{(1+l+\sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2 - \sigma^2\lambda_2^2}} \tag{34}$$

Similarly, using Eqn. (26) the skin-friction drag per unit length on the lower fixed plate due to the clean gas is obtained as

$$D_c = \mu V\alpha \left[\frac{\sinh\alpha \cdot \cos\alpha + \cosh\alpha \cdot \sin\alpha}{\sinh^2\alpha \cdot \cos^2\alpha + \cosh^2\alpha \cdot \sin^2\alpha} \times \cos\omega t \right.$$

$$\left. - \frac{\sinh\alpha \cdot \cos\alpha - \cosh\alpha \cdot \sin\alpha}{\sinh^2\alpha \cdot \cos^2\alpha + \cosh^2\alpha \cdot \sin^2\alpha} \cdot \sin\omega t \right]$$

$$+ 2\mu\pi^4 V \sum_{n=1}^{\infty} n^4 \cdot (-1)^n \cdot \frac{e^{-n^2\pi^2 t}}{(n^4\pi^4 + \omega^2)} \tag{35}$$

7. Results

The velocities of the dusty gas, clean gas and the dust particle are calculated for the time periods $T = 12.57$ and $T = 2$ for different values of l and σ from eqns. (21), (26) and (25) respectively. Results are obtained for $l = 0.1$, 0.5 and 0.9 for $\sigma = 0.8$ and for $\sigma = 0.4$ and 1.5 when $l = 0.2$ (It is interesting to note that for $l = 0.1$ and $\sigma < 0.28$, the expression $(1 + l + \sigma n^2\pi^2)^2 - 4\sigma n^2\pi^2$ of Eqns. (21) and (25) will be negative so that one gets imaginary velocities, which is impossible. Therefore, one has to start with value of $\sigma > 0.28$, in our case we have started from

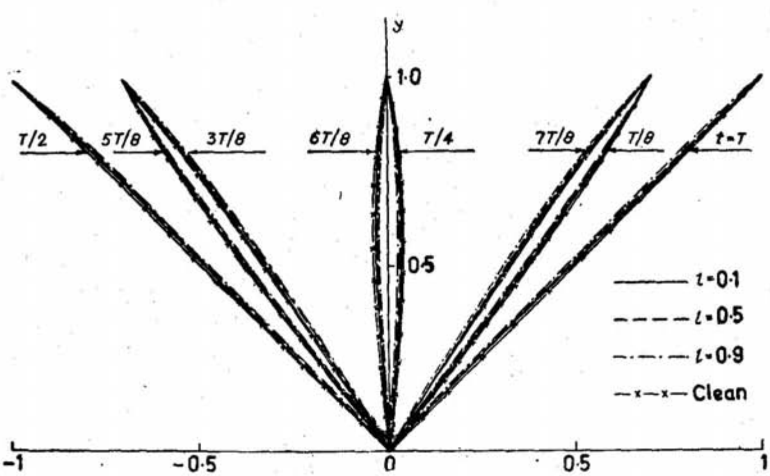

Figure 1. Velocity distributions for dusty and clean gas varying l from 0.1 to 0.9 for $\sigma = 0.8$ at different times within one complete oscillation for $T = 12.57$.

$\sigma = 0.4$). The velocity profiles for the dusty gas, clean gas and the particle are drawn at times $\frac{T}{8}$, $\frac{2T}{8}$, $\frac{3T}{8}$, $\frac{4T}{8}$, $\frac{5T}{8}$, $\frac{6T}{8}$, $\frac{7T}{8}$ and T where T is the time period for one complete oscillation of the upper plate.

In Figures 1 and 2, the velocity profiles for the dusty and clean gas are drawn for $l = 0.1$, 0.5, 0.9 when $\sigma = 0.8$ and for $\sigma = 0.4$ and 1.5 when $l = 0.2$ respectively for $T = 12.57$. It is found from Figure 1 that as the mass concentration l increases, the velocity of the dusty gas decreases and the velocity of the clean gas is greater than that of the dusty gas. This means that due to the presence of the dust particles, the velocity of the gas is decreased.

