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Dusty Viscous Flow Through a Cylinder of Rectangular Cross-Section Under Time Dependent Pressure Gradient

E. RUKMANGADACHARI

Department of Mathematics, P. R. Govt. College
Kakinada-533001.

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Abstract. Unsteady laminar flow of a dusty, viscous, incompressible fluid through a cylindrical tube of rectangular cross-section is studied when the pressure gradient varies (i) exponentially and (ii) harmonically w.r.t. time. The velocity fields for the fluid and the dust particles have been obtained. Results in limiting cases are derived. Flux and drag have been calculated. Flow through a tube of square section and flow between parallel plates have been obtained as special cases.

1. Introduction

The study of the motion of dusty viscous fluids has recently attracted a number of workers since the publication of Saffman's investigations[1], which reveal the effect of the dust particles on the stability of the laminar flow of an incompressible fluid with uniform mass concentration of dust particles. Such situations arise, for example, in problems of fluidization; in sedimentation; in the use of dust in gas-cooling systems; in the movement of dustladen air; and in tidal waves.

Michael and Miller[2] and Mathur[3] *et al.* have studied the plane parallel flows, while Michael and Norey[4] Sambasiva Rao[5], Tewari and Bhattacharjee[6], Newal Kishore and Pandey[7] and Rukmangadachari[8,9] have all considered dusty fluid flows in circular tubes under various situations. Rukmangadachari[10,11], has recently investigated flow of dusty fluid through pipes of elliptic and triangular cross-sections.

In this paper, we have discussed the flow of a dusty viscous fluid through a cylindrical tube of rectangular cross-section under the influence of a pressure gradient (i) varying harmonically with time and (ii) varying exponentially with time. Expressions for the velocities of the fluid and dust particles have been given in the form of infinite series and results in the extreme cases deduced. Physical quantities of interest viz., flux and drag have been evaluated. Results for the particular case of flow between parallel plates have been deduced and those relating to a tube of square section are obtainable by putting $b = a$.

2. Formulation and Solution of the Problem

Taking the axis of cylinder as z-axis and assuming the number density N to be constant, Saffman's equations[1] for the present problem reduce to

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w + \frac{KN}{\rho}(w^* - w);\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial y^2}{\partial y^2} \tag{1}$$

$$m\frac{\partial w^*}{\partial t} = K(w - w^*). \tag{2}$$

where w and w^* are the components of velocity in z-direction; and the other symbols have their usual meaning. Eliminating w^* from Eqn. (1) with the help of Eqn. (2), one gets

$$\tau\frac{\partial^2 w}{\partial t^2} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(\tau\frac{\partial p}{\partial t} + p\right) + [\nu\tau\nabla^2 - (1+f)]\frac{\partial w}{\partial t} + \nu\nabla^2 w \tag{3}$$

where $\tau(=m/K)$ and $f(=mN/\rho)$ are respectively the relaxation time and mass concentration of dust.

Periodic pressure gradient

Case (i) Assuming the pressure gradient to be periodic with period $2\pi/\sigma$, and substituting†

$$\left(-\frac{1}{\rho}\frac{\partial p}{\partial z},\ w,\ w^*\right) = [\alpha,\ \phi(x, y),\ \phi^*(x, y)]\ e^{i\sigma t}, \tag{4}$$

(where α is a real constant), in Eqns. (1) and (2), we obtain

$$\phi^* = \frac{1}{1 + i\sigma\tau}\phi \tag{5}$$

and

$$\nabla^2\phi - \lambda^2\phi + \alpha/\nu = 0 \tag{6}$$

where

$$\lambda^2 = \frac{i\sigma}{\nu}\left(\frac{1 + f + i\sigma\tau}{1 + i\sigma\tau}\right) \tag{7}$$

Now, putting

$$\phi = \frac{\alpha}{\nu\lambda^2} - \psi, \tag{8}$$

we have

$$(\nabla^2 - \lambda^2)\psi = 0 \tag{9}$$

Taking the rectangular coordinates so that the cross-section of the cylinder is defined by $x = \pm a$, $y = \pm b$, the boundary conditions are

$$\psi = \alpha/\nu\lambda^2, \text{ when } x = \pm a,\ y = \pm b \tag{10}$$

Solution of Eqn. (9) may be assumed in the form

†The convention that real parts are to be understood whenever complex expressions are quoted for physical quantities, is adopted.