In Figure 3, one finds similar results for the time period $T = 2$, as in Figure 1 for $T = 12.57$, only the effect of disturbance is more prominent here. Now, the velecity

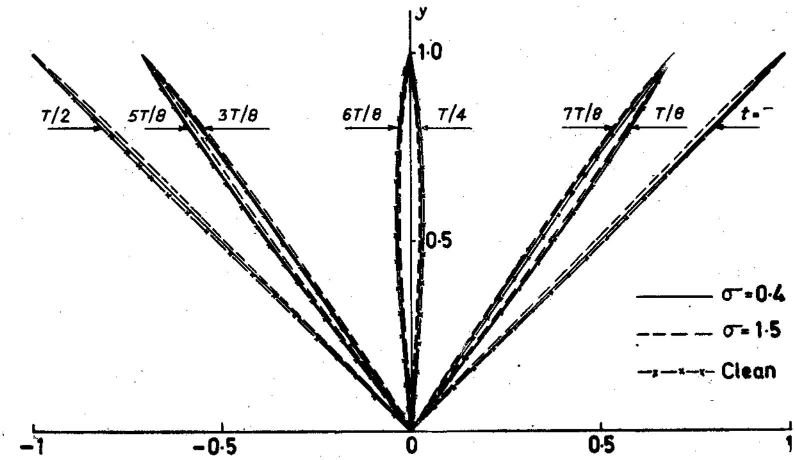

Figure 2. Velocity distributions for dusty and clean gas varying σ from 0.4 to 1.5 for $l = 0.2$, when $T = 12.57$.

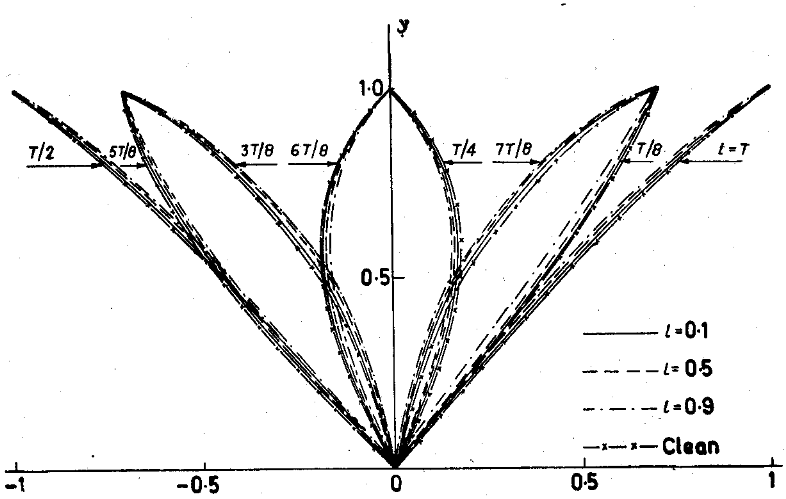

Figure 3. Velocity distributions for dusty and clean gas varying l from 0.1 to 0.9 for $\sigma = 0.8$, when $T = 2$.

profiles for the dusty and clean gas are shown in Figure 4 for $T = 2$ varying σ from 0.4 to 1.5 when $l = 0.2$. It is observed that as the time relaxation parameter σ increases, the velocity of the dusty gas increases which is again similar in nature with Figure 2.

To study the velocity profiles for the dust particle, similar set of values (varying l and σ as previously) have been obtained for the both $T = 2$ and $T = 12.57$ and are plotted in Figures 5–6 and in Figures 7–8 respectively. In Figure 5, one finds that as the mass concentration l increases, the velocity of the dust particle gradually decreases while Figure 6 shows that dust particle velocity decreases with increase in time relaxation parameter σ. Figures 7 and 8 represent similar results for $T = 12.57$. Here also the effect of oscillation is more prominent in case of $T = 2$, which is obvious. Thus

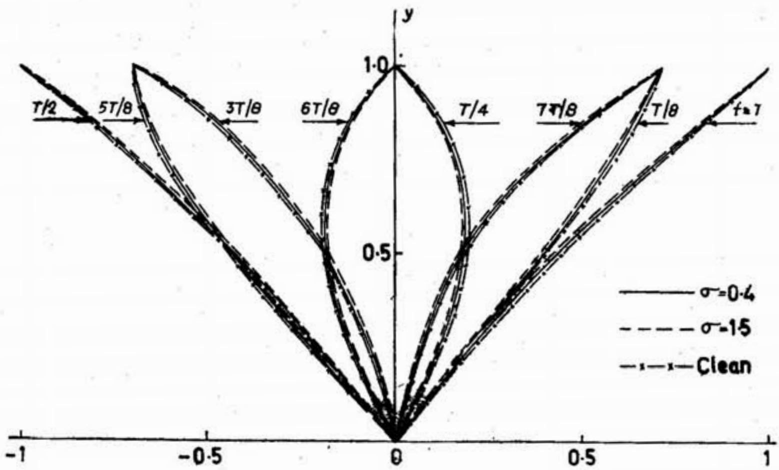

Figure 4. Velocity distributions for dusty and clean gas varying σ from 0.4 to 1.5 for $l = 0.2$, when $T = 2$.

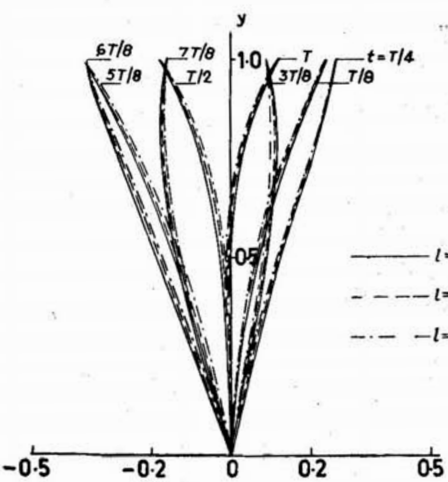

Figure 5. Velocity distributions for dust particles varying l from 0.1 to 0.9 for $\sigma = 0.8$, when $T = 2$.

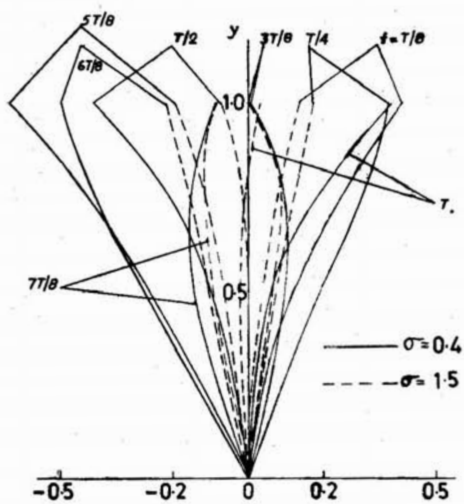

Figure 6. Velocity distributions for dust particles varying σ from 0.4 to 1.5 for $l = 0.2$, when $T = 2$.

from Figures 2 and 8 and 4 and 6, it follows that the difference between the velocities of the dusty gas and dust particle at a particular point (at a particular time) gradually increases as the relaxation time parameter increases, which excellently agrees with Saffman's theory[1].