$$\psi = \sum A_m \cos \beta_m x \cosh \gamma_m y + \frac{\alpha}{\nu\lambda^2}\left(\frac{\cosh \lambda x}{\cosh \lambda a}\right) \tag{11}$$

where

$$\gamma_m^2 = \lambda^2 + \beta_m^2 \tag{12}$$

The first of the boundary conditions (10) will be satisfied if

$$\beta_m = \frac{2m+1}{2a}\pi \tag{13}$$

and the second yields

$$\sum A_m \cos \beta_m x \cosh \gamma_m b = -\frac{\alpha}{\nu\lambda^2}\left(\frac{\cosh \lambda x}{\cosh \lambda a} - 1\right)$$

Multiplying this by $\cos \beta_m x$, both sides, and integrating w.r.t. 'x' between '$-a$' and 'a', we get

$$A_m = (-1)^m \frac{2\alpha \operatorname{sech} \gamma_m b}{a\nu\beta_m(\lambda^2 + \beta_m^2)} \tag{14}$$

Consequently, the velocity of the fluid and that of the dust particles are respectively

$$w = e^{i\sigma t}\left[\frac{\alpha}{\nu\lambda^2}\left(1 - \frac{\cosh \lambda x}{\cosh \lambda a}\right) + \frac{2\alpha}{a\nu}\sum_{m=0}^{\infty}{}' \frac{(-1)^{m+1}}{\beta_m(\lambda^2 + \beta_m^2)}\right.$$
$$\left.\times \frac{\cosh \gamma_m y}{\cosh \gamma_m b}\cos \beta_m x\right] \tag{15}$$

$$w^* = w\left(\frac{e^{i\sigma t}}{1 + i\sigma\tau}\right) \tag{16}$$

where Σ' indicates that the first term of the sum is to be multiplied by $\frac{1}{2}$.

Case (ii) (a) Increasing exponential pressure gradient

Taking $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}, w, w^*\right) = [\alpha, \phi(x, y), \phi^*(x, y)]\, e^{\sigma t}$,

where α is a real constant and $\sigma > 0$, and proceeding as in *Case (i)*, we obtain the velocity of fluid and that of dust particles respectively, as

$$w = e^{\sigma t}\left[\frac{\alpha}{\nu\lambda^2}\left(1 - \frac{\cosh \lambda x}{\cosh \lambda a}\right) + \frac{2\alpha}{a\nu}\sum_{m=0}{}' \frac{(-1)^{m+1}\cos \beta_m x \cosh \gamma_m y}{\beta_m(\lambda^2 + \beta_m^2)\cosh \gamma_m b}\right] \tag{17}$$

$$w^* = w\left(\frac{e^{\sigma t}}{1 + \sigma\tau}\right),\ \lambda^2 = \frac{\sigma}{\nu}\left(\frac{1 + f + \sigma\tau}{1 + \sigma\tau}\right) \tag{18}$$

(b) Decreasing exponential pressure gradient

Now, we take

$$\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}, w, w^*\right) = [\alpha, \phi(x, y), \phi^*(x, y)]\, e^{-\sigma t}$$

($\sigma > 0$) and proceed as above to obtain the expressions for the velocity of fluid and dust particles respectively, as

$$w = \frac{\alpha e^{-\sigma t}}{a\nu\lambda^2}\left[\sum{}' \left\{\frac{2(-1)^m}{\beta_m} - \frac{1}{\cos\lambda a}\left(\frac{\sin(\lambda-\beta_m)a}{\lambda-\beta_m} + \frac{\sin(\lambda+\beta_m)a}{\lambda+\beta_m}\right)\right\}\right.$$

$$\left.\times \frac{\cosh\gamma_m y}{\cosh\gamma_m b}\cos\beta_m x - \left(1 - \frac{\cos\lambda x}{\cos\lambda a}\right)\right], \lambda^2 \neq \beta_m^2 \tag{19}$$

$$w^* = w\left(\frac{1}{1-\sigma\tau}\right), \; \lambda^2 = \frac{\sigma}{\nu}\left(\frac{1+f-\sigma\tau}{1-\sigma\tau}\right), \; \gamma_m^2 = \beta_m^2 - \lambda^2 \tag{20}$$