Lastly, the drags, for dusty and clean gas, on the lower plate due to the oscillation of the upper plate have been calculated from Eqns. (34) and (35) respectively for $T = 12.57$ and $T = 2$. For both time periods, results have been obtained varying l and σ, as done for gas and particle velocities previously, and are given in Tables 1 and 2. From Table 2, drags for clean gas and dusty gas (for $l = 0.2$ and $\sigma = 0.4$ and for $l = 0.9$ and $\sigma = 0.8$) are plotted in Figure 9. Tables show that drags for dusty gas for other values of l and σ, are quite similar in nature. One also finds (mainly from tables)

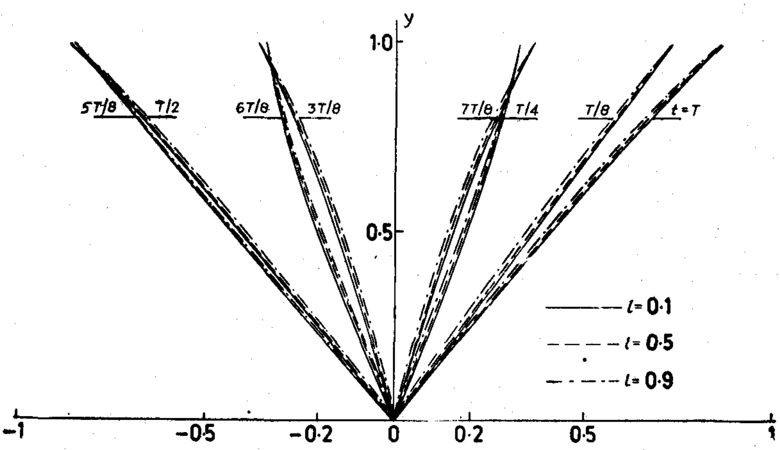

Figure 7. Velocity distributions for dust particle varying l from 0.1 to 0.9 for $\sigma = 0.8$, when $T = 12.57$.

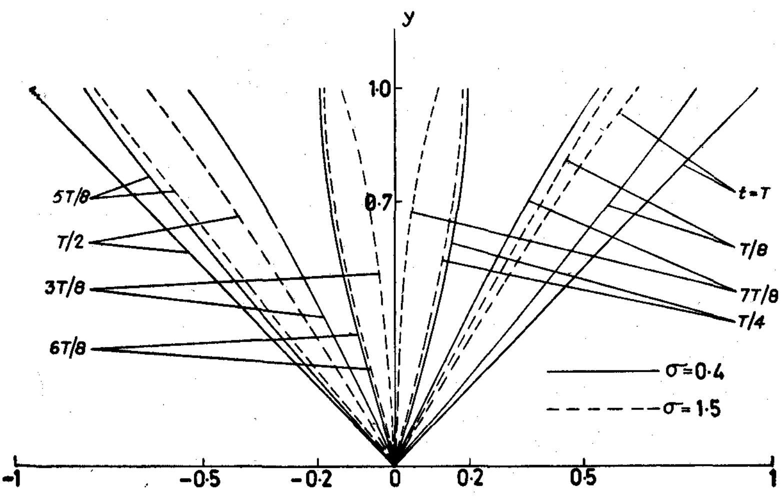

Figure 8. Velocity distribution for dust particle varying σ from 0.4 to 1.5 for $l = 0.2$, when $T = 12.57$.

Table 1. Dusty and clean drags for $T = 12.57$

t	Dusty drags: $\sigma = 0.8$, $l = 0.1$	$l = 0.2$	$l = 0.5$	$l = 0.9$	Dusty drags: $l = 0.2$, $\sigma = 0.4$	$\sigma = 0.6$	$\sigma = 1$	$\sigma = 1.2$	$\sigma = 1.5$	Clean drags
0.31	0.8938	0.8818	0.8470	0.8039	0.8652	0.8756	0.8858	0.8887	0.8917	0.9062
1.57	0.7613	0.7602	0.7561	0.7495	0.7663	0.7626	0.7588	0.7589	0.7573	0.7624
3.14	0.0893	0.0955	0.1135	0.1360	0.0984	0.0971	0.0939	0.0925	0.0907	0.0831
4.71	—0.6376	—0.6304	—0.6086	—0.5799	—0.6304	—0.6302	—0.6307	—0.6314	—0.6324	—0.6450
6.28	—0.9915	—0.9817	—0.9762	—0.9603	—0.9900	—0.9887	—0.9870	—0.9866	—0.9862	—0.9952
6.60	—0.9933	—0.9906	—0.9824	—0.9706	—0.9932	—0.9918	—0.9898	—0.9891	—0.9885	—0.9959
7.86	—0.7646	—0.7666	—0.7724	—0.7789	—0.7696	—0.7681	—0.7653	—0.7642	—0.7628	—0.7624
9.43	—0.0898	—0.0965	—0.1161	—0.1414	—0.0985	—0.0975	—0.0954	—0.0942	—0.0926	—0.0831
11.00	0.6376	0.6302	0.6081	0.5789	0.6304	0.6301	0.6304	0.6309	0.6317	0.6450
12.57	0.9915	0.9877	0.9762	0.9601	0.9900	0.9887	0.9869	0.9864	0.9860	0.9952

Table 2. Dusty and clean drags for $T = 2$

t	Dusty drags: $\sigma = 0.8$, $l = 0.1$	$l = 0.2$	$l = 0.5$	$l = 0.9$	Dusty drags: $l = 0.2$, $\sigma = 0.4$	$\sigma = 0.6$	$\sigma = 1$	$\sigma = 1.2$	$\sigma = 1.5$	Clean drags
0.05	—0.2167	—0.2142	—0.2067	—0.1971	—0.2070	—0.2120	—0.2153	—0.2161	—0.2168	—0.2193
0.25	0.7492	0.7386	0.7079	0.6694	0.7236	0.7330	0.7423	0.7449	0.7477	0.7600
0.50	0.4463	0.4400	0.4217	0.3992	0.4392	0.4388	0.4412	0.4424	0.4438	0.4529
0.75	—0.2498	—0.2436	—0.2260	—0.2043	—0.2258	—0.2376	—0.2471	—0.2492	—0.2512	—0.2561
1.00	—0.8113	—0.7962	—0.7529	—0.6998	—0.7741	—0.7875	—0.8059	—0.8059	—0.8101	—0.8268
1.15	—0.9317	—0.9156	—0.8695	—0.8126	—0.8989	—0.9083	—0.9209	—0.9248	—0.9290	—0.9481
1.25	—0.8990	—0.8844	—0.8423	—0.7902	—0.8738	—0.8791	—0.8886	—0.8919	—0.8957	—0.9140
1.50	—0.4607	—0.4556	—0.4405	—0.4213	—0.4640	—0.4573	—0.4555	—0.4559	—0.4569	—0.4660
1.75	0.2471	0.2394	0.2177	0.1917	0.2162	0.2314	0.2439	0.2468	0.2493	0.2551
2.00	0.8099	0.7936	0.7471	0.6904	0.7691	0.7838	0.8001	0.8046	0.8093	0.8267

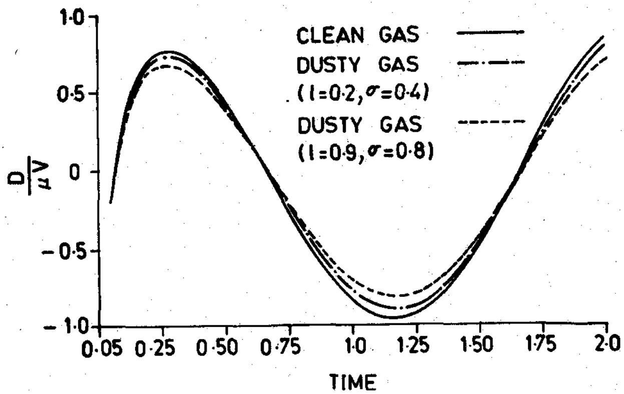

Figure 9. Drags on the lower plate for dusty and clean gas, when $T = 2$.

that as the value of l decreases for a fixed value of σ and with the increase of σ for a fixed l, the value of the drag for the dusty gas gradually approaches the value of the drag for the clean gas.

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