3. Flux and Drag

The volume of fluid discharged per unit time over any cross-section of the cylinder is given by

$$Q = \int_S w\,dS = 4\int_0^a\int_0^b w\,dx\,dy \tag{21}$$

and the drag per unit length on the surface of the walls of the cylinder is given by

$$D = \oint_C \tau_{xz}\,ds = 4\mu\left[\int_0^a \left(\frac{\partial w}{\partial y}\right)_{y=b} dx + \int_0^b \left(\frac{\partial w}{\partial x}\right)_{x=a} dy\right] \tag{22}$$

where τ_{xz} and τ_{yz} are the shearing stress components in z-direction on the surfaces $x = a$ and $y = b$ respectively.

(i) *Periodic pressure gradient*

Using the expression (15) for the velocity of fluid under periodic pressure gradient, the flux 'Q' and the drag 'D' are obtained as

$$Q = 4\,e^{i\sigma t}\left[\frac{\alpha b}{\nu\lambda^3}(\lambda a - \tanh\lambda a) - \frac{2\alpha}{a\nu}\sum_{m=0}^{\infty}{}' \frac{\tanh\gamma_m b}{\beta_m^2\;\gamma_m^3}\right] \tag{23}$$

$$D = -4\mu\,e^{i\sigma t}\left[\frac{\alpha b}{\nu\lambda}\tanh\lambda a + \frac{2\alpha\lambda^2}{a\nu}\sum_{=0}^{\infty}{}' \frac{\tanh\gamma_m b}{\beta_m^2\;\gamma_m^3}\right] \tag{24}$$

when $|\lambda|$ *is small*: For small $|\lambda|$, which implies small $|\gamma_m|$, we obtain the flux and skin friction drag as

$$Q_S = -\frac{8\alpha b}{a\nu} e^{i\sigma t} \sum_{m=0}^{\infty}{}' \frac{1}{\beta_m^2 \ \gamma_m^2} \tag{25}$$

$$D_S = -e^{i\sigma t}\left[\frac{\alpha ab}{\nu} + \frac{2\alpha b\lambda^2}{a\nu}\sum_{m=0}^{\infty}{}' \frac{1}{\beta_m^2 \ \gamma_m^2}\right] \tag{26}$$

where we have taken the dimensions of the cross-section to be small ; and

$\tanh \lambda a \simeq \lambda a$ and $\tanh \gamma_m b \simeq \gamma_m b$

Since, for each m,

$$\frac{1}{\beta_m^2 \ \gamma_m^2} = \frac{1}{\beta_m^4}\left(1 + \frac{\lambda^2}{\beta_m^2}\right)^{-1} \simeq \frac{1}{\beta_m^4} - \frac{\lambda^2}{\beta_m^6} \tag{27}$$

the Eqns. (25) and (26) can be put in the following forms, on taking the real parts and retaining terms up to $0(\,|\,\lambda\,|^2)$ only.

$$Q_s = Q_{s,c} + Q_{s,d} = [SR\cos(\sigma t - T)]_c + [-SM\sigma\beta^{-1}\cos(\sigma t + \gamma)]_d \tag{28}$$

$$D_s = D_{s,c} + D_{s,d} = [\rho SR^*\cos(\sigma t + T^*)]_c + [\rho S(R^*\sin T^*) \times f\beta^{-1}\cos(\sigma t + \gamma)]_d \tag{29}$$

where $[\ ]_c$ and $[\ ]_d$ denote respectively the clean viscous flow part and dusty viscous flow part of flux or drag, as the case may be; and where

$$\left.\begin{aligned} L &= R\cos T & \sigma\tau &= \beta\cos\gamma \\ N\sigma/\nu &= R\sin T & 1 &= \beta\sin\gamma \\ R &= (L^2 + M^2\sigma^2/\nu^2)^{1/2} & \beta &= (1 + \sigma^2\tau^2)^{1/2} \\ T &= \tan^{-1}(M\sigma/L\nu) & \gamma &= \cot^{-1}\sigma\tau \end{aligned}\right\} \tag{30}$$

$$\tfrac{1}{2}\nu = R^*\cos T^* \quad a^2 L = \sum_{m=0}^{\infty}{}' \frac{1}{\beta_m^4} = \left(\frac{2a}{\pi}\right)^4\left[-\tfrac{1}{2} + \frac{\pi^4}{96}\right]$$

$$L\sigma/b = R^*\sin T^* \quad a^2 M = \sum_{m=0}^{\infty}{}' \frac{1}{\beta_m^6} = \left(\frac{2a}{\pi}\right)^6\left[-\tfrac{1}{2} + \frac{(2^6-1)\,\pi^6}{84(6)!}\right]$$

$$R^* = (\nu^2/4 + L^2\sigma^2/b^2)^{1/2}$$

$$T^* = \tan^{-1}(2L\sigma/b\nu); \ S = -\frac{8\alpha ab}{\nu}.$$

Here we have used the formula (Gradshteyn & Ryzhik[12])

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^{2n}} = \frac{(2^{2n}-1)\,\pi^{2n}}{2(2n)!} \mid B_{2n} \mid \tag{31}$$

where B_n are the Bernoulli's constants.

The velocity field for the dusty fluid for small values of $\mid \lambda \mid$, deduced from (15), is

$$w_s = e^{i\sigma t}\Big[\frac{\alpha}{2\nu}(a^2 - x^2) + \frac{2\alpha}{a\nu} \sum_{m=0}^{\infty}{}' \frac{(-1)^{m+1}\cos\beta_m x}{\beta_m \gamma_m^2} \times \{1 - \tfrac{1}{2}\gamma_m (b^2 - y^2)\}\Big] \tag{32}$$

where terms higher than $0(\mid \lambda \mid^2)$ have been neglected.

When $\mid \lambda \mid$ *is large* : In case $\mid \lambda \mid$ is large, we have

$$\frac{1}{\beta_m^2 \gamma_m^3} \simeq \frac{1}{\lambda^3 \beta_m^2}\left(1 - \frac{\beta_m^2}{\lambda^2 + \beta_m^2}\right)^{3/2} \simeq \frac{1}{\lambda^3 \beta_m^2} \tag{33}$$

Neglecting terms of $0(\mid \lambda \mid^{-3})$ and higher, we obtain the expressions for the flux and drag as

$$Q_L = (S\sigma/2\nu)\sin \sigma t - (S\sigma/2\nu) f \beta^{-1} \cos(\sigma t + \gamma) \tag{34}$$

$$D_L = -4\rho\alpha b \, Re\,(e^{i\sigma t}/\lambda) \quad (Re = \text{Real part of}) \tag{35}$$

(ii) (b) *Decreasing exponential pressure gradient*

After a little simplification, we can put Eqn. (19) as

$$w = \frac{\alpha e^{-\sigma t}}{a\nu\lambda^2}\left[\sum_{m=0}^{\infty}{}' \frac{2(-1)^{m+1}}{\beta_m \gamma_m^2} \lambda^2 \frac{\cosh \gamma_m y}{\cosh \gamma_m b} \cos \beta_m x + \frac{\cosh \lambda x}{\cosh \lambda a} - 1\right] \tag{36}$$

Using this expression, the flux and drag per unit length of the cylinder are got as

$$Q' = -\frac{8\alpha e^{-\sigma t}}{a\nu}\left[\tfrac{1}{2}\lambda b(\lambda a - \tan \lambda a) + \sum_{m=0}^{\infty}{}' \frac{\tanh \gamma_m b}{\beta_m^2 \gamma_m^3}\right] \tag{37}$$

$$D' = \frac{\alpha e^{-\sigma t}}{a\nu\lambda}\left[2\lambda^3 \sum_{m=0}^{\infty}{}' \frac{\tanh \gamma_m b}{\beta_m^2 \gamma_m^3} - \tan \lambda a\right] \tag{38}$$

Similarly, we can get the corresponding expressions in the case of increasing exponential pressure gradient.

Further, if we put $b = a$ in the expressions got above, the results for the flow through a cylinder of square cross-section are obtained.

4. Flow Between Parallel Plates

Letting $b \to \infty$, we obtain from Eqns. (15) and (16), the velocities of the fluid and dust particles respectively for flow between parallel plates $x = \pm a$ under the influence of periodic pressure gradient, as

$$w_p = \frac{\alpha e^{i\sigma t}}{\nu\lambda^2}\left(1 - \frac{\cosh \lambda x}{\cosh \lambda a}\right) \tag{39}$$

$$w_p^* = w_p\left(\frac{1}{1 + i\sigma\tau}\right) \tag{40}$$

For small values of $|\lambda|$ when the distance between the plates is small, we have up to $0(|\lambda|^4)$

$$w_{p,s} = \frac{\alpha e^{i\sigma t}}{2\nu}(a^2 - x^2) - \frac{\lambda^2\alpha e^{i\sigma t}}{24\nu}(x^2 - 6a^2x^2 + 5a^4) \tag{41}$$

The first part gives the velocity field for the clean fluid, up to zeroth order of $|\lambda|$, whose profile, as it is well-known has the parabolic form.

5. Deductions

1. By making $f \to 0$ in all the expressions, the dust particles are removed from the flow quantities and we regain the corresponding expressions for the clean fluid flow.
2. It is observed that the velocity of the dusty particles is always smaller than that of the dusty fluid, while the latter itself is smaller than that of the clean fluid.
3. The velocity of dust particles vanishes along with the fluid velocity on the boundary. This shows that the dust particles adhere to the boundary.
4. From Eqns. (28) and (29), we note that the presence of dust particles in the fluid is to decrease the flux and increase the drag on the walls of the cylinder by amounts

$$Q_{s,d} = SM\sigma f\beta^{-1}\cos(\alpha t + \gamma) \tag{42}$$

$$D_{s,d} = \rho S(R^* \sin T^*) f\beta^{-1}\cos(\sigma t + \gamma) \tag{43}$$

respectively, which can be expected from the physical considerations, qualitatively.

Now, for small τ, we have

$$\beta^{-1} = (1 + \sigma^2\tau^2)^{-1/2} \simeq 1 - \tfrac{1}{2}\sigma^2\tau^2 \tag{44}$$

which shows that the additional drag due to the presence of dust decreases as τ increases up to a critical level, given by $\tau_{\text{crit}} = \sqrt{2}/\sigma$. This agrees with Saffman's observation that, when τ is small, which corresponds to the case of the dust being fine, the effective kinematic viscosity is reduced. Also, for exponential pressure gradient the corresponding result is $\tau'_{\text{crit}} = 1/\sigma$ so that the critical value of τ for periodic pressure gradient is $\sqrt{2}$ times greater than that for exponential pressure gradient.

Again, for large τ, we have

$$f\beta^{-1} = f(\sigma\tau)^{-1}\left(1 + \frac{1}{\sigma^2\tau^2}\right)^{-1/2} \simeq f(\sigma\tau)^{-1}\left(1 - \frac{1}{2\sigma^2\tau^2}\right)$$

$$= s\sigma^{-1} + 0\left((\sigma\tau)^{-3}\right) \tag{45}$$

where $s = f\sigma^{-1} = KN/\rho$. The additional drag $D_{s,d}$ varies with 's' in the case of coarse dust.

The convergence of the infinite series involved in the solutions can be easily established as in [ref. 11].

Long time after completing the solution of the above problem, the author has recently noticed that the problem of vibrating membrane with its edge vibrating in a prescribed manner is similar to the one considered in case (i) above. If we, therefore, write $\lambda^2 = -m^2$, Eqns. (9) and (10) become

$$\nabla^2\psi + m^2\psi = 0; \tag{46}$$

$$\psi = -\frac{\alpha}{\nu m^2}, \text{ when } x = \pm a,\ y = \pm b \tag{47}$$

and the solution may be written, directly following Seth[13], as

$$\psi = -\frac{\alpha}{\nu m^2}\left[\frac{\cos mx}{\cos ma} + \sum_{n=0}^{\infty} \frac{(-1)^n\, 2m^2}{a\lambda_n(m^2 - \lambda_n^2)} \frac{\cos\sqrt{m^2 - \lambda_n^2}\, y}{\cos\sqrt{m^2 - \lambda_n^2}\, b} \cos\lambda_n x\right] \tag{48}$$

where

$$\lambda_n = (2n + 1)\,\pi/2a$$

which agrees with the result in case (i) of periodic pressure gradient above.

6. Numerical Results and Discussion

With a view to have an insight into the behaviour of the various parameters involved in the expressions for physical quantities, numerical work has been done and the results are presented in Tables 1 to 4 and Figs. 1 and 2.

In Table 1 the variations of the clean fluid flux $Q_s^{(c)}$ and the dusty fluid flux Q_s (Eqn. 28) and in Table 2 the variations of the clean fluid drag $D_s^{(c)}$ and the dusty fluid drag D_s (Eqn. 29) against σ are presented for different values of mass concentration of dust f. They show that flux increases with frequency parameter σ both for clean and dusty fluids. Also, flux decreases when dust is added to the clean fluid and the decrease is more as mass concentration of dust f increases. Further, we notice that drag decreases with σ for clean fluid as also for dusty fluid and that drag increases with f, though at high frequencies the increase is small.

Table 1. Values of clean fluid flux $Q_s^{(c)}$ and dusty fluid flux Q_s against σ for different mass concentrations of dust f.

	f \ σ =	6	8	10	12	14	16
$Q_s^{(c)}$	0	1.010551	1.237483	1.488993	1.738488	1.957119	2.115751
	0.2	0.979925	1.211472	1.469889	1.727729	1.955591	2.034712
	0.4	0.949299	1.185462	1.450785	1.716969	1.954063	1.953672
Q_s	0.6	0.918672	1.159452	1.431681	1.706210	1.952535	1.872636
	0.8	0.888054	1.133482	1.412576	1.695458	1.951006	1.791657
	1.0	0.857420	1.107431	1.361448	1.684691	1.949479	1.710555

Table 2. Values of clean fluid drag $D_s^{(c)}$ and dusty fluid drag D_s against σ for different mass concentrations of dust f.

	f \ σ =	6	8	10	12	14
$D_s^{(c)}$	0	3.205892	2.665550	2.018926	1.291853	0.513262
	0.2	3.212355	2.669657	2.021345	1.292988	0.513401
	0.4	3.218819	2.673764	2.023764	1.294124	0.513539
D_s	0.6	3.225282	2.677871	2.026183	1.295259	0.513677
	0.8	3.231746	2.671977	2.028602	1.296394	0.513815
	1.0	3.238209	2.686135	2.031021	1·297530	0.513953

Table 3. Values of the clean fluid drag $D_s^{(c)}$ and dusty fluid drag D_s against t for different mass concentrations of dust f.

	f \ t =	0 (sec)	50 (sec)	100 (sec)	150 (sec)	200 (sec)
$D_s^{(c)}$	0	4.0	3.987177	3.904201	3.752565	3.534854
	0.2	4.010145	3.997680	3.914877	3.763228	3.545314
	0.4	4.020289	4.008183	3.925554	3.773889	3.555774
D_s	0.6	4.030434	4.018686	3.936230	3.784551	3.566234
	0.8	4.040579	4.029189	3.946906	3.795213	3.576695
	1.0	4.050723	4.039692	3.957583	3.805876	3.587155

Table 4. Values of velocities of clean fluid $w^{(c)}_{p,s}$ and dusty fluid $w_{p,s}$ for the flow between parallel plates when the distance '$2a$' between them and the frequency of oscillation σ are small.

	$t = 100$ (sec)		150 (sec)		200 (sec)	
x	$w^{(c)}_{p,s}$	$w_{p,s}$	$w^{(c)}_{p,s}$	$w_{p,s}$	$w^{(c)}_{p,s}$	$w_{p,s}$
0	0.42788000	0.37519450	0.38008750	0.32754500	0.32571250	0.27410000
0.2	0.41118413	0.36101067	0.36550445	0.31546717	0.31344618	0.26434218
0.4	0.36088685	0.31804721	0.32144507	0.27872171	0.27639214	0.23446564
0.6	0.27635917	0.24506820	0.24697869	0.21577265	0.21328509	0.18266109
0.8	0.17035040	0.14001374	0.14055419	0.12406005	0.12209959	0.10591309

Table 3 shows the values of drag against t, from which it is noticed that drag decreases as time t increases. The rate of increase in drag with f is almost uniform for all t.

Table 4 gives the values of velocities of clean fluid $w^{(c)}_{p,s}$ and dusty fluid $w_{p,s}$ (Eqn. 41) for the flow between parallel plates. This clearly shows the reduction in velocity with the addition of dust, the dusty fluid becoming slower with increase in the value of f. The velocity for clean and dusty fluids reduces as 't' increases.

From Fig. 1, we observe that the flux curves for the clean and the dusty fluids become closer with the increase in σ. The clean and the dusty fluid curves are closer for small f and are far apart for large f when the frequency σ is small.

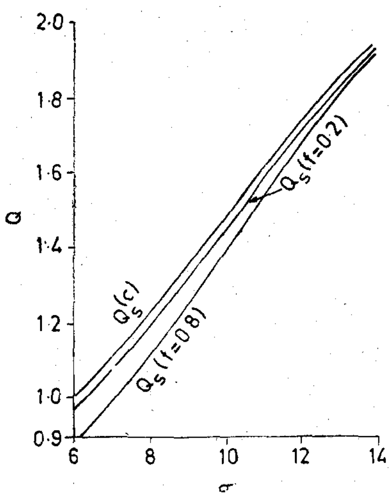

Figure 1. Graph shows flux Q against σ, where $Q_s^{(c)}$ = clean fluid flux and Q_s = dusty fluid flux.

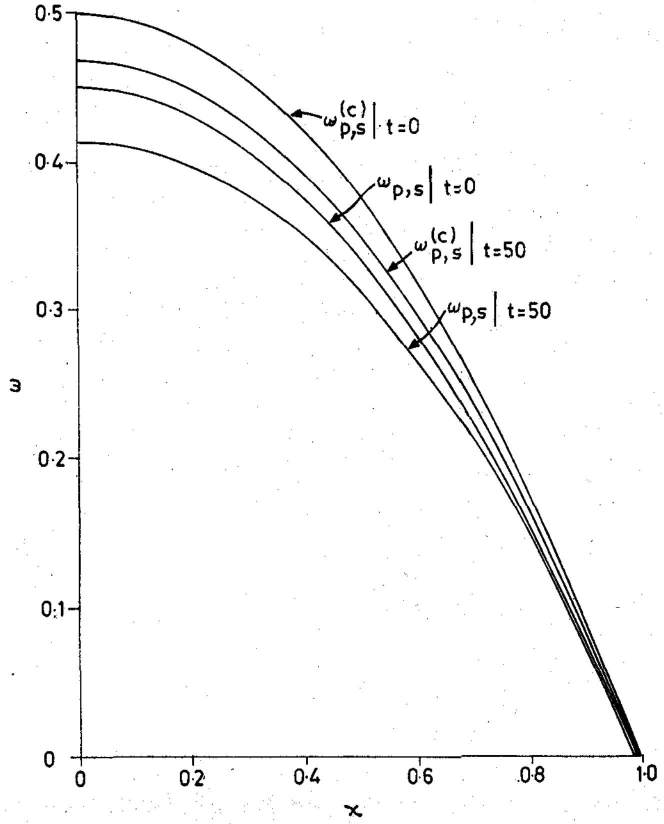

Figure 2. Velocities of clean and dusty fluids flowing between parallel plates.

Finally, Fig. 2 shows clearly that the addition of dust reduces the speed of the fluid flow. The parabolic curve of the classical nondusty fluid becomes blunt at the mid-point between the parallel plates with the addition of dust, which shows that the dust has a damping effect on the fluid flow.

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